Export Subsidies and Timing of Decision-Making

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Abstract

This paper examines how the timing of decision-making affects the strategic trade policy. Extending the analysis on the strategic trade policy to the sequential-move game, I show some interesting results as follows: First, it is shown that when the governments decide the subsidies simultaneously before the Stackelberg competition, the leader firm loses its first-mover advantage. Second, it is shown that under the sequential-move game in which the government that can subsidize the leader firm decides the subsidy at first, the profit of the leader firm is less than that of the follower in the Stackelberg model.

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1 Introduction

This paper examines how the timing of decision-making affects the strategic trade policy. I analyze the relationship between the different timing of the decision-making by exporting firms and their subsidizing governments and its impact on the export subsidy policy.

Although WTO reorganized from the GATT in 1995 and the FTAs tend to increase rapidly nowadays, the export subsidy policy have yet been practiced in many countries as a strategic means to induce more domestic surplus from exportation. In the WTO agreements, the export subsidies to the manufactured products are prohibited _per se_, but as for the agricultural products, the WTO members are in the very act of negotiating the reduction of the subsidy rate.

From the theoretical point of view, since Brander and Spencer (1985) elucidate the strategic effect of subsidy policy, many articles has dealt with the exporting subsidies in the context of the strategic trade policy. Using the third country model, Brander and Spencer (1985) analyze the rent-shifting effect of the export subsidy and the strategic interaction between the export subsidies. They show that the export subsidy functions effectively to raise the domestic welfare, but it implies that the strategic subsidy choices by both governments in the exporting countries fall into the prisoner’s dilemma. Eaton and Grossman (1986) extend the model of Brander and Spencer (1985) to allow the different conjectural variations that includes Cournot case.

Although the above two representative papers and their successors deal with the general demand structure and illuminate the strategic aspects on the trade policy clearly, their papers have limited to the analyses of only the situation in which the choices of the strategic variables by the competitive firms are made simultaneously. Extending the simultaneous-move game to the sequential one on output choice and also subsidy choice, I present a new point of view about
the strategic subsidy policy that is influenced by the timing of decision-making.

In the actual international trade policy, for instance, it may take place that the governments of the developed countries first determine the subsidy levels in advance before the governments of the developing countries determine the subsidy levels and vice versa. Because of the different abilities between governments to implement and enforce the trade policy, there exists the time lag on the subsidy decisions by governments. I introduce the difference on the timing of decision-making on output and subsidy level into the model.

As for the sequential-move game on strategic trade policy, there are several articles that it should be referred. In the two-country model, Syropoulos (1994) shows that the governments may choose tariffs sequentially under perfect competition. Collie (1994) shows that the domestic government sets tariff at first and then the foreign government sets export subsidy under Cournot quantity competition. In the third-country model, Arvan (1991) concludes that demand uncertainty may cause the sequential-move of the policy choice by governments. Shivakumar (1993) introduces the export quota and shows that the restricted quantity competition and demand uncertainty bring the sequential decision of trade policy by governments. Recently, Ohkawa, Okamura and Tawada (2002) endogenize the timing of government intervention under international oligopoly. Their paper is closely related with my paper in the sense that the sequential-move game by governments is analyzed in the third-country model. Different from my concern, however, they focus on the relationship between the number of firms and the endogenous timing of the policy decision by governments and they do not deal with Stackelberg competition between firms.

Comparing between the simultaneous and sequential moves by firms and governments, I show some interesting results. First, when the governments decide the export subsidies simulta-
ously under the Stackelberg competition, the original leader firm loses its first-mover advantage.

Different from the Cournot model, under the Stackelberg model, the subsidy policy by the government that can subsidize the leader firm is almost nullified. Second, under the sequential-move game in which the government that can subsidize the leader firm decides the subsidy level at first, the profit of the leader is less than that of the follower, although the first-mover advantage of the government is maintained and the leader produces more than the follower. The paper presents one of the theoretical foundations on the significance that the timing of policy decision has on the effectiveness of trade policy.

The remainder of the article is organized as follows. Section 2 describes the model. Section 3 derives subsidy, output, profit and domestic welfare in the equilibrium and analyze the relationship between the different structures. In Section 4, I summarize the calculating results about variables under each case and make the comparative statics with regard to the different structures on timing. Section 5 is the concluding remarks.

2 The model

Two identical firms, one from country $i$ and one from country $j$, produce and sell in a third country. I consider the imperfect quantity competition model in the third country à la Brander and Spencer (1985). Both firms produce only for the third market. They produce homogeneous goods. The firm in country $i$ ($j$) is denoted by the index $i$ (respectively $j$).

Firm $i$ (firm $j$) produces quantity $q_i$ (resp. $q_j$). The total quantity is $Q \equiv q_i + q_j$. The inverse demand function is denoted by $P(Q) \equiv a - bQ$ and the constant marginal cost is denoted by $c_i$. It is assumed that $a > c_i$ and $b > 0$. Government $i$ that lies in country $i$ can implement the per unit
export subsidy, \( s_i \geq 0 \). The profit of firm \( i \) is denoted by \( \pi^i(q_i, q_j; s_i, s_j) \equiv (P(Q) - c_i + s_i)q_i = (P(Q) - e_i)q_i \). The surplus of country \( i \) is denoted by \( G^i(s_i, s_j) \), which consists of the profit from the exporting firm \( i \) minus the cost of the export subsidy: \( G^i(s_i, s_j) \equiv \pi^i(q_i, q_j; s_i, s_j) - s_iq_i \). Government \( i \) maximizes this surplus. The solution concept is the subgame perfect equilibrium.

The timing of the game is as follows:

1st stage: Governments choose subsidy levels simultaneously or sequentially.

2nd stage: Firms choose output levels simultaneously or sequentially.

Subsidy policies can be committed by both governments and can be observed by both firms before the competition stage in advance.

3 The analysis

In this section, I examine the subsidy, the output, the profit and the domestic welfare in the equilibrium. As the first step, I solve the subgame at the second stage. At first, I examine the Cournot quantity competition. Then I proceed to consider the Stackelberg duopoly.

3.1 The output choice at the second stage

3.1.1 Cournot competition

Given the subsidies \((s_i, s_j)\), both firms maximizes their profits. The first-order condition for firm \( i \) to maximize the profit is as follows: \( \pi^i_i = (a - b(q_i + q_j) - e_i) - bq_i = 0 \). The reaction function of firm \( i \) is \( q_i = R_i(q_j) = \frac{a - bq_j - e_i}{2b} \). By solving the intersection of the reaction functions,

\[ \text{The subscript } i \text{ of the profit denotes the partial derivative by } q_i. \text{ Because of the concavity of the profit function with regard to } q, \text{ the second-order condition is satisfied throughout the following analysis.} \]
the Cournot output level is obtained as follows:

\[
(q_i^C(s_i, s_j), q_j^C(s_i, s_j)) = \left( \frac{a - 2e_i + e_j}{3b}, \frac{a - 2e_j + e_i}{3b} \right). \tag{1}
\]

If there is no subsidy, the Cournot outcome is as follows: \((q_i^C(0, 0), q_j^C(0, 0)) = \left( \frac{a - 2e_i + e_j}{3b}, \frac{a - 2e_j + e_i}{3b} \right)\).

