Welfare-Reducing Trade Liberalization

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Abstract

Recent literature on the workhorse model of intra-industry trade has explored heterogeneous cost structures at the firm level. These approaches have proven to add realism and predictive power. This paper shows, however, that this added realism also implies that there may exist a positive bilateral tariff that maximizes national and world welfare. Applying one of the simplest specifications possible, namely a symmetric two-country intra-industry trade model with fixed export costs that are heterogeneous across firms, we find that the reciprocal reduction of small tariffs reduces welfare.

JEL: F12, F13, F15

Key Words: Intra-industry trade, monopolistic competition, heterogeneous firms, welfare, protectionism

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1 Introduction

Recently firm-level heterogeneity has been introduced to intra-industry trade models, e.g. Schmitt and Yu (2001), Melitz (2003), Helpman, Melitz and Yeaple (2004) or Yeaple (2005). These specifications, where firms are heterogeneous with respect to their cost structures, have provided important new insights, frequently reconciling theory with the stylized facts of international trade. For example Schmitt and Yu (2001) resolve the puzzle of scale economies and the volume of intra-industry trade by introducing firm-level heterogeneous fixed exporting costs. Melitz (2003) features firm-level heterogeneous marginal costs and analyzes intra-industry reallocations, showing that additional gains from trade stem from the induced productivity improvements. Helpman, Melitz and Yeaple (2004) introduce firm-level heterogeneities and are able to capture the exporting-versus-FDI decision of firms. Finally, Yeaple (2005) derives firm heterogeneity from labor force heterogeneities and arrives at realistic predictions concerning the productivity of exporting firms and the effects of trade on the skill premium.

However, thus far the literature has not fully examined the effects of these new – and more realistic – assumptions on the welfare effects of trade policies such as tariffs. In this paper, we examine this issue by introducing bilateral ad valorem tariffs into a simple symmetric two-county Krugman-type (1980) intra-industry trade model with firm-level heterogeneous fixed costs of exporting as in Schmitt and Yu (2001). A second contribution of the present paper is it to extend the literature by presenting the case of firm-level heterogeneous fixed export costs with an entry mechanism à la Melitz (2003), i.e. firms decide entry subject to sunk costs and based on expected profits, knowing only the distribution of firm heterogeneity in the population but not their own realization. The present model employs several assumptions that promote free trade as a welfare optimum: there are no wasteful (e.g. iceberg) trade costs, the firm-specific fixed costs of exporting are less than the cost of creating a new variety, and all firm profits and tariff revenues are redistributed in a lump sum fashion to consumers. Still, we find that in this model there is in fact too much trade in the free trade equilibrium. More resources


2Arguable, the entry mechanism in Schmitt and Yu (2001), where firms’ entry decision is based on reaching breakeven on their home market operation alone, can be problematic because there exist export profits that do not trigger industry entry. Jørgensen and Schröder (2005b) find that the possibility of welfare reducing tariff liberalization established in the present paper for the more complex Melitz (2003)-type entry mechanism does also occur for the Schmitt and Yu (2001) entry mechanism.
are used on the exporting/importing activity than is welfare-optimal, measured as total consumer utility. National and world welfare increases when imposing small bilateral tariffs. The welfare maximizing tariff is strictly positive, less than 1 and increases in the degree of product differentiation (love of variety). Thus, reciprocal trade liberalization, in particular the reduction of small tariffs, will be welfare-reducing. The underlying mechanism is that even though small bilateral tariffs reduce the number of traded varieties, the total number of available varieties in both countries is maintained or rather increases slightly. Any tariff reduces the number and volume of traded varieties. For a small tariff fairly inefficient exporters cease their trading activity, and paired with the volume reduction in exports/imports, the total of saved resources more than compensates consumers via the entry and larger volumes of home varieties. However, for large tariffs, a further increase in the tariff forces fairly efficient exporters out of the trading activity, replacing cheaply generated varieties (i.e. imported from abroad) with expensively generated varieties (i.e. produced at home).

