Patterns of specialization and the geography of regions

Gilles Spielvogel*

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Preliminary and incomplete version

Abstract

This paper elaborates an economic geography model able to explain some stylized facts present in numerous developing countries. The model focuses on agricultural vs. manufacturing specialization of regions in presence of transport costs for both types of goods. Gains from interregional trade are assessed and the effects of labor mobility on regional inequalities are evaluated.

Keywords: economic geography, developing countries, interregional trade, regional inequalities.

1 Introduction

In many developing countries, and especially in Sub-Saharan Africa, internal transportation networks have stagnated or improved very slowly since the 1960’s. In some cases, they have even shrunked: for instance, the total road network of Burkina Faso was about 17,000 kilometers at the end of the 1960’s and less than 13,000 kilometers at the end of the 1990’s. The quality of roads in Africa have also deteriorated over time: in 1992, about 17 per cent of Sub-Saharan Africa’s primary roads were paved, but by 1998 the figure had fallen to 12 per cent.

Is is also well known that rural-urban migration has been a major engine of urban growth in African countries since the 1950’s (Williamson, 1989).1 As a consequence, many countries exhibit very unbalanced spatial structures: urban primacy is very high in virtually all African countries, as well as in many developing and emerging countries in other regions, and manufacturing production is usually extremely concentrated in big cities. For instance, according to the 1984 census of Ghana, the manufacturing share of employment in the Accra district was 19.1%, almost twice that of the rest of the country (10.2%) (Ghana Statistical Service, 1987). The case of neighboring Côte d’Ivoire is even more striking: in 1988, the share of the active population in food and general goods industries was 9.5% in Abidjan, while it was less than 1.9% in the rest of the country and about 5% in the other urban areas (Institut National de la Statistique, 1988).

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*Sciences Po Paris, IRD-DIAL and University of Lille 2. E-mail: spielvogel@dial.prd.fr.

1At least until the 1990’s: though urban growth remains high, it has significantly slowed down in recent years and, in several countries, rural-urban migration flows seem to be now lower than in the previous decades.
In line with the spatial concentration of industrial activity, persistent regional inequalities are also a common characteristic of many low and middle income countries: some examples are Northeast vs. Southeast of Brazil, North vs. South in Ghana and Côte d’Ivoire, etc. Finally, an obvious observation is that most developing countries have a significant share of their population involved in agricultural activities.

This combination of high internal transport costs, important regional inequalities, high share of agriculture in GDP and spatial concentration of non-agricultural activities imply that traditional economic geography models may not be adequate to explain the spatial distribution of activities in many developing countries. Indeed, most traditional economic geography models (Krugman, 1991, Puga, 1999) focus on manufacturing concentration and assume that agricultural labor is specific and tied to the land. Moreover, agricultural transport costs are generally assumed to be null, which simplifies the analysis in the Dixit-Stiglitz-Krugman framework. While some works have introduced transport costs for agricultural goods (Calmette et Le Pottier, 1995, Davis, 1998, Fujita et al., 1999, chap. 7, among others), rural-urban migration is usually not addressed.

This paper therefore aims at elaborating a simple model of economic geography able to take these stylized facts into account. The basic idea is to start from Ethier (1982)’s model of international trade. We then introduce transport costs for both the agricultural and the manufacturing goods and consider the effects of labor mobility between regions. The agglomeration force is provided by increasing returns to scale at the sectoral level in manufacturing production due to efficiency gains as workers specialize in a smaller range of intermediate tasks (Duranton et Puga, 2004).

In the next section, we present the model. We then turn to the analysis of the autarkic equilibrium in section 3 and of the different specialization patterns in section 4. Section 5 deals with interregional inequalities and gains from trade and section 6 with the role of labor mobility.

2 The model

We consider an economy made up of two regions, named $C$ and $I$. There exist three goods: the manufacturing good ($M$), which production benefits from increasing returns to scale, the agricultural good ($A$), produced with a constant returns to scale technology, and a numeraire good, produced with constant returns as well. Trade of both the agricultural and manufacturing goods between the two regions is costly, while the numeraire can be costlessly traded. When one unit of agricultural good is shipped from one region to the other, only $\frac{1}{\tau_A}$ unit arrives, and for one unit of the manufacturing good, one obtains $\frac{1}{\tau_M}$ unit in the other region (with $\tau_A$, $\tau_M > 1$).

The regions are endowed with two production factors, which can be thought of as two different types of labor: one is used in agriculture and manufacturing ($L$), while the other is specific to the numeraire sector ($N$). We assume that each worker supplies inelastically one unit of labor. Endowments of region $r$ ($r = C, I$) are denoted $L_r$ and $N_r$. In the remaining of the paper, we
will assume that the population $N$ is relatively small compared to the number of $L$ workers and we will therefore be mainly interested by what happens to type-$L$ workers.

### 2.1 The numeraire good

The numeraire is produced with a constant returns technology. Thus, the amount $N_r$ represents at the same time the input and the output of the sector in region $r$. Income in numeraire obtained by workers in this sector is therefore 1. Since the factor $N$ can only be used to produce numeraire, both regions necessarily produce this good. The numeraire being homogenous and transported costlessly, its price is 1 in both regions.