The total quantity is \(Q^C = \frac{2a - e_i - e_j}{3b}\), the price is \(P(Q^C) = \frac{a + e_i + e_j}{3}\), and the profit margin is \(P(Q^C) - e_i = \frac{a - 2e_i + e_j}{3} = bq_i^C\). The Cournot profit is calculated as follows:

\[
(\pi_i^C(s_i, s_j), \pi_j^C(s_i, s_j)) = (b(q_i^C)^2, b(q_j^C)^2) = (\frac{(a - 2e_i + e_j)^2}{9b}, \frac{(a - 2e_j + e_i)^2}{9b}). \tag{2}
\]

### 3.1.2 Stackelberg competition

Under the Stackelberg competition, I consider that firm \(i\) is the Stackelberg leader and firm \(j\) is the follower without loss of generality. Anticipating the reaction function of firm \(j\), firm \(i\) maximizes the following objective: \(\pi^i(q_i, R_j(q_i))\). The f.o.c. is \(\pi^i + \pi^j R'_j(q_i) = 0\). The Stackelberg output pairs are as follows:

\[
(q_i^S(s_i, s_j), q_j^S(s_i, s_j)) = \left( \frac{a - 2e_i + e_j}{2b}, \frac{a - 3e_j + 2e_i}{4b} \right). \tag{3}
\]

For (1) and (3) to be positive, it is assumed that \(a - 2e_i + c_j > 0\) and \(a - 3e_j + 2e_i > 0\). With no subsidy, the Stackelberg outcome is as follows: \((q_i^S(0, 0), q_j^S(0, 0)) = \left( \frac{a - 2e_i + e_j}{2b}, \frac{a - 3e_i + 2e_j}{4b} \right)\).

It is well-known that \(q_i^C(s_i, s_j) < q_i^S(s_i, s_j)\) and \(q_j^C(s_i, s_j) > q_j^S(s_i, s_j) \ \forall (s_i, s_j)\). The total quantity is \(Q^S = \frac{3a - 2e_i - e_j}{4b}\) and the price is \(P(Q^S) = \frac{a + 2e_i + e_j}{4}\). It is satisfied that \(Q^S > Q^C\) and \(P(Q^C) > P(Q^S)\). The profit margin is \(P(Q^S) - e_i = \frac{a - 2e_i + e_j}{4} = b \frac{q_i^S}{2}\) and \(P(Q^S) - e_j = \frac{a - 3e_j + 2e_i}{4} = bq_j^S\).

The profit under the Stackelberg competition is calculated as follows:

\[
(\pi_i^S(s_i, s_j), \pi_j^S(s_i, s_j)) = \left( \frac{b}{2}(q_i^S)^2, b(q_j^S)^2 \right) = (\frac{(a - 2e_i + e_j)^2}{8b}, \frac{(a - 3e_j + 2e_i)^2}{16b}). \tag{4}
\]
It is satisfied that $\pi^C(s_i, s_j) < \pi^S(s_i, s_j)$ and $\pi^{Cj}(s_i, s_j) > \pi^{Sj}(s_i, s_j) \forall (s_i, s_j)$.

For the following analysis, I present the comparative statics:

$$\frac{\partial q^C(s_i, s_j)}{\partial s_i} = \frac{2}{b}, \quad \frac{\partial q^C(s_i, s_j)}{\partial s_j} = -\frac{1}{2b},$$
$$\frac{\partial q^S(s_i, s_j)}{\partial s_i} = \frac{3}{4b}, \quad \frac{\partial q^S(s_i, s_j)}{\partial s_j} = -\frac{1}{2b}.$$

### 3.2 The subsidy decision at the first stage

At the first stage, government $i$ maximizes the domestic surplus, that is, $\max_{s_i \geq 0} G^i(s_i, s_j) \equiv \pi^i(q_i, q_j; s_i, s_j) - s_i q_i$. The f.o.c. for government $i$ is as follows:

$$\frac{\partial G^i(s_i, s_j)}{\partial s_i} = \frac{\partial \pi^i(q_i, q_j; s_i, s_j)}{\partial s_i} - q_i - s_i \frac{\partial q_i}{\partial s_i} = 0, \quad (5)$$

if $s_i \geq 0$ (interior solution). If $\frac{\partial G^i(s_i, s_j)}{\partial s_i} < 0$, the solution is $s_i = 0$ (corner solution).\(^2\)

I classify the different timing of the decision-making between firms and between governments into five cases. In Case A and Case B, I examine the unilateral and the bilateral intervention by government(s) under the Cournot competition. In Case C and Case D, I examine the unilateral and the bilateral intervention by government(s) under the Stackelberg competition. In Case E, the situation in which all players sequentially decide is analyzed. In the following subsection, I investigate all cases in turn. See Figure 1. The superscripts, $C$ and $S$, stand for Cournot and Stackelberg equilibrium respectively.

**Figure 1 around here**

### A. unilateral intervention under Cournot competition

First, I examine the unilateral intervention case in which only government $i$ subsidizes under the Cournot competition. As $s_j = 0$, government $i$ maximizes $G^{Ci}(s_i, 0)$. By solving the f.o.c.,

\(^2\)Throughout the following analysis, the s.o.c. is satisfied and the solution is unique and stable.
the optimal subsidy level is obtained as follows:

\[ s_{i}^{uC} = \frac{a - 2c_i + c_j}{4}. \]  

(6)

The Cournot output is as follows:

\[ (q_{i}^{C}(s_{i}^{uC}, 0), q_{j}^{C}(s_{i}^{uC}, 0)) = \left(\frac{a - 2c_i + c_j}{2b}, \frac{a - 3c_j + 2c_i}{4b}\right) = (q_{i}^{S}(0, 0), q_{j}^{S}(0, 0)). \]  

(7)

This result is summarized immediately in the following proposition.

**Proposition 1.** Under the Cournot competition, the unilateral intervention by government \(i\) changes the market structure from the Cournot duopoly to the Stackelberg one in which firm \(i\) is the leader.

This proposition is just a corollary of Proposition 3 in Brander and Spencer (1985). The optimal subsidy has the profit-shifting effect and moves the Cournot competition to the Stackelberg leader-follower position. It is immediately shown that \(q_{i}^{C}(s_{i}^{uC}, 0) > q_{i}^{C}(0, 0)\) and \(q_{j}^{C}(s_{i}^{uC}, 0) < q_{j}^{C}(0, 0)\). As a result of the unilateral subsidy, the profit of firm \(i\) that is subsidized by the government raises, that is, \(\pi_{i}^{C}(s_{i}^{uC}, 0) = \pi_{i}^{S}(0, 0) > \pi_{i}^{C}(0, 0)\) and \(\pi_{j}^{C}(s_{i}^{uC}, 0) = \pi_{j}^{S}(0, 0) < \pi_{j}^{C}(0, 0)\). Also, it is obvious that the surplus in country \(i\) (\(j\)) expands (resp. contracts) by the subsidy of government \(i\), that is, \(G_{i}^{C}(s_{i}^{uC}, 0)(= \max_{s_{i}} G_{i}^{C}(s_{i}, 0)) > G_{i}^{C}(0, 0)\) and \(G_{j}^{C}(s_{i}^{uC}, 0)(= \pi_{j}^{C}(s_{i}^{uC}, 0)) < G_{i}^{C}(0, 0)(= \pi_{j}^{C}(0, 0))\).

**B. bilateral intervention under Cournot competition**

I analyze the bilateral intervention under the Cournot competition. I examine the simultaneous and sequential decision of subsidy in turn.
B-1. simultaneous decision of subsidy

Consider the simultaneous decision of subsidies \((s_i, s_j)\) by both governments. Given \(s_j\), government \(i\) maximizes \(G^C_i(s, s_j) = \pi_i(q^C_i(s, s_j), q^C_j(s, s_j); s_i, s_j) - s_i q^C_i\). Like Case A, the f.o.c. is arranged as \(s_i = \frac{b q^C_i}{2 q^C_i} = \frac{a - 2c_i + c_j}{6}\). The reaction function is derived as \(s_i = R_i(s_j) = -s_j + \frac{a - 2c_i + c_j}{4}\). The subsidy level in the equilibrium is obtained as follows:

\[
s_i^{b_C} = \frac{a - 3c_i + 2c_j}{5}. \tag{8}
\]

Substituting this subsidy level, the Cournot output in the equilibrium is obtained:

\[
(q^C_i(s_i^{b_C}, s_j^{b_C}), q^C_j(s_i^{b_C}, s_j^{b_C})) = \left(\frac{2(a - 3c_i + 2c_j)}{5b}, \frac{2(a - 3c_j + 2c_i)}{5b}\right). \tag{9}
\]

Comparing the subsidy under the unilateral and bilateral cases, the lemma is derived.