The finding of welfare-reducing tariff liberalization contradicts much of the existing literature, see e.g. Markusen and Venables (1988), Fukushima and Kim (1989), Lockwood and Wong (2000). Also, in intra-industry trade models, bilateral tariffs are usually welfare-reducing, e.g. Gros (1987), Jørgensen and Schröder (2005a).³ The central difference between these models and the present model is that the earlier work assumes firms to be homogeneous in their cost structure. However, Melitz (2003), Falvey, Greenaway and Yu (2004) and Baldwin and Forslid (2004), all using a Melitz-type (2003) framework with firm-level heterogeneous marginal costs, examine, inter alia, iceberg trade cost reductions, which are often interpreted to represent trade liberalization, and find, in line with earlier literature, an overall welfare gain. Furthermore, Melitz (2003) and Baldwin and Forslid (2004) note the possibility for an anti-variety effect. Yet, this situation, in contrast to the anti-variety effect in the present model, only emerges once the fixed costs of exporting are larger than the fixed costs of pure domestic production, and thus the export activity of a firm ties up more resources than an additional domestic variety would require. This situation is explicitly ruled out in the present model. The possibility of welfare-reducing trade liberalization is, however, found in Montagna (2001), in a framework where firms have heterogeneous marginal costs. Yet, a welfare loss occurs as a special case when trade allows relatively inefficient firms to enter and when consumers’ taste for variety is sufficiently

³On the other hand, small unilateral tariffs may increase welfare (Gros, 1987), and unilateral tariffs can induce a home market effect in the presence of transportation costs (Helpman and Krugman, 1989).
low.

The next section presents the model. In Section 3, we derive the welfare
effect of imposing bilateral ad valorem tariffs, and discuss the results. Section
4 concludes.

2 The Model

The starting point is a standard Krugman-type (1980) model of intra-
industry trade, yet with the feature of firm-level heterogeneous fixed costs
of exporting as in Schmitt and Yu (2001). Consumers in two identical coun-
tries, home and foreign, love variety and have identical preferences, in which
all consumption goods, $c$, enter symmetrically. Utility is given by

\[ U = \sum v(c_i) \]

\[ = \sum c_i^\theta, \quad \theta \in (0, 1). \]

More specifically we can write (1) as

\[ U = \sum_{i_d=1}^{N_d} c_{d,i_d}^\theta + \sum_{i_t=1}^{N_t} c_{t,i_t}^\theta + \sum_{i_f=1}^{N_f} c_{f,i_f}^\theta, \]

where $c_{d,i_d}$ is consumption of variant $i_d$ of non-exported domestic products,
$c_{t,i_t}$ is consumption of variant $i_t$ of the exported domestic products and $c_{f,i_f}$ is consumption of variant $i_f$ of imported products.\footnote{Since all goods enter symmetrically and since all firms behave identically within the two categories trading and non-trading, we can omit subscript $i$ where unnecessary.} The number of variants actually produced ($n_d$, $n_t$, and $n_f$) is assumed to be large, although smaller than $N_d$, $N_t$, and $N_f$. Furthermore, denoting foreign variables by $\ast$, the
symmetry of the setup implies $n_t = n_f = n_f = n_t^\ast$ and that trade is balanced.

Firms

Firms can produce their specific variant for the home market alone or for both
the home and foreign market. The decision to export is firm-endogenous,
where some but not all firms will export. Each firm produces with the same
constant marginal cost $\beta$ and a fixed cost $\alpha$, both expressed in terms of
labor, $L$, which is the only factor of production and is remunerated at the
economy-wide wage rate $w$. When exporting, a firm faces an additional firm-
specific fixed export cost, $a_i$, heterogeneous across firms and, for simplicity,
assumed to be uniformly distributed on the interval \([0, \alpha]\), with \(F(.)\) denoting the distribution function which is public knowledge. Finally, to enter, firms face an initial fixed entry cost \(f\), which is measured in monetary units and sunk, and where \(0 < f < \frac{\alpha}{2}\). In order to focus on the problem at hand we avoid several of the complexities of modelling sunk entry costs and the probability of firm ‘death’ as presented in Melitz (2003), and instead apply an alternative version, simply envisaging two separate rounds. In particular, production and sales for the home market (and the fixed costs \(\alpha\) and \(f\)) are sunk in the sense that they are assumed to simply occur prior to an exporting round, in which the individual \(a_i\)'s are revealed and export production – if the firm chooses so – and sales take place.