### 2.2 Agriculture

The agricultural sector produces a homogenous good using a constant returns technology. Output is $q_A = L_A$, where $L_A$ is agricultural employment. The sector being competitive, we assume marginal cost pricing:

$$P_A = w$$

where $P_A$ represents the price of the agricultural good in terms of the numeraire and $w$ is the current wage rate for $L$ workers.

### 2.3 Manufacturing

The manufacturing good is a CES aggregate of different tasks:

$$q_M = \left[ \int_0^n T(h)^\rho \, dh \right]^{1/\rho},$$

where $n$ is the number of available tasks.

These tasks are being produced directly by workers. When one worker devotes a share $l(h)$ of its time to producing task $h$, his output is

$$T(h) = l(h)^{1+\alpha},$$

where $\alpha > 0$ indicates the level of increasing returns due to specialization of the worker$^3$.

Workers in the manufacturing sector first choose which tasks they want to do, then fix the prices of these tasks. If two workers choose the same task during the first stage, they become Bertrand competitors during the second stage and therefore obtain no income from this task. However, if a worker is the only supplier of a given task, he obtains a positive income. As a consequence, an perfect sub-game equilibrium must be such that no task is produced by more than one worker. Moreover, we assume that $\eta = \frac{1}{\rho} - 1 - \alpha > 0$: advantages due to the complementarity of tasks are higher than those due to workers’ specialization. This implies

$^3$Since each worker supplies one unit of labor, we have $\int l(h) \, dh = 1$. 

that, at the equilibrium, each task will be performed. Therefore, there exists only one perfect
sub-game equilibrium where each task of \([0, n]\) is performed by exactly one worker. With \(n\) tasks
and \(L_M\) workers in the sector, this means that each worker must allocate a share \(L_M/n\) of its
labor unit to each of the \(n/L_M\) tasks he performs. Without loss of generality, we assume \(n = 1\).
The final good production can then be rewritten as:

\[
q_M = n^n L_M^{1+\alpha} = L_M^{1+\alpha}.
\]

We assume free-entry for firms assembling the various tasks. As a consequence, the sector’s
profit is null at the equilibrium, which implies that the price of the final good is equal to average
cost:

\[
P_M = \frac{w L_M}{q_M} = w L_M^{-\alpha}.
\]  

\[2\]  

2.4 Preferences

Agents of type \(L\) and \(N\) have the same preferences, represented by the utility function:

\[
U = c q_0^\nu q_A^\beta q_M^\mu, \text{ with } c = \nu^{-\nu} \beta^{-\beta} \mu^{-\mu} \text{ and } \nu + \beta + \mu = 1,
\]  

\[3\]  

where \(q_0, q_A\) and \(q_M\) are respectively the numeraire, agricultural good and manufacturing good
consumptions.

3 Autarkic equilibrium

Let us start with the determination of one region’s autarkic equilibrium (region \(C\), for instance).
Due to the form of the utility function, demand of each good in this region is equal to a certain
share of the global income divided by the price of this good. Equilibrium between supply and
demand for agricultural and manufacturing goods in region \(C\) therefore implies:

\[
L_M C^{1+\alpha} = \mu Y_C / P_M C \text{ and } L_A C^{\gamma} = \beta Y_C / P_A C
\]  

\[4\]  

where \(Y_C\) is the global income of region \(C\). The relative demand price in the region is therefore:

\[
\left(\frac{P_M C}{P_A C}\right)_D = \frac{\mu L_A C^{\gamma}}{\beta L_M C^{1+\alpha}}.
\]  

\[5\]  

Moreover, the relative supply price is:

\[
\left(\frac{P_M C}{P_A C}\right)_S = L_M C^{-\alpha}.
\]  

\[6\]  

General equilibrium requires that the supply price equals the demand price, leading to
\(L_M C / \mu = L_A C / \beta \gamma\). Assuming that interregional labor mobility is not possible at this stage,
full employment in region \(C\) implies \(L_C = L_A C + L_M C\), which permits to obtain the optimal
allocation of $L$ workers in autarky:

$$L_{AC} = \frac{\beta \gamma}{\beta \gamma + \mu} L_C \quad \text{and} \quad L_{MC} = \frac{\mu}{\beta \gamma + \mu} L_C.$$  \tag{7}

The equilibrium relative price can then be determined:

$$\frac{P_M}{P_A} = \gamma (\beta \gamma)^{-1} \mu^{-\alpha} (\beta \gamma + \mu)^{1+\alpha-\gamma} L_C^{\gamma-1-\alpha}.$$  \tag{8}

The equilibrium on the numeraire market $N_C = \nu Y_C$ permits to obtain the real prices of the two other goods, necessary to determine the equilibrium real wage and the welfare level of $L$ agents:

$$P_{MC} = \mu^{-\alpha} \frac{N_C}{\nu} \left( \frac{\beta + \mu}{L_C} \right)^{1+\alpha} \quad \text{and} \quad P_{AC} = w = \frac{N_C}{\nu} \frac{\beta + \mu}{L_C}.$$  \tag{9}

Both regions having identical technologies and preferences, and potentially different endowments, the welfare differential for type $L$ agents is:

$$\frac{U_{aut}}{U_{aut}} = \left( \frac{N_C}{N_I} \right)^{\nu} \left( \frac{L_I}{L_C} \right)^{\nu-\alpha \mu}.$$  \tag{10}

If $N_C = N_I$, type-$L$ agents have the same welfare level when $L_C = L_I$. If $N_C$ and $N_I$ are not equal and $\alpha \mu > \nu$, identical utility levels requires that the $L$-rich region (relatively to the other region) must be $N$-poor. On the contrary, if $\alpha \mu < \nu$, the $L$-rich region must be $N$-rich in order to have equal welfare. Note that the autarkic price ratio is, as in Ethier (1982),

$$\frac{P_{MC}}{P_{AC}} \frac{P_{MI}}{P_{AI}} = \left( \frac{L_I}{L_C} \right)^{\alpha} :$$

the relative price of manufacturing in terms of agricultural goods is higher in the smaller region. It this ratio is used as a comparative advantage indicator, it appears that the biggest region has a comparative advantage in production of the increasing returns good.