**Lemma 1.** The subsidy under the unilateral intervention is larger than under the bilateral intervention. That is, \(s_i^{u_C} > s_i^{b_C}\).

This lemma implies that under the bilateral intervention, there is the strategic interaction about the subsidy setting between governments and as a result, the impact on which the subsidy affects the output choice of the firm is smaller than under the unilateral intervention.

Then I compare the output levels under the unilateral and bilateral interventions.

**Proposition 2.** Under the Cournot competition,

(a) the output level of firm \(i\) (\(j\)) under the bilateral intervention is smaller (resp. larger) than under the unilateral intervention by government \(i\). That is,

\[
q^C_i(s_i^{u_C}, 0) = q^C_i(0, 0) > q^C_i(s_i^{b_C}, s_j^{b_C}), q^C_j(s_i^{u_C}, 0) = q^C_j(0, 0) < q^C_j(s_i^{b_C}, s_j^{b_C}).
\]

(b) when each firm has almost identical cost, the output level under the bilateral intervention is larger than under no intervention. That is, if \(a - 8c_j + 7c_i > 0\), \(q^C_i(0, 0) < q^C_i(s_i^{b_C}, s_j^{b_C})\).

\[^3\text{If the cost is identical, this condition is satisfied.}\]
This proposition implies that by strategic substitutes on output competition, subsidizing by rival government results in the output reduction of the own firm. As the reaction functions of both firms shift outwards by subsidizing bilaterally, as a result, the output competition under the bilateral intervention becomes more severe than without intervention.

As for the profit of the firm, as $\pi_{Ci} = b(q_i^{C})^2$, the relation about the profit size is immediately obtained. By Proposition 2, it is obtained that $\pi_{Ci}(s_i^{uC}, 0) > \pi_{Ci}(s_i^{bCC}, s_j^{bCC})$ and $\pi_{Cj}(s_j^{uC}, 0) < \pi_{Cj}(s_i^{bCC}, s_j^{bCC})$. When the cost is almost identical, $\pi_{Ci}(0, 0) < \pi_{Ci}(s_i^{bCC}, s_j^{bCC})$. They imply that subsidizing by government $i$ ($j$) makes the profit of firm $i$ larger (resp. smaller) and the bilateral intervention makes the profits of both firms larger than without intervention.

Finally, I examine the effect of the subsidy on the surplus. By direct calculation, it is obtained that $G_{Ci}(0, 0) = \frac{(a-2c_i+c_j)^2}{96}$, $G_{Ci}(s_i^{uC}, 0) = \frac{(a-2c_i+c_j)^2}{86}$, $G_{Cj}(s_i^{uC}, 0) = \pi_{Cj}(s_i^{uC}, 0) = \frac{(a-3c_j+2c_i)^2}{166}$, $G_{Cj}(s_i^{bCC}, s_j^{bCC}) = \frac{2(a-3c_j+2c_i)^2}{296}$. It is necessarily satisfied that $G_{Ci}(s_i^{uC}, 0) > G_{Ci}(0, 0)$ and $G_{Cj}(s_i^{bCC}, s_j^{bCC}) > G_{Cj}(s_i^{uC}, 0)$. If the cost is almost identical, it is satisfied that $G_{Ci}(0, 0) > G_{Ci}(s_i^{bCC}, s_j^{bCC})$. The bilateral intervention falls into the prisoner’s dilemma for governments. This is just a corollary of Proposition 5 in Brander and Spencer (1985). See Table 1.

**Proposition 3.** Under the Cournot competition, when each firm has almost identical cost, the surplus under the bilateral intervention is smaller than under no intervention. That is, the bilateral intervention falls into the prisoner’s dilemma for governments.

| Table 1 around here |

**B-2. sequential decision of subsidy**

Next I consider the sequential decision of subsidy as the sequential-move game for governments. Government $i$ decides $s_i$ at first and then government $j$ decides $s_j$ after observing $s_i$.  

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The follower government $j$ decides the subsidy $s_j = R_j(s_i) = \frac{-s_i + a - 2c_j + c_i}{4}$ given $s_i$. Government $i$ induces this reaction and solves the following maximization problem: $\max_{s_i \geq 0} G^C_i(s_i, R_j(s_i))$.

Solving the f.o.c., I obtain the subsidy levels as follows:

$$s_{i}^{bSC} = \frac{a - 3c_i + 2c_j}{3}, \quad s_{j}^{bSC} = R_j(s_{i}^{bSC}) = \frac{a - 4c_j + 3c_i}{6}. \quad (10)$$

By substituting $(s_{i}^{bSC}, s_{j}^{bSC})$, the Cournot output is obtained as follows:

$$(q_{i}^{C}(s_{i}^{bSC}, s_{j}^{bSC}), q_{j}^{C}(s_{i}^{bSC}, s_{j}^{bSC})) = (\frac{a - 3c_i + 2c_j}{2b}, \frac{a - 4c_j + 3c_i}{3b}). \quad (11)$$

For $q_{j}^{C} > 0$, it is assumed that $a - 4c_j + 3c_i > 0$ throughout the following analysis. Comparing the subsidy levels under the unilateral and bilateral cases, I obtain the lemma.

**Lemma 2.** The subsidy under the unilateral intervention is smaller than that of the leader government under the bilateral intervention when the cost is almost identical. The subsidy under the unilateral intervention is larger than that of the follower government under the bilateral intervention. That is, $s_{i}^{uC} < s_{i}^{bSC}$ if $a - 6c_i + 5c_j > 0$ and $s_{j}^{uC} > s_{j}^{bSC}$.

Different from Case B-1, when the sequential decision of subsidy is made by governments under the bilateral intervention, the subsidy of the first-mover government is larger and that of the follower government is smaller than under the unilateral case.

Then I compare the output levels under the unilateral and bilateral interventions.

**Lemma 3.** Under the Cournot competition,

(a) whether the output of firm $i$ under the bilateral sequential intervention is smaller than under the unilateral intervention by government $i$ depends on the relative sizes of the firm’s cost. That is, $q_{i}^{C}(s_{i}^{uC}, 0) \gtrless q_{i}^{C}(s_{i}^{bSC}, s_{j}^{bSC}) \iff c_{i} \gtrless c_{j}$.

When the cost is almost identical, the output of firm $j$ under the bilateral intervention is
smaller than under the unilateral intervention by government $i$. That is, if $a - 7c_j + 6c_i > 0,$
$q_j^C(s_i^{uC},0) < q_j^C(s_i^{bSC},s_j^{bSC})$.

When the cost is almost identical, the output of firm $j$ under the bilateral intervention is smaller than under the unilateral intervention by government $j$. That is, if $a - 6c_i + 5c_j > 0,$
$q_j^C(0,s_j^{uC}) > q_j^C(s_i^{bSC},s_j^{bSC})$.

(b) When the cost is almost identical, the output of firm $i$ under the bilateral intervention is larger than under no intervention. That is, if $a - 5c_i + 4c_j > 0,$
$q_i^C(0,0) < q_i^C(s_i^{bSC},s_j^{bSC})$.

Whether the output of firm $j$ under the bilateral intervention is smaller than under no intervention depends on the relative sizes of the firm’s cost. That is, $q_j^C(0,0) \lesssim q_j^C(s_i^{bSC},s_j^{bSC}) \iff c_i \lesssim c_j$.

As a corollary of this lemma, under the identical costs, it is satisfied that $q_i^C(s_i^{uC},0) = q_i^C(s_i^{bSC},s_j^{bSC})$ and $q_j^C(0,0) = q_j^C(s_i^{bSC},s_j^{bSC})$. When the cost is identical, the output of firm $i$ under the bilateral intervention by government $i$ is equal to that under the unilateral intervention by government $i$. And also the output of firm $j$ under the bilateral intervention by government $j$ is equal to that under no intervention. The subsidy policy by the government has two effects on output. The one is to shift the reaction function outwards and have the home firm the advantage on output competition under strategic substitutes. The second is to adjust to competitive distortion by the cost difference. If the cost is identical, the second effect does not appear and the output is adjusted at the level of the Stackelberg leader by subsidizing.