Trade is costly. Both countries charge the same ad valorem tariff \(\tau \in (0, 1)\) on imports, i.e. a bilateral tariff. The presence of fixed export costs and the tariff creates an asymmetry between trading and non-trading firms, and hence, the profit functions of a pure domestic firm only servicing the home market, and an exporting home firm servicing both markets, are

\[
\pi_d = p_d x_d - (\alpha + \beta x_d) w, \\
\pi_z = p_t x_t + (1 - \tau) p_z x_z - (\alpha + a_i + \beta (x_t + x_z)) w,
\]

where \(x_d\) is the production of a pure domestic firm, and \(x_t\) and \(x_z\) are the output of an exporting firm to the home and the foreign market respectively. Finally, various market-clearing relations complete the model: goods market clear \(L_{c_{d,i}} = x_{d,i} d\), \(L_{c_{t,i}} = x_{t,i} t\) and \(L^* c_{f,i} f = x_{z,i} f\), where the foreign index \(i_f\) and the home index \(i_t\) denote one and the same variant; income expenditure clearing \(L w + R = p_d x_d n_d + p_t x_t n_t + p_z x_z n_f\), where \(R\) denotes the profits of all domestic firms (excluding \(f\)) and all tariff revenues assumed to be lump-sum redistributed to consumers; and similar relations for the foreign country.

**Prices and quantities**

Maximization of (2) leads to the familiar inverse demand functions, e.g. \(p_d = \frac{\theta c}{\theta - 1}\) for any non-traded home good \(i_d\), and similar for traded products, given that the number of products is large. Then, profit maximization of (3) with respect to \(x_d\) and maximizing (4) with respect to \(x_t\) and \(x_z\) results in the

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5The fixed entry costs capture some form of fee payment required in order to enter the industry and are bound by the maximum expected profits of export activity, which – as will become clear below – are equal to \(\frac{\alpha}{2}\). The costs \(f\) could also represent the threshold return (premium) demanded by entrepreneurs in order to cover the risk (since \(a_i\) draws are uncertain) they take when establishing a firm, or the costs of bank lending.

6We are grateful to Marc Melitz for pointing out this short-cut.
price

\[ p_d = p_t = \frac{\beta w}{\theta} \]  

(5)

\[ p_z = \frac{\beta w}{(1 - \tau)\theta} = \frac{p_d}{1 - \tau} \]  

(6)

for sales on the home and the foreign market respectively. Since \( p_t = p_d \) consumers do not distinguish between non-traded home products and traded home products; and hence, sales quantities of trading firms on their home market must be identical to that of non-trading firms, i.e. \( x_d = x_t \). Yet, exported goods are more expensive than domestically produced goods and by symmetry \( p_z = p^*_z \), i.e. the price that a home firm charges abroad is the same as the price charged by foreign exporters on our home market. In equilibrium, maximization of utility (2) requires that the ratio of the marginal utility of an extra consumption unit equals the price ratio, i.e. \( \frac{\theta_c}{\theta} = \frac{p_d}{p^*_z} = 1 - \tau \).

Utilizing the goods market clearing conditions, this implies

\[ x_z = x^*_z = x_d(1 - \tau)^{\frac{1}{1-\tau}}. \]  

(7)

Thus exporting firms charge the same price on their home market and have the same sales volume as non-trading firms, but charge higher prices and sell less of their variety on the foreign market. By the same token, domestic consumers pay more and consume less of imported product varieties compared to domestically produced varieties.

With these relations in place production scale can be determined as driven by free entry/exit. Firms know the distribution of \( a_i \)'s and the relation given in (7). Furthermore, there must exists some cut-off level, \( \bar{a} \), of the firm specific fixed export costs denoting the firm that is exactly indifferent between engaging in exports and being a non-trading firm. Then, entry of firms occurs until expected profits equal fixed entry cost \( f \), in particular

\[ \pi^{exp} = F(\bar{a})\pi_d + ((1 - F(\bar{a}))\pi_z = f. \]  

Using (3) and (4) and realizing that the expected fixed cost of exporting must be \( \bar{a}^2 \) the equation reads:

\[ \bar{a}^2 \left( p_d x_t + (1 - \tau)p_z x_z - \left( \alpha + \frac{\bar{a}}{2} + \beta(x_t + x_z) \right) w \right) 
\]

\[ + \left( 1 - \frac{\bar{a}}{\alpha} \right) (p_d x_d - (\alpha + \beta x_d) w) = f. \]  

(8)