4 Specialization and trade

Let us now consider trade of goods between the two regions. There exist different possible allocation of production between the two regions of the economy: the agricultural good and the manufacturing good may be produced either in both regions, or in only one, though the agents’ preferences imply that all goods must be produced; a good that is not produced locally must therefore be imported. These different patterns of specialization naturally map into different possible allocations of type-$L$ labor between agriculture and manufacturing within each region. In order to determine under which circumstances theses different specialization patterns – and therefore intraregional labor allocation patterns – may or may not be feasible, it is particularly
useful to use allocation curves, developed by Ethier (1982).\footnote{Allocation curves are not frequently used in the trade literature. See however Francois et Nelson (2002) for a recent example.}

First, it is useful to list the different possible specialization patterns, assuming, without loss of generality, that region $C$ is always exporting the manufacturing good while region $I$ exports the agricultural good:

1. $C(M) - I(A)$: each region is fully specialized in its export sector.

2. $C(M) - I(A, M)$: region $C$ is fully specialized in manufacturing production but region $I$ produces this good as well.

3. $C(A, M) - I(A)$: region $C$ produces both goods and region $I$ is fully specialized in agriculture.

4. $C(A, M) - I(A, M)$: both regions produce both goods, but the export sector in each region is bigger than in autarky.

### 4.1 Allocation curves

We can now define region $C$’s allocation curve: it is the set of couples $(L_{MC}, L_{MI})$ such that region $C$ is in equilibrium on the interregional market, i.e. such that the relative supply price of region $C$ is equal to the relative demand price of the economy. It is not necessary to go back to the relative supply price since it is identical to the autarky supply price. However, the demand price is now unique for the whole two-region economy. In order to compute this price, let us consider the most general case, where both regions produce both goods. The equilibrium on the agricultural good market is:

\[
L_{AI} + \tau_A L_{AC} = \beta \left( \frac{Y_I}{P_{AI}} \right) + \beta \left( \frac{\tau_A Y_C}{P_{AC}} \right) = \beta \left( \frac{Y_I + Y_C}{P_{AI}} \right)
\]  

(12)

where $Y_C$ and $Y_I$ are global incomes of the two regions. The LHS of this equality is the sum of agricultural supplies of the two regions, evaluated in region $I$ (region $C$ produces $L_{AC}$, but obtaining that amount in $C$ is also possible if one has $\tau_A$ times this quantity in region $I$). The RHS represents the global demand for the agricultural good: on the first line, region $C$’s demand is evaluated in region $I$, just as for supply. Moreover, if region $C$ manages to produce the agricultural good while region $I$ is exporting this good, we must necessarily have $P_{AC} = \tau_A P_{AI}$: the local price of $A$ in region $C$ must be equal to the CIF price of the good exported from region $I$. Indeed, if $P_{AC}$ is smaller than $\tau_A P_{AI}$, exports from $I$ cannot compete, while if $P_{AC}$ is higher, local output will not be purchased. Taking this condition into account, one easily gets the second line of equation (12).
Similarly, the equilibrium on the manufacturing good market is:

\[ L_{MC}^{1+\alpha} + \tau_M L_{MI}^{1+\alpha} = \mu \left( \frac{Y_C}{P_{MC}} \right) + \mu \left( \frac{\tau_M Y_I}{P_{MI}} \right) \]

\[ = \mu \left( \frac{Y_C + Y_I}{P_{MC}} \right) \]

where global supply and demand are evaluated in region C. Since region I is supposed to produce M while it is exported by C, we must have \( P_{MI} = \tau_M P_{MC} \), for the same reason as above.

Eliminating global income \( Y_C + Y_I \) between the two equations (12) and (13) and replacing \( P_AI \) by \( P_AC/\tau_A \), one gets the relative demand price, evaluated in region C:

\[ \left( \frac{P_{MC}}{P_{AC}} \right)_D = \frac{\mu}{\beta} \frac{L_{AI}/\tau_A + L_{AC}}{L_{MC}^{1+\alpha} + \tau_M L_{MI}^{1+\alpha}}. \]

(14)

It is important to note that the relative demand price is unique but, due to the presence of transport costs, its expression is different in the two regions. A proportional relation exists between the demand price expressed in C and the demand price expressed in I:

\[ \left( \frac{P_{MC}}{P_{AC}} \right)_D = \frac{1}{\tau_A \tau_M} \left( \frac{P_{MI}}{P_{AI}} \right)_D. \]

Logically, in the trade equilibrium, when region C exports the manufacturing good and region I exports the agricultural good, the local relative price of manufacturing is lower in region C.