As for the profit of the firm, the relation about the profit size is immediately obtained with $\pi^C_i = b(q_i^C)^2$. By Lemma 3, it is obtained that $\pi^C_i(s_i^{uC},0) \gtrless \pi^C_i(s_i^{bSC},s_j^{bSC})$ if $c_i \gtrless c_j$, $\pi^C_j(s_i^{uC},0) < \pi^C_j(s_i^{bSC},s_j^{bSC})$ if $a - 7c_j + 6c_i > 0$, and $\pi^C_j(0,s_j^{uC}) > \pi^C_j(s_i^{bSC},s_j^{bSC})$ if $a - 6c_i + 5c_j > 0$. Moreover it is satisfied that $\pi^C_i(0,0) < \pi^C_i(s_i^{bSC},s_j^{bSC})$ if $a - 5c_i + 4c_j > 0$,
and $\pi^{Cj}(0, 0) \geq \pi^{Cj}(s_i^{bSC}, s_j^{bSC})$ if $c_i \leq c_j$.

When the cost is almost identical, the firm that is subsidized by the leader government prefers the unilateral intervention to the bilateral one when the firm’s cost is higher than the rival’s and vice versa. On the other hand, the firm that is subsidized by the follower government always prefers the bilateral intervention to the unilateral intervention by the rival government, although this firm always prefers the unilateral intervention by its home government to the bilateral intervention. Comparing no intervention with bilateral one, firm $i$ always prefers the bilateral intervention to no intervention, although the firm $j$ prefers the bilateral intervention to no intervention if the cost is lower and vice versa.

Finally, I examine the effect of the subsidy on the surplus. By direct calculation, it is obtained that $G^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-3c_i+2c_j)^2}{12b}$ and $G^{Cj}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-4c_i+3c_j)^2}{18b}$. It is necessarily satisfied that $G^{Ci}(s_i^{uC}, 0) > G^{Ci}(0, 0)$ and $G^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-3c_i+2c_j)^2}{12b} > G^{Ci}(0, s_j^{uC}) = \frac{(a-3c_i+2c_j)^2}{16b}$.

If the cost is almost identical, $G^{Cj}(s_i^{bSC}, s_j^{bSC}) < G^{Cj}(s_i^{bSC}, 0)$, $G^{Ci}(0, 0) > G^{Ci}(s_i^{bSC}, s_j^{bSC})$ and $G^{Cj}(0, 0) > G^{Cj}(s_i^{bSC}, s_j^{bSC})$ are obtained. Different from Case B-1, when the cost is almost identical, the leader government chooses to intervene and the follower government chooses not to intervene. The first-mover advantage of government $i$ on the choice of subsidy can deter the rival follower government from exercising the subsidy. I state this result in the following proposition. See Table 2.

**Proposition 4.** Under the Cournot competition, when the cost is almost identical, the surplus under the bilateral intervention is smaller than under no intervention. In the equilibrium, the result is that only the leader government $i$ intervenes and the follower government $j$ does not intervene. The prisoner’s dilemma under the bilateral intervention does not occur.
C. unilateral intervention under Stackelberg model

I proceed to examine the unilateral intervention under the Stackelberg model.

C-1. unilateral intervention of government $i$

I examine the case in which government $i$ whose firm $i$ is the Stackelberg leader intervenes. As $s_j = 0$, government $i$ maximizes $G^{S_i}(s_i, 0) = \pi^i(q_i^S(s_i, 0), q_j^S(s_i, 0); s_i, 0) - s_i q_i^S$. Solving the f.o.c., I obtain that the subsidy is zero, $s_i^{uS} = 0$. The Stackelberg output in the equilibrium is as follows:

$$\left(q_i^S(s_i^{uS}, 0), q_j^S(s_i^{uS}, 0)\right) = \left(q_i^S(0, 0), q_j^S(0, 0)\right) = \left(\frac{a - 2c_i + c_j}{2b}, \frac{a - 3c_j + 2c_i}{4b}\right).$$ (12)

In Case C, another corollary of Prop. 3 in Brander and Spender (1985) is derived. The optimal subsidy moves to the Stackelberg leader-follower position.

**Proposition 5.** Under the Stackelberg competition in which firm $i$ is the leader, the unilateral intervention by the government of the leader firm $i$ is of no use. That is, there is no subsidy.

This proposition is just a corollary of Proposition 3 in Brander and Spencer (1985). The optimal subsidy has the profit-shifting effect and moves the Cournot competition to the Stackelberg leader-follower position. Under the Stackelberg competition, the government of the leader firm $i$ has nothing to do. The profit of firm $i$ that is not subsidized by the government is $\pi^{S_i}(s_i^{uS}, 0) = \pi^{S_i}(0, 0) = \frac{b}{2}(q_i^S)^2$ and $\pi^{S_j}(s_i^{uS}, 0) = \pi^{S_j}(0, 0) = b(q_j^S)^2$. The surplus is $G^{S_i}(s_i^{uS}, 0) = G^{S_i}(0, 0) = \pi^{S_i}(0, 0)$ and $G^{S_j}(s_i^{uS}, 0) = \pi^{S_j}(0, 0)$.
C-2. unilateral intervention of government \( j \)

I examine the case in which government \( j \) whose firm \( j \) is the Stackelberg follower intervenes. As \( s_i = 0 \), government \( j \) maximizes \( G^{S_j}(0, s_j) = \pi^j(q^S_j(0, s_j), q^S_i(0, s_j); 0, s_j) - s_j q^S_j \). By solving the f.o.c., the subsidy level is as follows:

\[
s^u_j = \frac{a - 3c_j + 2c_i}{3}.
\]

(13)
The Stackelberg output in the equilibrium is as follows:

\[
(q^S_i(0, s^u_j), q^S_j(0, s^u_j)) = \left( \frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b} \right).
\]

(14)

This output level is equivalent to that in Case B-2. The following proposition is obtained.

**Proposition 6.** Under the Stackelberg competition in which firm \( i \) is the leader, the unilateral intervention by the government of the follower firm \( j \) yields the same result on output as the bilateral sequential intervention under Cournot competition in which government \( j \) first moves and then government \( i \) moves. That is, \((q^S_i(0, s^u_j), q^S_j(0, s^u_j)) = (q^C_j(s^{bSC}_i, s^{bSC}_j), q^C_j(s^{bSC}_i, s^{bSC}_j))\).

This proposition implies that the subsidy of the government works as if it changes the competition mode from Stackelberg to Cournot. The optimal subsidy improves the Stackelberg-follower position to the Cournot one. Even if firm \( j \) is the Stackelberg follower the optimal subsidy by government \( j \) makes the disadvantage of follower reduce to some extent.

Substituting (14) into \( \pi^S_i(s_i, s_j) = \frac{b}{2}(q^S_i)^2 \) and \( \pi^S_j(s_i, s_j) = b(q^S_j)^2 \), the profit of the firm is immediately obtained: \( \pi^S_i(0, s^u_j) = \frac{(a - 4c_i + 3c_j)^2}{18b} \) and \( \pi^S_j(0, s^u_j) = \frac{(a - 3c_j + 2c_i)^2}{4b} \). It is shown that \( \pi^S_i(s^u_i, 0) = \frac{(a - 2c_i + c_j)^2}{8b} > \pi^S_i(0, s^u_j) \) and \( \pi^S_j(s^u_j, 0) = \frac{(a - 3c_j + 2c_i)^2}{15b} < \pi^S_j(0, s^u_j) \).