\footnote{We depart here significantly from Schmitt and Yu (2001), where firms determine entry subject to reaching breakeven on their home market operation. Instead we follow Melitz (2003), namely firms determine their entry subject to expected profits and sunk cost, accordingly some firms will make profits and some losses in equilibrium.}
Inserting from above (8) can be solved for $x_d$ to yield:

$$x_d = \frac{\theta}{(1 - \theta) \beta} \frac{\bar{a}^2 + 2f \alpha + 2 \alpha^2}{2(\alpha + \bar{a}(1 - \tau)^{1/\beta})},$$

which also is the home market production scale of exporting firms ($x_i$) and can be plugged into (7) to determine $x_z$. Note that in autarky (where the tariff is the prohibitive $\tau = 1$ and accordingly $\bar{a} = 0$) the production scale reaches the textbook case, namely $x_d|_{\text{autarky}} = \frac{\theta}{(1 - \theta) \beta} (\alpha + f)$.

The indifferent firm

With the prices and quantities derived above, it is straightforward to identify the firm just indifferent between becoming an exporting firm and becoming a pure domestic firm. This firm is characterized by a fixed cost of exporting $\bar{a}$ such that $\pi_{z, \bar{a}} = \pi_d$ must hold. Since profits (losses) stemming from home market sales are the same for both types of firms, the condition becomes $(1 - \tau) p_z x_z - (\bar{a} + \beta x_z) w = 0$, i.e. the indifferent firm makes zero profits from the exporting activity. After setting in $p_z$, and $x_z$ from above one can solve:

$$\bar{a} = \left(\sqrt{\alpha^2 + 2\alpha(f + \alpha)(1 - \tau)^{2/\beta}} - \alpha\right) (1 - \tau)^{1/\beta}.$$

All firms $i$ such that $a_i \in [0, \bar{a}]$ make non-negative profits from exporting, while all firms $i$ such that $a_i \in [\bar{a}, \alpha]$ are non-trading firms. Notice that by (10) we have $\bar{a} > 0$ and that in the free trade situation ($\tau = 0$) we have $\bar{a}|_{\tau=0} < \alpha$. The reason for $\bar{a}|_{\tau=0} < \alpha$ is as follows. With zero tariffs the sales scale on the home and foreign market are identical ($x_d = x_z$). Since exporting promises expected profits this scale is competed so small (via entry) that the home sales will not break-even, accordingly the exporting activity of a firm with a fixed export costs $a_i = \alpha$ must also result in negative profits, hence also with free trade ($\tau = 0$) some firms are non-trading firms choosing to minimize their losses by refraining from exporting. Furthermore, $\bar{a}$ decreases in the tariff rate (see appendix A.1), implying that the least efficient (high $a_i$) firms will cease their trading activity in response to a tariff increase.

The number of firms

The total number of firms at home, $n = n_t + n_d$, is determined via the income expenditure clearing condition $Lw + R = p_d x_d n_d + p_t x_t n_t + p_f x_f n_f$, where $n_t = n_f$, where $n_t = F(\bar{a}) n$, $n_d = (1 - F(\bar{a})) n$, and where $R$ is the
redistributed tariff income and fixed entry cost \( f \) – or equivalently total firm profits excluding entry investment (see (8)). One gets:

\[
n = \frac{L(1 - \theta)}{\alpha + f\theta + \frac{\bar{a}_2}{2\alpha}}. \tag{11}
\]

Because of trade, consumers also have access to foreign varieties, in particular due to symmetry \( n_t = n_t^* = n_f = F(\tilde{n})n \) and accordingly the number of varieties available on the home market are given by \( \tilde{n} = n + n_f \).

### 3 Welfare results

Total consumer utility is our measure of welfare. Given goods market clearing and (2), we can write \( \sum U = n_d x^\theta_d + n_t x^\theta_t + n_f x^\theta_f \), and setting in values from above and simplifying gives:

\[
\sum U = \frac{L(1 - \theta) \left( \sqrt{g} \left(1 - \tau \right)^{\frac{1}{\sqrt{g}}} + \alpha \left(1 - \tau \right)^{\frac{\beta}{\sqrt{g}}} - \alpha \left(1 - \tau \right)^{\frac{1}{\sqrt{g}}} \right) h^\theta}{f(1 + \theta) + \alpha \left(2 + \left(1 - \tau \right)\frac{\beta}{\sqrt{g}} \right) - \sqrt{g} \left(1 - \tau \right)^{\frac{1}{\sqrt{g}}}}, \tag{12}
\]

where \( g = \alpha \left( \alpha + 2(f + \alpha)(1 - \tau)\frac{\beta}{\sqrt{g}} \right) \),

and \( h = \frac{\theta(\sqrt{g} - \alpha)}{\beta(1 - \theta)(1 - \tau)^{\frac{1}{\sqrt{g}}}} \).