Region C’s allocation curve is then obtained by equalizing the relative supply price of region C (equation (6)) to the relative demand price expressed in C (equation (14)) and replacing \( L_{AC} \) by \( L_C - L_{MC} \) and \( L_{AI} \) by \( L_I - L_{MI} \):

\[ \Gamma_C(L_{MC}, L_{MI}) = L_{MC}^{-\alpha} - \frac{\mu}{\beta} \frac{(L_I - L_{MI})/\tau_A + (L_C - L_{MC})}{L_{MC}^{1+\alpha} + \tau_M L_{MI}^{1+\alpha}} = 0. \]

(15)

When transport costs are null (\( \tau_A = \tau_M = 1 \)), one gets the “classical” allocation curve exposed in Ethier (1982).

The allocation curve of region I is defined similarly as region C’s (the set of couples \( (L_{MC}, L_{MI}) \) such that region I is in equilibrium on the interregional market) and is obtained by equalizing the relative supply price of region I to the relative demand price expressed in I. However, it is useful to use its expression in C, in order to make it directly comparable to region C’s curve:

\[ \Gamma_I(L_{MI}, L_{MC}) = \frac{L_{MI}^{-\alpha}}{\tau_A \tau_M} - \frac{\mu}{\beta} \frac{(L_I - L_{MI})/\tau_A + (L_C - L_{MC})}{L_{MC}^{1+\alpha} + \tau_M L_{MI}^{1+\alpha}} = 0. \]

(16)

The relative demand price being unique, equations (15) and (16) imply the equalization of both supply prices to this demand price. This condition is met when both curves intersect in the plan \( (L_{MC}, L_{MI}) \), thus defining a trade equilibrium where both regions produce both goods.
and where region C exports the manufacturing good.\(^5\)

However, as soon as one or the other region is fully specialized, a trade equilibrium where region C exports the manufacturing good is also possible when supply prices are not equal to the demand price. For instance, if region C is fully specialized in M, the supply price of region C will be smaller than the demand price and region I’s supply price. Similarly, if region I is fully specialized in good A, the supply price of this region is higher than the demand price and region C’s supply price. Table 1 sums up the relationships between supply and demand prices in the different possible trade cases presented above.

<table>
<thead>
<tr>
<th>Case</th>
<th>(\frac{P_{MC}}{P_{AC}}) subscripts</th>
<th>(\frac{P_{MC}}{P_{AC}}) subscripts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: (C(M) - I(A))</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
</tr>
<tr>
<td>Case 2: (C(M) - I(A, M))</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
</tr>
<tr>
<td>Case 3: (C(A, M) - I(A))</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
</tr>
<tr>
<td>Case 4: (C(A, M) - I(A, M))</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
<td>(\frac{P_{MC}}{P_{AC}}) subscripts</td>
</tr>
</tbody>
</table>

Figure 1 represents the allocation curve of region C. The curve \(OBDFX\) is a graph of \(\Gamma_{C}\) (equation (15)). The supply price is higher than the demand price above the curve and lower below. Moreover, the total type-L labor endowment being represented by \(E(L_C, L_I)\), the parts of \(OBDFX\) lying outside the rectangle \(OL_IEL_C\) are irrelevant. On the contrary, the segments \([LI]\) and \([FLC]\) are compatible with equilibrium when there is full specialization (segment \([LI]\) when region C is fully specialized in A, and segment \([FLC]\) when it is fully specialized in M). The full allocation curve of region C is therefore made up of \(LIOB\) and \(DFLC\).\(^6\)

Figure 2 represents both allocation curves (\(LIÖZX\) for region C and \(L_CÖZL_I\) for I). The point A represents the autarkic equilibrium\(^7\) and the interregional equilibria are given by intersections of the two curves: in this example, the points Z and X are trade equilibria such that region C exports the manufacturing good.

### 4.2 Existence and stability conditions of the different equilibria

The different types of equilibrium presented in table 1 can all be represented in figures similar to figure 2. Case 4 \((C(A, M) - I(A, M))\) is located at any intersection of the two allocation curves

\(^5\)It is important to note that, contrary to the case where there are no transport costs (Ethier, 1982), the allocation curves defined here are only valid if region C exports the good M. In order to obtain the other case, one just needs to reverse the regions’ names.

\(^6\)When dealing with the specific case where region C exports the good M, the segment \([LI]\) is of course incompatible with equilibrium.

\(^7\)Its coordinates are \((\frac{\mu}{\mu+\nu}L_C, \frac{\mu}{\mu+\nu}L_I)\).
being strictly inside the rectangle $OL_EL_C$ (i.e. such that both regions produce both goods). The coordinates of these points being such that $L_{MI} = (\tau_A \tau_M)^{-1/\alpha} L_{MC}$, it is obvious that there is at most one equilibrium of this kind. This corresponds to point $Z$ in Figure 2. As we will see below, this equilibrium is unstable and inefficient and its existence conditions are therefore uninteresting. However, it is useful to know its position in the plan $(L_{MC}, L_{MI})$; its coordinates are obtained after solving $\Gamma_C = \Gamma_I$:

$$Z \left( \frac{(\tau_A \tau_M)^{\frac{1+\alpha}{\alpha}}}{\tau_M + (\tau_A \tau_M)^{\frac{1+\alpha}{\alpha}}} \frac{\mu}{\beta + \mu} \left( L_C + \frac{L_I}{\tau_A} \right), \frac{\tau_A \tau_M^{\frac{1+\alpha}{\alpha}}}{\tau_M + (\tau_A \tau_M)^{\frac{1+\alpha}{\alpha}}} \frac{(L_C + \frac{L_I}{\tau_A})}{\beta + \mu} \right)$$ (17)