Finally, the surplus is obtained that \( G^{S_i}(0, s^u_j) = \pi^S_i(0, s^u_j) = \frac{(a - 4c_i + 3c_j)^2}{18b} < G^S_i(s^u_i, 0) = \pi^S_i(s^u_i, 0) = \frac{(a - 2c_i + c_j)^2}{8b} \), and \( G^{S_j}(0, s^u_j) = b(q^S_j)^2 - s^u_j q^S_j = \frac{(a - 3c_j + 2c_i)^2}{12b} > G^{S_j}(s^u_j, 0) = \pi^S_j(s^u_j, 0) = \frac{(a - 3c_j + 2c_i)^2}{16b} \). When the cost is identical, the following proposition is obtained:
Proposition 7. Consider the Stackelberg competition in which firm \( i \) is the leader. Suppose that the cost is identical. When the government of the follower firm \( j \) unilaterally intervenes, the profit of the follower firm \( j \) (the leader firm \( i \)) is larger (resp. smaller) than that of the leader firm \( i \) (resp. the follower firm \( j \)) when the government of the leader firm \( i \) unilaterally intervenes. That is,

\[
\pi^S_{ij}(0, s^u_{ij}) = \frac{(a-c)^2}{8b} > \pi^S_{ij}(s^u_{ij}, 0) = \frac{(a-c)^2}{18b}, \quad \pi^S_{ii}(0, s^u_{ij}) = \frac{(a-c)^2}{8b} < \pi^S_{ij}(s^u_{ij}, 0) = \frac{(a-c)^2}{18b}.
\]

When government \( j \) unilaterally intervenes, the welfare of government \( j \) (i) is smaller than that of government \( i \) (resp. \( j \)) when government \( i \) unilaterally intervenes. That is,

\[
G^S_{ij}(0, s^u_{ij}) = \frac{(a-c)^2}{12b} < G^S_{ij}(s^u_{ij}, 0) = \frac{(a-c)^2}{8b}, \quad G^S_{ii}(0, s^u_{ij}) = \frac{(a-c)^2}{18b} < G^S_{ij}(s^u_{ij}, 0) = \frac{(a-c)^2}{16b}.
\]

Note that this proposition also holds when the cost is almost identical. Although it looks at first glance that the leader firm may enjoy higher profit when the government of the leader firm can subsidize than that of the follower when its government can subsidize, the above proposition shows that this view is not correct. This implication is derived from the fact that government \( i \) does not subsidize at all because the advantage of the Stackelberg leader has already acquired by firm \( i \) in Case C-1. On the other hand, this intuition is correct when the welfare is considered. Even if the government makes the follower firm recover the first-mover advantage by intervention, it takes an extra cost to subsidize it.

D. bilateral intervention under Stackelberg model

I examine the bilateral intervention under Stackelberg model. Both governments intervene.

D-1. simultaneous decision of subsidy

I consider the simultaneous decision of subsidy. Government \( i \) whose firm \( i \) is leader maxi-
mizes the following objective: Given $s_j, G^S(s_i, s_j) = \pi^i(q^S_i(s_i, s_j), q^S_j(s_i, s_j); s_i, s_j) - s_i q^S_i$. By the f.o.c., the reaction function is $s^b_{iCS} = R_i(s^b_{jCS}) = 0$. Government $j$ whose firm $j$ is the follower maximizes the following objective: Given $s_i, G^S_j(s_i, s_j) = \pi^j(q^S_j(s_i, s_j), q^S_i(s_i, s_j); s_i, s_j) - s_j q^S_j$. By the f.o.c., the reaction function is $s^b_{jCS} = R_j(s^b_{iCS}) = \frac{-2s_i + a - 3c_j + 2c_i}{3}$. I solve the intersection of the reaction function.

$$s^b_{iCS} = 0, \quad s^b_{jCS} = \frac{a - 3c_j + 2c_i}{3}. \quad (15)$$

Note that $s^b_{iCS} = s^uS_i = 0$ and $s^b_{jCS} = s^uS_j = \frac{a - 3c_j + 2c_i}{3}$.

The optimal Stackelberg output level is obtained as follows:

$$(q^S_i(s^b_{iCS}, s^b_{jCS}), q^S_j(s^b_{iCS}, s^b_{jCS})) = \left(\frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b}\right). \quad (16)$$

It is satisfied that $q^S_i(s^b_{iCS}, s^b_{jCS}) = q^S_i(0, s^uS)$ and $q^S_j(s^b_{iCS}, s^b_{jCS}) = q^S_j(0, s^uS)$.

In the case of the simultaneous decision of subsidy levels, the subsidy policy of the government to the leader firms does not work. Different from the Cournot model, under the Stackelberg model, the subsidy policy of the government of the leader firm is nullified. In Case D-1, the Stackelberg leader and follower behave as if they do in Case C-2.

**Proposition 8.** Consider the Stackelberg competition in which firm $i$ is the leader. The result in the equilibrium under Case D-1 is the same as that under Case C-2. The government whose firm is the leader does not subsidize its firm.

Also the profit and the welfare are the same as that under Case C-2. See also Table 3.

D-2. sequential decision of subsidy ($s_i \rightarrow s_j$)

I examine the sequential decision of subsidy under which the government $i$ of the Stackelberg leader $i$ moves at first and then the government $j$ of the follower $j$ decides $s_j$. Observing $s_i$, the
follower government \( j \) decides the subsidy \( s_j = R_j(s_i) \). Government \( j \) whose firm \( j \) is the follower maximizes the following objective: Given \( s_i \), \( G^{S_j}(s_i, s_j) = \pi^j(q_j^S(s_i, s_j), q_i^S(s_i, s_j); s_i, s_j) - s_j q_j^S \).

By direct calculation, the reaction function is obtained as \( s_j^{bSIS} = R_j(s_i^{bSIS}) = \frac{-2s_i+a-3c_j+2c_i}{3} \).

The reaction function is obtained by the same procedure as in Case D-1.

The leader government induces this reaction and solves the following maximization problem:

\[
G^{S_i}(s_i, R_j(s_i)) = \pi^i(q_i^S(s_i, R_j(s_i)), q_j^S(s_i, R_j(s_i); s_i, R_j(s_i)) - s_i q_i^S(s_i, R_j(s_i)).
\]

By the f.o.c., the following equality is obtained: \( s_i = \frac{b}{4} q_i = \frac{a-2(c_i-s_i)+(c_j-R_j(s_i))}{8} \).

Under the sequential decision, the optimal subsidy level is as follows:

\[
s_i^{bSIS} = \frac{a - 4c_i + 3c_j}{8}, \quad s_j^{bSIS} = R_j(s_i^{bSIS}) = \frac{a - 5c_j + 4c_i}{4}.
\] (17)

Note that if the cost is almost identical, \( s_j^{bSIS} > s_i^{bSIS} \), that is, the subsidy to the follower is larger than that to the leader.

Substituting \( s_i \), the Stackelberg output level is obtained as follows:

\[
(q_i^S(s_i^{bSIS}, s_j^{bSIS}), q_j^S(s_i^{bSIS}, s_j^{bSIS})) = \left( \frac{a - 4c_i + 3c_j}{2b}, \frac{3(a - 5c_j + 4c_i)}{8b} \right).
\] (18)

Note that when the cost is almost identical, the output of the leader is larger than that of the follower, \( q_i^S(s_i^{bSIS}, s_j^{bSIS}) > q_j^S(s_i^{bSIS}, s_j^{bSIS}) \).

As for the profit, because it is satisfied that \( (\pi^S_i, \pi^S_j) = (\frac{b}{2}(q_i^S)^2, b(q_j^S)^2) \), the profit is calculated as \( \pi^S_i(s_i^{bSIS}, s_j^{bSIS}) = \frac{(a-4c_i+3c_j)^2}{8b} \) and \( \pi^S_j(s_i^{bSIS}, s_j^{bSIS}) = \frac{9(a-5c_j+4c_i)^2}{64b} \). When the cost is identical, it is worth noting that \( \pi^S_i(s_i^{bSIS}, s_j^{bSIS}) = \frac{(a-c)^2}{8b} < \pi^S_j(s_i^{bSIS}, s_j^{bSIS}) = \frac{9(a-c)^2}{64b} \).

In other words, the profit of the follower is larger than that of the leader.