The following results can be stated.

**Proposition 1.** Total consumer utility under free trade exceeds that under autarky, yet, there exists a strictly positive bilateral tariff, \( \hat{\tau} \) that maximizes total national (and world) consumer utility. In particular, \( \sum U|_{\tau=0} > \sum U|_{\text{autarky}} \) and \( \frac{\partial \sum U}{\partial \tau}|_{\tau=0} > 0 \).

For proof, see appendix A.2. To illustrate proposition 1, consider figure 1 which plots (12) as a function of \( \tau \) for various values of \( \theta \).\(^8\) To the right, for \( \tau \) close to 1 we are in the autarky situation. To the left, for \( \tau = 0 \), we are in the free trade situation, and welfare in both countries is clearly above the autarky level. However, imposing a small bilateral tariff increases welfare until we reach the welfare maximizing bilateral tariff, \( \hat{\tau} \), beyond which welfare starts to decrease towards the autarky level. What proposition 1 implies is in fact that there is too much trade in the free trade situation. National and world welfare increases when imposing small bilateral tariffs.

\(^8\)The parameter values are \( \alpha = 2.2, \beta = 0.9, f = 1, L = 100 \)
The welfare maximizing bilateral ad valorem tariff is strictly positive, less than 1 and increases in the degree of product differentiation, $\theta$, (love of variety). Accordingly, trade liberalization, in particular the bilateral reduction of tariffs smaller than $\hat{\tau}$, will be welfare-reducing.

\[ \sum U \]

\[ \theta = 0.2 \]

\[ \theta = 0.25 \]

\[ \theta = 0.35 \]

Figure 1: The welfare effect of bilateral tariffs

What drives this finding of small bilateral tariffs being able to increase welfare? To illustrate the intuition for the result it is useful to break down the contributing factors. First, examine the number of firms given in (11) and in particular the number of varieties available on the home market given by $\tilde{n} = n + n_f$. It turns out that with the imposition of a small bilateral tariff, the exit of trading firms and therewith the loss of $n_t$ and $n_f$ is compensated by the entry of additional pure domestic firms $n_d$, in fact slightly increasing $\tilde{n}$ at first before it falls for larger tariffs.\(^9\) Accordingly, within the consumption basked foreign products have been replaced with home products. The second contribution to an utility increase stems from the changes in the output volumes, $x_d$ and $x_z$ that can be consumed. As can be seen from (9) and (7) a tariff increases the output volume of domestic varieties available to domestic consumers and reduces the output volume directed at the foreign market (and hence, the consumption volume of each imported variety).\(^10\) Thus even

\(^9\)Formal proof of $\frac{\partial \tilde{n}}{\partial \tau}\big|_{\tau=0} > 0$ is given in appendix A.3. Appendix A.4 shows a plot of the number of firms and $\tilde{n}$.

\(^10\)Appendix A.5 shows a plot of the output volumes $x_d$ and $x_z$. 
if the number of available varieties was just constant (and not increasing),
then the pure shift from foreign varieties to home varieties paired with an
increase in the amounts consumed of each home variety would constitute an
utility increase.

To see the logic of these changes in the number of available varieties and
the consumption volumes, consider the following reasoning. A small bilat-
eral tariff, reduces the number of imported varieties and – via the imposed
price increase of foreign products – the import volume of all remaining vari-
eties. However, overall a small tariff still increases welfare because the least
efficient exporters are the first to cease their trading activity. Paired with
the additional resources saved by reducing the trading activity of all remain-
ing exporting firms, this frees enough resources for the production of more
home varieties in larger quantities. That is the tariff reduces the volume of
each remaining importer/exporter but converts it into additional domestic
entry and consumption. However, beyond the welfare maximizing bilateral
tariff, \( \hat{\tau} \), a further increase in the tariff further cuts imported volumes, and
more importantly, it forces fairly efficient exporters out of the trading ac-
tivity. Thus, additional variants produced relatively cheaply (i.e. by foreign
exporters who have fairly low fixed export costs) are replaced with variants
produced relatively expensively (i.e. by new home producers incurring the
fixed production cost, \( \alpha \)).