Case 3 ($C(A, M) - I(A)$) implies that region $I$ is fully specialized in agriculture while region $C$ products both goods. This case corresponds to point $X$ in Figure 2. Case 1 ($C(M) - I(A)$) would happen if $X$ was located on the right of $L_C$. The allocation curve of region $C$ would then cut the segment $[EL_C]$ and the equilibrium would be such that $L_{MC} = L_C$. It is therefore
Figure 2: Allocation curves of regions $C$ et $I$

possible to distinguish these two cases according to the position of $X$ with regards to $L_C$:

$$L_{MC}(X) < L_C \iff \frac{L_C}{L_I} > \frac{\mu}{\beta \tau_A} \iff C(A, M) - I(A) \iff L_{MC} = \frac{\mu}{\beta + \mu} (L_C + \frac{L_I}{\tau_A})$$  \hspace{1cm} (18)

$$L_{MC}(X) > L_C \iff \frac{L_C}{L_I} < \frac{\mu}{\beta \tau_A} \iff C(M) - I(A) \iff L_{MC} = L_C. \hspace{1cm} (19)$$

Finally, Case 2 ($C(M) - I(A, M)$) implies that region $C$ is fully specialized in manufacturing production and that region $I$ produces both goods. Such an equilibrium is necessarily located on $[EL_C]$ and can exist only if region $I$’s allocation curve crosses this segment. Figure 3 is an example of this case. In appendix A, we show that, when there is at least one intersection between region $I$’s allocation curve and $[EL_C]$, there are exactly two. We then show that a necessary and sufficient existence condition for these two equilibria is:

$$\frac{L_I}{L_C} > \frac{(1 + \alpha)(\beta + \mu)}{\alpha \mu} \left( \frac{\alpha \beta}{\tau_M(\beta + \mu)} \right)^{\frac{1}{\tau + n}}. \hspace{1cm} (20)$$

Note that the existence of such an equilibrium implies the simultaneous existence of a stable
Case 1 equilibrium (though the reverse is not true).

As we have seen above, the allocation curve delimits two zones: below the curve, the relative supply price is lower than the relative demand price, which implies that the manufacturing supply must increase, and above the curve, the supply price is higher than the demand price, implying that the manufacturing sector must contract. As a consequence, the two allocation curves delimit four zones in the rectangle $OL_1E_LC$ where the evolution of the manufacturing labor force is indicated by arrows. The equilibrium in $Z$ is therefore always unstable. On the contrary, the equilibria corresponding to Cases 1 and 3 (i.e. points $L_C$ and $X$) are always stable. For Case 2, points $S$ and $U$ being the two equilibria, it is simple to see that $U$ is unstable, while $S$ is unstable (see Figure 3).

Starting from the autarkic equilibrium, opening to trade will lead to a stable trade equilibrium such that region $C$ exports the manufacturing good only if the point $A$ is located southeast to point $Z$. Since $A$ and $Z$ belong to the line $D$ (see appendix A), this condition is met if:

$$\frac{L_C}{L_I} > (\tau_A\tau_M)^{\frac{1}{2}}.$$  \hfill (21)
Therefore, the region \( C \) must have a type-\( L \) population sufficiently larger than region \( I \)'s in order to be able to export the manufacturing good. The higher the transport costs, the higher the population difference or the intensity of increasing returns required for these trade equilibria to exist. An important consequence of this result is that the existence of a price differential in autarky is not a sufficient condition for a trade equilibrium to exist when trade is allowed: if the price differential is lower than \( \tau_A \tau_M \), both regions will indeed stay at the autarkic equilibrium. If the autarkic price differential is sufficient for trade to happen between the two regions, each region will export the good for which it has a comparative advantage.\(^8\)

Existence conditions of the different types of stable equilibrium are summed up in table 2.

**Table 2: Existence conditions for each type of equilibrium**

<table>
<thead>
<tr>
<th>Cas 1: ( C(M) - I(A) )</th>
<th>((\tau_A \tau_M)^{\frac{1}{\alpha}} &lt; \frac{L_C}{L_I} &lt; \frac{\mu}{\beta \tau_A})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cas 2: ( C(M) - I(A, M) )</td>
<td>((\tau_A \tau_M)^{\frac{1}{\alpha}} &lt; \frac{L_C}{L_I} &lt; \frac{\alpha \mu}{\beta \eta (1 + \alpha)(\beta + \mu)} \left( \frac{\tau_M (\beta + \mu)}{\alpha \beta} \right)^{-\frac{1}{1 + \alpha}})</td>
</tr>
<tr>
<td>Cas 3: ( C(A, M) - I(A) )</td>
<td>((\tau_A \tau_M)^{\frac{1}{\alpha}} &lt; \frac{L_C}{L_I} ) et ( \frac{\mu}{\beta \tau_A} &lt; \frac{L_C}{L_I})</td>
</tr>
<tr>
<td>Cas 4: ( C(A, M) - I(A, M) )</td>
<td>unstable equilibrium</td>
</tr>
</tbody>
</table>

**5 Interregional inequalities and gains from trade**

For each of the specialization patterns, it is possible to compute the welfare differential between workers of type \( L \) in both regions. Similarly, the determination of gains from trade for these workers requires to compare the welfare level they obtain in each case to the autarkic welfare level.