By \( G^{S_i} = \pi^S_i - s_i q_i \), the welfare is calculated as follows: \( G^{S_i}(s_i^{bSIS}, s_j^{bSIS}) = \frac{(a-4c_i+3c_j)^2}{16b} \) and \( G^{S_j}(s_i^{bSIS}, s_j^{bSIS}) = \frac{3(a-5c_j+4c_i)^2}{64b} \). When the cost is identical, it is satisfied that \( G^{S_i}(s_i^{bSIS}, s_j^{bSIS}) = \frac{(a-c)^2}{16b} > G^{S_j}(s_i^{bSIS}, s_j^{bSIS}) = \frac{3(a-c)^2}{64b} \).

With regard to the social welfare, the welfare of the leader
government is larger than that of the follower under the identical cost. The above result is summarized in the following proposition.

**Proposition 9.** Consider the Stackelberg competition in which firm $i$ is the leader. In the symmetric equilibrium under Case D-2, the profit of the leader is less than that of the follower. The welfare of the leader government $i$ is larger than that of the follower government $j$.

Although the leader produces more than the follower, the government of the leader subsidizes less than that of the follower. As a result, it is shown that the profit of the leader is less than that of the follower and the welfare of the first-mover government is larger than that of second-mover government. The result that the profit of the leader is smaller is a new viewpoint that is obtained from considering the timing of subsidy policy.

Further by comparing the surplus under no intervention with that under Case D-2, it is obtained that $G^{Si}(0, 0) = \frac{(a-2c_i+c_j)^2}{6b} > G^{Si}(s_i^bS_iS_i, s_j^bS_jS_j) = \frac{(a-4c_i+3c_j)^2}{16b}$ and $G^{Sj}(0, 0) = \frac{(a-3c_j+2c_i)^2}{16b} > G^{Sj}(s_i^bS_iS_i, s_j^bS_jS_j) = \frac{3(a-5c_i+4c_j)^2}{64b}$ if the cost is almost identical. Thus I am in the position to state the following proposition. Under the bilateral intervention, both governments obtain fewer surpluses than under no intervention.

**Proposition 10.** Under the Stackelberg competition in which firm $i$ is the leader, when the cost is almost identical, the surplus under the bilateral intervention in Case D-2 is smaller than under no intervention. That is, the bilateral intervention falls into the prisoner’s dilemma.

**D-3. sequential decision of subsidy ($s_j \rightarrow s_i$)**

Finally, I examine the sequential decision of subsidy under which the government $j$ of the Stackelberg follower $j$ moves at first and then the government $i$ of the leader $i$ decides $s_i$. This is
the adverse case of Case D-2 with regard to the timing of the decision-making by governments.

Observing \( s_j \), the follower government \( i \) decides the subsidy \( s_i = R_i(s_j) \). The follower government \( i \) whose firm \( i \) is the leader maximizes the following objective: Given \( s_j \), \( G_i(s_i, s_j) = \pi_i(q_i^S(s_i, s_j), q_j^S(s_i, s_j); s_i, s_j) - s_iq_i^S \). The reaction function is \( s_i^{bSjS} = R_i(s_j^{bSjS}) = 0 \). This is the same procedure as the leader government \( i \) in D-1 Case.

The leader government \( j \) induces this and solves the following maximization problem with substituting \( s_i^{bSjS} = R_i(s_j^{bSjS}) = 0 \):

\[
\max_{s_j \geq 0} G_j(0, s_j) = \pi_j(q_j^S(0, s_j), q_j^S(0, s_j); 0, s_j) - s_jq_j^S(0, s_j).
\]

Under the sequential decision, the optimal subsidy level is as follows:

\[
s_i^{bSjS} = 0, \quad s_j^{bSjS} = \frac{a - 3c_j + 2c_i}{3}.
\]

Substituting \( s_i \), the Stackelberg output level is obtained as follows:

\[
(q_i^S(s_i^{bSjS}, s_j^{bSjS}), q_j^S(s_i^{bSjS}, s_j^{bSjS})) = \left(\frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b}\right).
\]

The equilibrium in this case is the same as Case D-1 (and also C-2 case).

**Proposition 11.** Consider the Stackelberg competition in which firm \( i \) is the leader. The result in the equilibrium under Case D-3 is the same as that under Case D-1 (and also Case C-2). The government whose firm is the leader does not subsidize its firm.

In this case, the follower subsidy setter in the country where there is the Stackelberg leader firm has nothing to do, by the same reason as Case D-1 and C-2. Also the profit and the welfare are the same as that under Case D-1 and C-2. I compare the profit and the welfare in Case D-2 with that in Case D-3. When the cost is identical, the following inequalities are satisfied: \( \pi_i(s_i^{bSjS}, s_j^{bSjS}) < \pi_i(s_i^{bSiS}, s_j^{bSiS}) < \pi_j(s_i^{bSiS}, s_j^{bSiS}) < \pi_j(s_i^{bSjS}, s_j^{bSjS}) \), and \( G_i(s_i^{bSjS}, s_j^{bSjS}) < G_j(s_i^{bSiS}, s_j^{bSiS}) < G_i(s_i^{bSiS}, s_j^{bSiS}) < G_j(s_i^{bSjS}, s_j^{bSjS}) \). See also Table 3.
E. wholly sequential decision

Finally, I examine the wholly sequential decision. I consider the bilateral intervention of sequential decision-making: \( s_i \rightarrow q_i \rightarrow s_j \rightarrow q_j \). The equilibrium is solved by backward induction.

At the 4th stage, the f.o.c. for profit maximization of firm \( j \) given \((s_i, q_i, s_j)\) is as follows:
\[
\pi^j = (a - b(q_i + q_j) - e_j) - bq_j = 0.
\]
The reaction function is \( q_j = R^j(q_i, s_j) = \frac{a-bq_i-e_j}{2b} \). Note that this reaction function does not depend on \( s_i \) directly. At the 3rd stage, the subsidy decision by government \( j \) is determined by maximizing the following objective, \( G^j(s_i, q_i, s_j, R^j_s(q_i, s_j)) \). By solving the f.o.c., the solution is corner, \( s^{bs}_j = 0 \).

At the 2nd stage, the output choice by firm \( i \) is solved as follows: Given \( s_i \), inducing \( s^{bs}_i = 0 \) and \( q_j = R^j_s(q_i, 0) = \frac{a-3c_j + 2c_i}{4b} = q_j^S(s_i, 0) \). At the 1st stage, the subsidy decision by government \( i \) is determined by the same procedure as C-1 case. That is, there is no subsidy, \( s_i = 0 \).

It is satisfied that \((q_i^{bs}, q_j^{bs}) = (q_i^S(0, 0), q_j^S(0, 0))\). The profit and the welfare is as follows:
\[
\pi^S_i(0, 0) = G^S_i(0, 0) = \frac{(a-2c_i+c_j)^2}{8b} \quad \text{and} \quad \pi^S_j(0, 0) = G^S_j(0, 0) = \frac{(a-3c_j+2c_i)^2}{16b}.
\]

4 The comparison

In this section, I compare the different structures with regard to the timing of the decision-making on the subsidy by governments and the output by the firms. Before proceeding to
the analysis, it is convenient to digest the equilibrium outcome under the different timing of the decision-making in the table. See Table 3. In order to figure out the output level in the equilibrium graphically, see also Figure 2 and 3.\(^4\)

**Table 3 around here**

**Figure 2 and 3 around here**

By Table 3, I can examine how the different structures about the timing of the decision-making by firms and governments affects the efficiency of the subsidy policy. Throughout the following comparison, I limit the argument on the situation in which the cost is identical. I show the several noticeable comparisons as the following propositions.\(^5\)

**Proposition 12.** Consider the unilateral intervention of the government. When the firm faces the Cournot competition, the profit is larger than when it competes as the Stackelberg leader and it is equal to that when it competes as the Stackelberg follower. That is,

\[
\pi^{Ci}(s^C_i, 0) = \pi^{Sj}(0, s^S_j) = \frac{(a-c)^2}{4b} > \pi^{Si}(s^S_i, 0) = \frac{(a-c)^2}{8b}.
\]

This proposition implies that although it looks at first glance that the Stackelberg leader has more advantage than under Cournot competition, this impression is not correct. If the unilateral intervention of the government (and no intervention of the rival government) is guaranteed, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader. The reason is that the government has nothing to do under the Stackelberg competition, but as the government under the Cournot competition subsidizes the firm, the acquisition of this subsidy raises the firm’s profit. Whereas, the proposition also implies that the Stackelberg

\(^4\)For simplification, I illustrate only the case in which the cost is identical.