In line with this reasoning, it turns out that the total fixed costs per
available variety that occur to a country \((n\alpha + n_t^f)/\bar{n})\) as a function of
\( \tau \) is U-shaped. Thus a small bilateral tariff, by forcing expensive (high \( a_i \))
exporters/importers out reduces the amount of fixed cost that society has to
tie up in order to generate variety.

4 Conclusion

This paper examined the welfare impact of trade policy in an intra-industry
trade model with firm-level heterogeneity. This new type of specifications,
where firms are heterogeneous with respect to their cost structures, has gen-
erated important new insights, frequently reconciling theory with the stylized
facts of international trade, e.g. Schmitt and Yu (2001), Melitz (2003), Help-
man, Melitz and Yeaple (2004) or Yeaple (2005), but has not yet been used
to systematically examine trade policies.

Our model examines bilateral ad valorem tariffs in a symmetric two-
country intra-industry trade model, with firm-level heterogeneous fixed costs
of exporting. We find that in this model there is in fact too much trade in the
free trade equilibrium. More resources are used on the exporting/importing
activity than is welfare-optimal, measured as total consumer utility. There exists a strictly positive bilateral tariff that maximizes national and world welfare. Accordingly, trade liberalization, in particular the reciprocal reduction of small tariffs, is welfare-reducing. This contradicts much − if not all − of the existing literature. The underlying mechanism for our result is that even though small bilateral tariffs reduce the number of traded varieties, the number of available varieties in both countries increases. This mechanism is at work even though the fixed costs of creating a new domestic variety are always larger than the firm-specific fixed costs of exporting and even though there are no wasteful (e.g. iceberg) trade costs. Future research should address the welfare effects of trade policies for different forms of firm-level heterogeneity and for more types of trade barriers.
A Appendix

A.1 Derivative of $\frac{\partial \bar{a}}{\partial \tau}$

From (10) we have $\bar{a} = \left(\sqrt{\alpha^2 + 2\alpha(f + \alpha)(1 - \tau)^\frac{2}{1+\theta}} - \alpha\right) (1 - \tau)^\frac{1}{1+\theta}$. It follows immediately that:

$$\frac{\partial \bar{a}}{\partial \tau} = \alpha \left(\sqrt{\alpha(\alpha + 2(f + \alpha)(1 - \tau)^\frac{2}{1+\theta}) - \alpha}\right) (1 - \tau)^\frac{2-\theta}{1+\theta} \left(\frac{-1 + \theta}{\sqrt{\alpha(f + 3\alpha)}}\right) < 0. \quad (A.1)$$

A.2 Proof of Proposition 1.

Proof. Total consumer utility under free trade exceeds that under autarky; in particular, $\sum U|_{\tau=0} > \sum U|_{\text{autarky}}$.

Evaluating (12) at $\tau = 0$ gives:

$$\sum U|_{\tau=0} = L(1 - \theta) \frac{\sqrt{\alpha(2f + 3\alpha)} \left(\frac{\sqrt{\alpha(2f + 3\alpha)} - \alpha}{\beta(1-\theta)}\right)^\theta}{f + 3\alpha + f\theta - \sqrt{\alpha(2f + 3\alpha)}} \quad (A.2)$$

Under autarky (where the tariff is prohibitive and accordingly $\bar{a} = 0$) we have $x_d|_{\text{autarky}} = \frac{\theta(\alpha + f)}{(1-\theta)^3}$ and accordingly $n_d|_{\text{autarky}} = n|_{\text{autarky}} = \frac{L(1-\theta)}{\alpha + f\theta}$. Setting in these values, total utility under autarky is given by:

$$\sum U|_{\text{autarky}} = \frac{L(1 - \theta) \left(\frac{(f + \alpha)\theta}{\beta(1-\theta)}\right)^\theta}{\alpha + f\theta} \quad (A.3)$$