**Case 1: \( C(M) - I(A) \)**

In the case where both regions are fully specialized, the general equilibrium implies the following prices:

\[
P_{MC} = \frac{\mu N_C + N_I}{\nu L_C^{1+\alpha}}, P_{AI} = \frac{\beta N_C + N_I}{\nu L_I} \quad \text{and} \quad w_C = P_{MC} L_C^{\alpha} = \frac{\mu N_C + N_I}{\nu L_C}. \tag{22}
\]

\(^8\)In Ethier’s sense: the more populated region has a comparative advantage in producing the increasing returns good, and the smaller region a comparative advantage in the production of the other good.
Welfare levels of type-$L$ workers in each region at the equilibrium are therefore:

\[
U_C = w_C(\tau_A P_{AI})^{-\beta} P_{MC}^{-\mu} \\
= \left(\frac{N_C + N_I}{\nu}\right)^{\beta-\beta} \mu^{1-\mu} \tau_A^{-\beta} L_C^{\mu(1+\alpha) - 1} L_I^\beta
\]  
(23)

\[
U_I = w_I P_{AI}^{-\beta}(\tau_M P_{MC})^{-\mu} \\
= \left(\frac{N_C + N_I}{\nu}\right)^{\beta-\beta} \mu^{1-\mu} \tau_M^{-\mu} L_C^{\mu(1+\alpha) - 1} L_I^\beta
\]  
(24)

As a consequence, the welfare differential between the two regions at the equilibrium is, in this case:

\[
\frac{U_C}{U_I} = \frac{\mu \tau_M^\mu L_I}{\beta \tau_A^\beta L_C}
\]  
(27)

Existence conditions of this equilibrium imply $L_C/L_I < \mu/\beta \tau_A$. Therefore, we must have $L_C/L_I < \mu \tau_M^\mu/\beta \tau_A^\beta$, and necessarily $U_C/U_I > 1$. Workers of type $L$ living in the region specialized in manufacturing production are therefore always better-off than those living in the agricultural region.

Welfare ratios between trade and autarky are:

\[
\frac{U_C}{U_C^{\text{aut}}} = \left(\frac{N_C + N_I}{N_C}\right)^{\nu} \left(\frac{\beta + \mu}{\beta} \frac{\alpha^{\mu-\nu}}{L_I \left(\frac{\beta}{\beta \tau_A \mu L_C}\right)^{\beta}}\right)
\]  
(28)

\[
\frac{U_I}{U_I^{\text{aut}}} = \left(\frac{N_C + N_I}{N_I}\right)^{\nu} \left(\frac{\beta + \mu}{\beta} \frac{\alpha^{\mu-\nu}}{\tau_M^{\mu} \left(\frac{\beta L_C}{\beta L_I}\right)^{(1+\alpha)\mu}}\right)
\]  
(29)

Since one must have $\mu L_I/\beta \tau_A L_C > 1$, a sufficient condition for gains from trade for region $C$’s type-$L$ workers is $\alpha \mu > \nu$ (which does not exclude that they may gain from trade even when this condition is not met). On the contrary, $\alpha \mu > \nu$ is not a sufficient condition for agents in region $I$ to gain from trade.

**Case 2: $C(M)-I(A, M)$**

When region $C$ is fully specialized in manufacturing production while region $I$ produces both goods, equilibrium prices are as follows:

\[
P_{MC} = \frac{\mu}{\nu} \frac{N_C + N_I}{L_C^{1+\alpha} + \tau_M L_M^{1+\alpha}}, \quad P_{AI} = \frac{\beta N_C + N_I}{\nu L_{AI}}, \quad P_{MI} = \frac{\mu}{\nu} \frac{N_C + N_I}{L_{MI}^{1+\alpha} + \tau_M L_M^{1+\alpha}}, \quad w_C = P_{MC} L_C^\alpha = \frac{\mu}{\nu} \frac{(N_C + N_I) L_C^\alpha}{L_C^{1+\alpha} + \tau_M L_M^{1+\alpha}}.
\]  
(30)
Welfare levels of type-$L$ agents in each region are therefore:

\[
U_C = w_C (\tau_A P_{AI})^{-\beta} P_{MC}^{-\mu} \\
= \left( \frac{N_C + N_I}{\nu} \right)^{\nu} \beta^{-\beta} \mu^{1-\beta} \tau_A^{-\beta} L_C^\alpha (L_C^{1+\alpha} + \tau_M L_{MI}^{1+\alpha})^{\mu-1} L_{AI}^{-\beta} 
\]

(31)

\[
U_I = w_I P_{AI}^{-\beta} P_{MC}^{-\mu} \\
= \left( \frac{N_C + N_I}{\nu} \right)^{\nu} \beta^{1-\beta} \mu^{-\mu} (L_C^{1+\alpha} + \tau_M L_{MI}^{1+\alpha})^{\mu-1} L_{AI}^{-\beta}. 
\]

(32)

As a consequence, the welfare differential between the two regions at the equilibrium is, in this case:

\[
\frac{U_C}{U_I} = \frac{\mu \tau_M^\mu}{\beta \tau_A^{\beta}} \frac{L_C^\alpha L_{AI}^{-\beta}}{L_C^{1+\alpha} + \tau_M L_{MI}^{1+\alpha}}. 
\]