\(^5\)Although I do not compare the welfare in the third country, it is worth noting that the third country’s welfare is reduced by the size of total quantity, \(Q\).
follower recovers the competitive position from the Stackelberg follower to the Cournot and it can compete on equal terms with the rival firm by subsidization of the government.

Next I compare the profits between two competitive forms under the bilateral intervention. At first, I compare Case B-1 with Case D-1.

**Proposition 13.** Consider the bilateral simultaneous intervention of the governments. When the firm faces the Cournot competition, the profit is larger than when it competes as the Stackelberg leader. The profit under the Cournot competition is smaller than when it competes as the Stackelberg follower. That is,

\[
\pi_{Si}(s_{i}^{bCS}, s_{j}^{bCS}) = \frac{(a-c)^2}{18b} < \pi_{Ci}(s_{i}^{bCC}, s_{j}^{bCC}) = \frac{4(a-c)^2}{25b} < \pi_{Sj}(s_{i}^{bCS}, s_{j}^{bCS}) = \frac{(a-c)^2}{4b}.
\]

This proposition is the extensive version of Proposition 12 to the bilateral intervention. Although it looks at first glance that the Stackelberg leader has more advantage than under Cournot competition, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader and moreover prefers to become the follower. The reason is similar to that of Proposition 12. Under the bilateral simultaneous intervention, the government that has the Stackelberg leader does not have any influence to change the market structure. As both governments under the Cournot competition subsidize the firm, their subsidies influence market structure and raise the firm’s profits. Whereas, the Stackelberg follower is fully supported by its government. It can compete on more advantageous terms than the rival firm in Case D-1.

This proposition suggests the following important assertion on the trade policy: When governments can intervene in its domestic firm with a certain policy instrument in advance, the difference of the competitive mode between firms such as Cournot or Stackelberg competition does not necessarily influence the firm’s advantage on the competition. Even if a firm is the
Stackelberg follower in the third-market, the subsidization by the government can get rid of the competitive disadvantage entirely.

Finally, I compare Case B-2 with Case D-2 and D-3.

**Proposition 14.** Consider the bilateral sequential intervention of the governments. Suppose that government $i$ first moves.

(i) When firm $i$ faces the Cournot competition, the profit is larger than when it competes as the Stackelberg leader. The profit of firm $j$ under the Cournot competition is smaller than when it competes as the Stackelberg follower. That is, 

$$\pi^C_i(s^bSC_i, s^bSC_j) = \frac{(a-c)^2}{4b} > \pi^S_i(s^bSiS_i, s^bSiS_j) = \frac{9(a-c)^2}{64b}.$$  

(ii) When firm $i$ faces the Cournot competition, the profit is equivalent when it competes as the Stackelberg follower. The profit of firm $j$ under the Cournot competition is larger than when it competes as the Stackelberg leader. That is, 

$$\pi^C_i(s^bSC_i, s^bSC_j) = \pi^S_i(s^bSiS_i, s^bSiS_j) = \frac{(a-c)^2}{4b} \text{ and } \pi^C_j(s^bSC_i, s^bSC_j) = \pi^S_j(s^bSjS_i, s^bSjS_j) = \frac{(a-c)^2}{18b}.$$  

Part (i) in this proposition states as follows: Under the bilateral sequential intervention, like the simultaneous intervention, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader. On the other hand, the firm prefers to become the follower rather than compete in the Cournot way. At first glance, it seems that the change of decision structure from Case B-2 to D-2 gives the firm that becomes the Stackelberg leader more advantage about output choice. However, this proposition implies that this shift of decision structure does not bring any advantage and in the contrary, the profit of the leader firm decreases. On the other hand, the firm that becomes the Stackelberg follower acquires more profit than under

$^6$Note that the index of $i$ is interchanged with that of $j$ in Case D-3, because government $i$ first moves and firm $i$ is the Stackelberg follower.
the previous Cournot competition.

The basic logic of the proposition is similar to that of Proposition 13, although this logic is a little bit complicated. Under the bilateral sequential intervention, the Stackelberg leader needs less subsidy than if this firm is engaged in Cournot competition, that is, $s_{ibSC}^{i} = \frac{a-c}{3} > s_{ibSiS}^{i} = \frac{a-c}{6}$, because the firm has already enjoyed the first-mover advantage. Whereas, the Stackelberg follower is supported by its government with great care and improves the competitive position, that is, $s_{j}^{bSC} = \frac{a-c}{6} < s_{j}^{bSiS} = \frac{a-c}{4}$. As a result of the asymmetric subsidy policy, it occurs that when both governments subsidize the firm under the Cournot competition, the profit of the leader (the follower) is less (resp. more) than when they do under the Stackelberg competition.

Part (ii) in this proposition states as follows: Under the bilateral sequential intervention, when the competition is in the Cournot way, the profit of the firm is equivalent to that when this firm competes as the Stackelberg follower. Moreover, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader in Case D-3. It implies that when government $i$ first moves, even if the competition form shifts from Cournot to Stackelberg and firm $i$ becomes the follower, its profit does not change. The profit of firm $j$ that becomes the leader in place of firm $i$ becomes less than under the previous Cournot competition. The shift of decision structure does not change the profit and only the profit of the firm that becomes the leader decreases.

It implies that it is desirable for both firms to shift the competitive mode from the Stackelberg competition under which the government of the Stackelberg follower first decides the subsidy, to the Cournot one under bilateral sequential intervention. This shift of the competitive mode avoids the excessive subsidy allocation by governments and saves the subsidy that does not have quite effect on the advantage of the domestic firm in the market.
These propositions insists that the difference in timing of policy execute (and announcement) by the government affects the profit of the firm and the resulting welfare. And also they insist that the government should exercise the trade policy, taking what is the competitive mode between firms into consideration.

5 The concluding remarks

This paper analyzes the relationship between the different timing of the decision-making by exporting firms and their subsiding governments and its impacts on the export subsidy. Two main results are as follows: First, when the governments decide the export subsidies simultaneously in advance under the Stackelberg quantity competition, the original leader in the output competition loses its first-mover advantage. Different from the Cournot model, under the Stackelberg model, the subsidy policy by the government that can subsidize the leader firm is nullified. Second, under the sequential-move game in which the government that can subsidize the leader firm decides the subsidy level at first, the profit of the leader is less than that of the follower, although the first-mover advantage is maintained. I conclude that the timing of decision-making affects the effectiveness on the export subsidy policy more significantly.

Although the paper mainly focuses on the theoretical aspect, the results in this paper are applicable to make some proper suggestions to the actual strategic trade policies. For example, in the realistic context of the international exporting competition, suppose that there is the leader firm that is the predecessor and lies in the dominant position in the exporting market. When the successor entries and the Stackelberg competition is made, the predecessor government of the leader firm may anticipate the successor’s government in deciding the trade policy. In
this situation, how does the government implement the strategic subsidy policy? Proposition 14 tells us that the predecessor government dares to make its firm acquire less profit than the rival firm with the less subsidy than the successor government and should save the subsidy in order to attain more welfare. Moreover, by Proposition 14, it is indicated that even if the government can choose the timing of subsidy policy, it should defend the position as the first-mover policy maker and maintain the first-mover advantage. On the other hand, Proposition 14 suggests also that in order to bring more profit to the successor firm, its government should announce the subsidy policy faster than the rival government of the leader and take the first-mover advantage if possible. This may present one of the reasons that the trade war is triggered.