Hence, we want to show that for all $\alpha > 0$, $0 < f < \frac{\alpha}{2}$, $\theta \in [0, 1]$:

$$\frac{\sqrt{\alpha(2f + 3\alpha)} \left(\sqrt{\alpha(2f + 3\alpha)} - \alpha\right)^\theta}{\alpha \left(f + 3\alpha + f\theta - \sqrt{\alpha(2f + 3\alpha)}\right)} > \frac{(f + \alpha)^\theta}{\alpha + f\theta} \quad (A.4)$$

Step 1: Define $s = \frac{L}{\alpha} \Leftrightarrow f = \alpha s$ and insert in (A.4), which leads to:

$$\frac{\sqrt{2s + 3} \left(\sqrt{2s + 3} - 1\right)^\theta}{s + 3 - \sqrt{2s + 3 + s\theta}} > \frac{(s + 1)^\theta}{1 + s\theta} \quad (A.5)$$
Step 2: Define \( v = \sqrt{2s + 3} \Leftrightarrow s = \frac{v^2 - 3}{2} \). As \( 0 < f < \frac{\alpha}{2} \) and \( s = \frac{L}{a} \) we have that \( \sqrt{3} < v < \sqrt{4} \). By substituting for \( s \) in (A.5) we get:

\[
\frac{v^2}{v^2 + 3 - 2v + \theta v^2 - 3\theta} > \frac{(v^2 - 1)^\theta}{\theta v^2 - 3\theta + 2}
\]  

Note, that \((v^2 - 1)^\theta = (v + 1)^\theta (v - 1)^\theta \). Since \( v > \sqrt{3} > 1 \), we have that \((v - 1)^\theta > 0 \) and (A.6) leads to:

\[
\frac{v^2}{v^2 + 3 - 2v + \theta v^2 - 3\theta} > \frac{(v^2 - 1)^\theta}{\theta v^2 - 3\theta + 2}
\]

\[
v (v^2 - 3) \theta + 2v > (\frac{v + 1}{2})^\theta (v^2 - 3) \theta + (\frac{v + 1}{2})^\theta (v^2 - 2v + 3)
\]  

Step 3: Define \( \text{LHS}(\theta) = v (v^2 - 3) \theta + 2v \) and \( \text{RHS}(\theta) = (\frac{v + 1}{2})^\theta (v^2 - 3) \theta + (\frac{v + 1}{2})^\theta (v^2 - 2v + 3) \).

\( \text{LHS}(\theta) \) is linear in \( \theta \) with a slope and an intercept that depend on \( v \). Furthermore, \( \text{LHS}(0) = 2v \) and \( \text{LHS}(1) = v^3 - v \). \( \text{RHS}(\theta) \) looks linear in \( \theta \) but it is multiplied with the factor \((\frac{v + 1}{2})^\theta \). It is evident that \( \text{RHS}(0) = v^2 - 2v + 3 \) and that \( \text{RHS}(1) = v^3 - v \). Hence, \( \text{LHS}(1) = \text{RHS}(1) \).

Now we want to show that \( \text{LHS}(0) > \text{RHS}(0) \) for all relevant \( v \). It is true since, \( \text{LHS}(0) > \text{RHS}(0) \Leftrightarrow 2v > v^2 - 2v + 3 \Leftrightarrow (1 - v)(v - 3) > 0 \) and \( \sqrt{3} < v < \sqrt{4} \).

Step 4: We want to show that \( \text{RHS}(\theta) \) is convex in \( \theta \) where \( \theta \in [0, 1] \) and \( \sqrt{3} < v < \sqrt{4} \). Differentiating \( \text{RHS}(\theta) \) with respect to \( \theta \) we get:

\[
\text{RHS}'(\theta) = (v^2 - 3) \left( \theta \left( \frac{v + 1}{2} \right)^\theta \ln \left( \frac{v + 1}{2} \right) + \left( \frac{v + 1}{2} \right)^\theta \right)
\]

\[
+ (v^2 - 2v + 3) \left( \frac{v + 1}{2} \right)^\theta \ln \left( \frac{v + 1}{2} \right)
\]