(33)

Since equilibrium must meet the condition (47), the welfare differential is rewritten:

\[
\frac{U_C}{U_I} = \frac{\tau_M^\mu}{\tau_A^{\beta}} \frac{L_C^\alpha}{L_C^{1+\alpha} + \tau_M L_{MI}^{1+\alpha}}. 
\]

(34)

Existence conditions for this equilibrium imply $L_{MI} < L_I < L_C$ and therefore $\tau_A \tau_M < (L_C/L_I)^\alpha < (L_C/L_{MI})^\alpha$. Then, $(L_C/L_{MI})^\alpha/\tau_M > \tau_A$, and since $\tau_M^\mu/\tau_A^{1-\beta} > 1$, we therefore have $U_C/U_I > 1$. Workers of type $L$ living in the region specialized in manufacturing production are therefore always better-off than those living in region $I$.

Welfare ratios between trade and autarky are:

\[
\frac{U_C}{U_{C aut}} = \frac{N_C + N_I}{N_C} \left( \frac{\beta + \mu}{\mu} \right)^{\alpha^{-\mu}} \left( \frac{\beta}{\mu} \right)^{\nu} \left( \frac{L_C}{L_{MI}} \right)^{\alpha(1-\mu)} \left( \frac{L_C}{L_{AI}} \right)^{\nu} \tau_A^{-\beta} \tau_M^{-\mu-1} 
\]

(35)

\[
\frac{U_I}{U_{I aut}} = \frac{N_C + N_I}{N_I} \left( \frac{\beta + \mu}{\mu} \right)^{\alpha^{-\mu}} \left( \frac{\beta}{\mu} \right)^{\nu} \left( \frac{L_C}{L_{MI}} \right)^{-\alpha\mu} \left( \frac{L_C}{L_{AI}} \right)^{\nu} 
\]

(36)

Case 3: $C(A, M)$–$I(A)$

Finally, in the case where region $I$ is specialized in agriculture while region $C$ produces both goods, equilibrium prices are:

\[
P_{MC} = \frac{\mu}{\nu} \frac{N_C + N_I}{(L_C + L_I)^{1+\alpha}} \ , \ P_{AI} = w_I = \frac{\beta + \mu}{\nu} \frac{N_C + N_I}{\tau_A L_C + L_I} \ 
\text{and} \ P_{AC} = w_C = \frac{\beta + \mu}{\nu} \frac{N_C + N_I}{L_C + L_I}. 
\]

(37)
Welfare levels of \( L \) agents in each region are therefore:

\[
U_C = w_C P_{AC}^{-\beta} P_{MC}^{-\mu} \\
= \left( \frac{N_C + N_I}{\nu} \right)^\nu (\beta + \mu)^{\nu-\alpha} \mu^{\alpha \mu} \left( L_C + \frac{L_I}{\tau_A} \right)^{\alpha \mu - \nu} \\
(40)
\]

\[
U_I = w_I P_{AI}^{-\beta} (\tau_M P_{MC})^{-\mu} \\
= \left( \frac{N_C + N_I}{\nu} \right)^\nu (\beta + \mu)^{\nu-\alpha} \mu^{\alpha \mu} \tau_A^{\beta-1} \tau_M^{-\mu} \left( L_C + \frac{L_I}{\tau_A} \right)^{\alpha \mu - \nu} \\
(41)
\]

\[
U_C = P_{AI}^{-\beta} (\tau_M P_{MC})^{-\mu} \\
(42)
\]

\[
U_I = \left( \frac{N_C + N_I}{\nu} \right)^\nu (\beta + \mu)^{\nu-\alpha} \mu^{\alpha \mu} \tau_A^{\beta-1} \tau_M^{-\mu} \\
(43)
\]

As a consequence, the welfare differential between the two regions at the equilibrium is, in this case:

\[
\frac{U_C}{U_I} = \tau_A^{1-\beta} \tau_M^{\mu} \\
(44)
\]

which higher than 1. Workers of type \( L \) in the agricultural region are worse off than those in region \( C \).

Welfare ratios between trade and autarky are:

\[
\frac{U_C}{U_C^{\text{aut}}} = \left( \frac{N_C + N_I}{N_C} \right)^\nu \left( \frac{L_I}{\tau_A L_C} + 1 \right)^{\alpha \mu - \nu} \\
(45)
\]

\[
\frac{U_I}{U_I^{\text{aut}}} = \left( \frac{N_C + N_I}{N_I} \right)^\nu \left( \frac{L_C}{L_I} + \frac{1}{\tau_A} \right)^{\alpha \mu - \nu} \tau_A^{\beta-1} \tau_M^{-\mu} \\
(46)
\]

A sufficient condition for agents in \( C \) to gain from trade is \( \alpha \mu > \nu \), while this condition is not sufficient for agents in \( I \).

For all possible specialization patterns, workers of type \( L \) in the region exporting the manufacturing good benefit from a higher welfare level than those in the region exporting the agricultural good.

6 **Interregional labor mobility**

The trade equilibria being such that there always exist a welfare differential between the two regions, let us assume that this gap may be reduced through type-\( L \) labor mobility from the “poor” region to the “rich” region. To what extent is this migration able to modify the specialization pattern?