Further extension in this paper can be considered. First, the more general model in which the demand and cost functions have the general forms can be analyzed. Second, product differentiation should be analyzed, although the basic results in the above analysis have remained unchanged. Thirdly, this result may be extended to strategic complements under which the optimal trade policy is to adopt the exporting tariff.
References


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<table>
<thead>
<tr>
<th>government $i$</th>
<th>no intervention</th>
<th>intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>government $j$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>no intervention</td>
<td>$(a-2c_i+c_j)^2$, $(a-2c_i+c_j)^2$</td>
<td>$(a-3c_i+2c_j)^2$, $(a-2c_i+c_j)^2$</td>
</tr>
<tr>
<td>intervention</td>
<td>$(a-2c_i+c_j)^2$, $(a-3c_i+2c_j)^2$</td>
<td>$2(a-3c_i+2c_j)^2$, $2(a-3c_i+2c_j)^2$</td>
</tr>
</tbody>
</table>

Table 1: the social surplus under no intervention, Case A and B-1

<table>
<thead>
<tr>
<th>government $i$</th>
<th>no intervention</th>
<th>intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>government $j$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>no intervention</td>
<td>$(a-2c_i+c_j)^2$, $(a-2c_i+c_j)^2$</td>
<td>$(a-3c_i+2c_j)^2$, $(a-2c_i+c_j)^2$</td>
</tr>
<tr>
<td>intervention</td>
<td>$(a-2c_i+c_j)^2$, $(a-3c_i+2c_j)^2$</td>
<td>$2(a-3c_i+2c_j)^2$, $(a-4c_i+3c_j)^2$</td>
</tr>
</tbody>
</table>

Table 2: the social surplus under no intervention, Case A and B-2
<table>
<thead>
<tr>
<th>Type</th>
<th>Subsidy</th>
<th>Output</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. no-subsidy Cournot</td>
<td>nothing</td>
<td>( \left( \frac{a-2c_i+c_j}{3b}, \frac{a-2c_i+c_j}{3b} \right) )</td>
<td>( a+c_i+c_j )</td>
</tr>
<tr>
<td>2. no-subsidy Stackelberg</td>
<td>nothing</td>
<td>( \left( \frac{a-2c_i+c_j}{2b}, \frac{a-2c_i+c_j}{2b} \right) )</td>
<td>( a+2c_i+c_j )</td>
</tr>
<tr>
<td>A. unilateral Cournot</td>
<td>( \left( \frac{a-2c_i+c_j}{4}, 0 \right) )</td>
<td>( \left( \frac{a-2c_i+c_j}{4}, \frac{a-3c_i+2c_j}{4} \right) )</td>
<td>( a+2c_i+c_j )</td>
</tr>
<tr>
<td>B. bilateral Cournot</td>
<td>B-1. simultaneous</td>
<td>( \left( \frac{a-3c_j+2c_i}{30}, \frac{a-3c_j+2c_i}{30} \right) )</td>
<td>( \frac{2(a-3c_j+2c_i)}{30}, \frac{2(a-3c_j+2c_i)}{30} )</td>
</tr>
<tr>
<td>B-2. sequential</td>
<td>( \left( \frac{-a-3c_j+2c_i}{3}, \frac{-a-3c_j+2c_i}{3} \right) )</td>
<td>( \frac{a-3c_j+2c_i}{20}, \frac{a-3c_j+2c_i}{20} )</td>
<td>( a+3c_j+2c_i )</td>
</tr>
<tr>
<td>C. unilateral Stackelberg</td>
<td>C-1. government i</td>
<td>( (0, 0) )</td>
<td>( \left( \frac{a-2c_i+c_j}{2b}, \frac{a-2c_i+c_j}{2b} \right) )</td>
</tr>
<tr>
<td>C-2. government j</td>
<td>( (0, \frac{a-2c_i+2c_j}{3}) )</td>
<td>( \left( \frac{-a-3c_j+2c_i}{30}, \frac{-a-3c_j+2c_i}{30} \right) )</td>
<td>( a+2c_i+3c_j )</td>
</tr>
<tr>
<td>D. bilateral Stackelberg</td>
<td>D-1. simultaneous</td>
<td>( (0, \frac{a-3c_j+2c_i}{3}) )</td>
<td>( \left( \frac{a-3c_j+2c_i}{30}, \frac{a-3c_j+2c_i}{30} \right) )</td>
</tr>
<tr>
<td>D-2. sequential gov. i</td>
<td>( \left( \frac{-a-3c_j+2c_i}{30}, \frac{-a-3c_j+2c_i}{30} \right) )</td>
<td>( \frac{a-3c_j+2c_i}{20}, \frac{a-3c_j+2c_i}{20} )</td>
<td>( a+3c_j+2c_i )</td>
</tr>
<tr>
<td>D-3. sequential gov. j</td>
<td>( (0, \frac{a-3c_j+2c_i}{3}) )</td>
<td>( \left( \frac{a-3c_j+2c_i}{30}, \frac{a-3c_j+2c_i}{30} \right) )</td>
<td>( a+2c_i+3c_j )</td>
</tr>
<tr>
<td>E. wholly sequential</td>
<td>( (0, 0) )</td>
<td>( \left( \frac{a-2c_i+c_j}{4}, \frac{a-2c_i+c_j}{4} \right) )</td>
<td>( a+2c_i+c_j )</td>
</tr>
</tbody>
</table>

Table 3: the equilibrium result
Cournot quantity competition
A. unilateral intervention
1st stage  
2nd stage  

B. bilateral intervention
B-1. simultaneous decision-making
1st stage  
2nd stage  

B-2. sequential decision-making
1st stage  
2nd stage  

E. Wholly sequential decision-making
1st stage  
2nd stage  

Stackelberg quantity competition
C. unilateral intervention
C-1. Only gov. i intervenes
1st stage  
2nd stage  

C-2. Only gov. j intervenes
1st stage  
2nd stage  

D. bilateral intervention
D-1. simultaneous decision-making
1st stage  
2nd stage  

D-2. sequential decision-making
1st stage  
2nd stage  

D-3. sequential decision-making
1st stage  
2nd stage  

Figure 1: decision node
Cournot competition
1. no intervention

\[ q_i = R_i(q_j), \quad \pi_i(q_i, q_j) \]

\[ q_j = R_j(q_i), \quad \pi_j(q_i, q_j) \]

\[ a - c \]

\[ 3b \]

Stackelberg competition
2. no intervention

\[ q_i = R_i(q_j), \quad \pi_i(q_i, q_j) \]

\[ q_j = R_j(q_i), \quad \pi_j(q_i, q_j) \]

\[ a - c \]

\[ 4b \]

A. unilateral intervention

\[ q_i = R_i(q_j), \quad \pi_i(q_i, q_j) \]

\[ q_j = R_j(q_i), \quad \pi_j(q_i, q_j) \]

B. bilateral intervention

B-1. simultaneous

\[ q_i = R_i(q_j), \quad \pi_i(q_i, q_j) \]

\[ q_j = R_j(q_i), \quad \pi_j(q_i, q_j) \]

B-2. sequential

\[ q_i = R_i(q_j), \quad \pi_i(q_i, q_j) \]

\[ q_j = R_j(q_i), \quad \pi_j(q_i, q_j) \]

C. unilateral intervention

C-1. government \( i \)

\[ q_i = R_i(q_j), \quad \pi_i(q_i, q_j) \]

\[ q_j = R_j(q_i), \quad \pi_j(q_i, q_j) \]

C-2. government \( j \)

\[ q_i = R_i(q_j), \quad \pi_i(q_i, q_j) \]

\[ q_j = R_j(q_i), \quad \pi_j(q_i, q_j) \]

Figure 2: reaction functions and equilibrium output levels
Stackelberg competition (continued)

D-1. simultaneous decision-making

E. Wholly sequential decision-making

D-2. sequential decision-making
(government $i$ moves first.)

D-3. sequential decision-making
(government $j$ moves first.)

Figure 3: reaction functions and equilibrium output levels (continued)