From (A.9) it follows that:

\[
= (v^2 - 3) \left( \ln \left( \frac{v + 1}{2} \right) \left( \theta \left( \frac{v + 1}{2} \right)^\theta \ln \left( \frac{v + 1}{2} \right) + \left( \frac{v + 1}{2} \right)^\theta \right) \right)
\]

\[
+ (v^2 - 3) \left( \frac{v + 1}{2} \right)^\theta \ln \left( \frac{v + 1}{2} \right)
\]

\[
+ (v^2 - 2v + 3) \ln \left( \frac{v + 1}{2} \right) \left( \frac{v + 1}{2} \right)^\theta \ln \left( \frac{v + 1}{2} \right)
\]

\[
\text{RHS}''(\theta)
\]
Form (A.10) it follows that $RHS''(\theta) > 0$ for all $\sqrt{3} < v < \sqrt{4}, \theta \in [0,1]$. Hence, $RHS(\theta)$ is convex, and therefor $LHS(\theta) > RHS(\theta)$. We have now show that $\sum U|_{\tau=0} > \sum U|_{autarky}$. 

Proof. There exists a strictly positive bilateral tariff that maximizes total national and world consumer utility; in particular $\frac{\partial \sum U}{\partial \tau}|_{\tau=0} > 0$.

By differentiation (12) with respect to $\tau$ and evaluating the expression in $\tau = 0$ we get:

$$\frac{\partial \sum U}{\partial \tau}|_{\tau=0} = \frac{2fL\alpha \left( f + 2\alpha - \sqrt{\alpha(2f+3\alpha)} \right) \theta \left( \frac{\sqrt{\alpha(2f+3\alpha)} - \alpha}{\beta(1-\theta)} \right)^{\theta-1}}{\sqrt{\alpha(2f+3\alpha)} \beta \left( f + 3\alpha - \sqrt{\alpha(2f+3\alpha)} + f\theta \right)^2}$$

(A.11) is positive as:

$$f + 2\alpha > \sqrt{\alpha(2f + 3\alpha)}$$

$$\Downarrow$$

$$f^2 + \alpha^2 + 2f\alpha > 0$$

\hfill \Box

A.3 Proof of $\frac{\partial \tilde{n}}{\partial \tau}|_{\tau=0} > 0$.

Proof. The number of available varieties increases for a small tariff. The number of varieties available on the home market is given by $\tilde{n} = n + n_f$.

From (11) and using the fact that $n_t = n^*_t = n_f = F(\tilde{n})n$ it follows that

$$\tilde{n} = \frac{L(1-\theta) \alpha - \alpha(1-\tau)^{\frac{1}{\theta-1}} + \sqrt{\alpha(\alpha + 2(f + \alpha)(1-\tau)^{\frac{2}{\theta-1}})(1-\tau)^{\frac{1}{\theta-1}}}}{f(1+\theta) + \alpha(2 + (1-\tau)^{\frac{\theta-2}{\theta-1}}) - \sqrt{\alpha(\alpha + 2(f + \alpha)(1-\tau)^{\frac{2}{\theta-1}})(1-\tau)^{\frac{1}{\theta-1}}}}$$

(A.12)

The derivative of $\tilde{n}$ in (A.12) with respect to $\tau$, and evaluated at the free trade situation, $\tau = 0$, gives:

$$\frac{\partial \tilde{n}}{\partial \tau}|_{\tau=0} = \frac{L(1-\theta)f \left( \sqrt{\alpha(2f + 3\alpha)} - \alpha \right)}{\sqrt{\alpha(2f + 3\alpha)} \left( f + 3\alpha - \sqrt{\alpha(2f + 3\alpha)} + f\theta \right)^2} > 0.$$  

(A.13) 

\hfill \Box
A.4 The number of firms and available varieties

Figure A.1 plots the number of firms, $n$, the number of pure domestic producers, $n_d$, and exporting producers, $n_t$, and the total number of available varieties, $\tilde{n}$ as a function of $\tau$. Other parameter values are $\alpha = 2.8$, $\beta = 0.5$, $\theta = 0.35$, $f = 1.2$, $L = 100$.

Figure A.1: Number of firms and available varieties
A.5 Production scale

Figure A.2 plots the production scale, \( x_d \), that is sold by domestic non-exporting and exporting firms on the domestic market, and the production scale, \( x_z \), sold by foreign exporters on the domestic market (which is identical to the sales that domestic exporters have on the foreign market). Other parameter values are \( \alpha = 2.8 \), \( \beta = 0.5 \), \( \theta = 0.35 \), \( f = 1.2 \), \( L = 100 \).

![Figure A.2: Output (production scale)](image)
References


Schmitt, Nicolas and Yu, Zhihao (2001), Economics of scale and the volume