As we have just seen, the welfare differential is in favor of region \( C \) in all specialization patterns. As a consequence, if type-\( L \) agents can migrate between regions, the population in \( C \) will necessarily grow as long as a welfare gap remains. Whatever the initial specialization pattern, the economy will therefore always end up at a point where \( L_C/L_I > \mu/\beta \tau_A \), i.e. in the equilibrium where \( I \) is fully specialized in agriculture and \( C \) is big enough to produce both goods (Case 3). In that case, the welfare gap does not depend on the population distribution
between regions and is always equal to $\tau A^{1-\beta} \tau M^\mu$.

The only possible way to reduce regional inequalities is therefore to reduce internal transport costs, for instance through improvements in the transportation networks. Moreover, in order to limit regional unbalances, it is highly preferable to reduce transport costs rapidly. Indeed, the longer the economy is confronted with a wide welfare gap between regions, the higher the chances that the manufacturing region becomes much bigger than the agricultural region.

7 Conclusion

TO DO
A  Some details on Case 2

Let us first establish that region I’s allocation curve is concave (the proof is of course valid as well for region C’s curve). The function $\Gamma_I(L_{MI}, L_{MC})$ and its partial derivatives are continuous on $[0, \frac{\mu}{\beta + \mu}(L_I + \tau_A L_C)] \times [0, \overline{L_{MC}}]$, where $\overline{L_{MC}}$ is the maximum value of $L_{MC}$ such that $\Gamma_I(L_{MI}, L_{MC}) = 0$ for $L_{MI} \in [0, \frac{\mu}{\beta + \mu}(L_I + \tau_A L_C)]$. Moreover, the partial derivative of $\Gamma_I$ with respect to $L_{MC}$ is non-zero over $[0, \frac{\mu}{\beta + \mu}(L_I + \tau_A L_C)] \times [0, \overline{L_{MC}}]$

$$\Gamma_I'_{L_{MC}} = \frac{\mu L_{MC}^{1+\alpha} + \tau_M L_{MI}^{1+\alpha} + (1 + \alpha)L_{MC}^{\alpha}(L_I - L_{MI})/\tau_A + (L_C - L_{MC})}{(L_{MC}^{1+\alpha} + \tau_M L_{MI}^{1+\alpha})^2} > 0$$

which permits, according to the implicit functions theorem, to deduce that there exists a continuous and differentiable function $g_I$ such that $\Gamma_I(L_{MI}, g_I(L_{MI})) = 0$. Then, we have:

$$g_I' = -\Gamma_I'_{L_{MI}}/\Gamma_I'_{L_{MC}} = \frac{\alpha}{\tau_A T_A} L_{MC}^{1-\alpha} L_{MI}^{-\alpha-2} v(g_I' L_{MI} - L_{MC}) (L_{MC} - g_I' L_{MI}).$$

Note $u$ and $v$ the numerator and the denominator of the above fraction, such that $g_I' = u/v$ ; $g_I''$ and $u'v - uv'$ then have the same sign :

$$u'v - uv' = \frac{\alpha(1 + \alpha)}{\tau_A T_A} L_{MC}^{\alpha-1} L_{MI}^{-\alpha-2} v(g_I' L_{MI} - L_{MC})(L_{MC} - g_I' L_{MI}).$$

The denominator $v$ of $g_I'$ being strictly positive, we have $u'v - uv' \leq 0$ and therefore $g_I'' \leq 0$, $\forall L_{MI} \in [0, \frac{\mu}{\beta + \mu}(L_I + \tau_A L_C)]$. The allocation curve of region I is therefore concave. Therefore, if region I’s allocation curve intersects $[EL_C]$, there are at most two intersections. Is it possible that there is only one intersection ? That would the case if the decreasing part of the curve would cut the line $(EL_C)$ above E. The intersections between the allocation curve and $(EL_C)$ are such that $\Gamma_I(L_{MI}, L_C) = 0$ (or $g_I(L_{MI}) = L_C$), that is:

$$\frac{L_C^{1+\alpha} L_{MI}^{-\alpha}}{\tau_A T_A} + \frac{L_{MI}}{\tau_A} = \frac{\mu(L_I - L_{MI})}{\beta \tau_A}.$$  \hspace{1cm} (47)

This equality can only be satisfied if $L_{MI} < L_I$, i. e. if any intersection between the allocation curve and the line $(EL_C)$ is located below E. Therefore, if we neglect the case where the allocation curve is tangential to $[EL_C]$, there exist always two intersections when there is at least one. Each of these points is a trade equilibrium. These equilibria exist if region I’s allocation curve cuts $[EL_C]$, i. e. if its maximum $[\overline{L_{MC}}, \overline{L_{MI}}]$, is on the right of $[EL_C]$ (i. e. outside the rectangle $OL_1EL_C$). This condition is written $\overline{L_{MC}} > L_C$, or:

$$g_I(g_I^{-1}(0)) > L_C \iff \frac{L_C}{L_I} < \frac{\alpha \mu}{(1 + \alpha)(\beta + \mu)} \left( \frac{\tau_M (\beta + \mu)}{\alpha \beta} \right)^{\frac{1}{\alpha + 1}}.$$

From Figure 3, it is easy to see that one of the intersections between $\Gamma_I$ and $[EL_C]$ is a stable equilibrium $(S)$, while the other is unstable $(U)$.  

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References


