Productivity Improvements and Falling Trade Costs: Boon or Bane?*

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Abstract

This paper looks at two features of globalization, namely productivity improvements and falling trade costs, and explores their effect on welfare in a monopolistic competition model with heterogenous firms and technological asymmetries. Contrary to received wisdom, and for reasons unrelated to adverse terms of trade effects, we show that there is good reason to expect improvements in a partner's productivity to hurt us. Moreover, falling trade costs can raise welfare in the technologically advanced country while reducing it in the backward one if it is backward enough.

1 Introduction

Should a country welcome productivity improvements in its trading partners or should it be apprehensive? Should all countries welcome falling trading costs or are their welfare effects asymmetric across countries with some gaining and others losing? This is a question of fundamental importance today as globalization results in the spread of technology from the North to the South and falling trade costs and trade barriers improve market access. The standard mantra from trade economists has been that, by and large, such changes are beneficial for the economy as a whole, though some segments of society gain and others

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lose. *We argue below, that though there are always gains from trade, improvements in a partner's productivity hurt us (for a new and different reason) and falling trade costs may hurt the laggard country while helping the advanced one.*

Traditional trade models (whether Ricardian or a variant of Heckscher-Ohlin) offer the basic insight that gains from trade arise when a country faces prices different from its autarky prices. Thus, aside from distributional issues, these models suggest that, ceteris paribus, one would prefer to trade with a country that is different rather than a country which is similar, and with a large country rather than a small one. Moreover, these models suggest that improvements in a trading partners productivity will benefit a country. For example, in the standard Ricardian model with a continuum of goods, productivity improvements by a trading partner raise the welfare of all agents as they weakly raise the real income of domestic labor, the only factor, in terms of each and every good. See Dornbush, Samuelson and Fischer (1977).[1] Also, a fall in trading costs tends to raise welfare as the price of imports falls which raises the real income of labor in terms of each good.

In a richer version of the Ricardian model, Krugman (1986) argues that technological catch up by the followers may hurt the leaders, while technological progress by the leaders helps all countries. The results follow from a combination of terms of trade and real income effects. Progress in the follower country results in greater competition with the leaders exports. This has adverse terms of trade effects for the leader which creates the possibility of welfare losses for the leader. However, technological improvements by the leader raise welfare in both countries. Though the leader suffers adverse terms of trade effects, the productivity improvements more than compensate for them, while the follower country gains since the price of the technologically advanced goods it imports falls. These

[1] The introduction of nonhomothetic preferences (see Matsuyama (2000)) does not change this result.

adverse terms of trade effects are one way for exogenous changes such as productivity improvements or falling trade costs to reduce welfare. However, this is not the channel by which we obtain our results.

Monopolistic competition models with economies of scale where countries have access to the same technology (for example, Helpman and Krugman (1985)) offer a further insight into the effects of trade and technological change. Trade increases market size, which results in a greater variety of products as well as lower prices for the products offered as firms are better able to exploit economies of scale in large markets. In this manner, trade can improve not just aggregate welfare, but the welfare of all agents.[2] However, even in these models, the size of countries plays a crucial role in the determination of gains from trade: the larger the trading partner, the greater the increase in market size due to trade and the greater the gains from trade. In this model, productivity improvements in a trading partner raise welfare as they raise effective market size![3]

Most recently, Melitz and Ottaviano (2003) highlight the role of market potential in trade. They consider a single factor (labor) monopolistic competition model with firm level heterogeneity. Countries differ in their size and in their trade costs but all firms, whether domestic or foreign, draw from the same Pareto productivity distribution. In other words, they have access to the same technological possibilities. Their work has implications for the effect of changing country size, unilateral, bilateral and preferential liberalization. They show that the larger country gains more from trade than the smaller one.[4] The larger

[2] In the simple HOS model, trade always results in trade-offs: some agents gain while others lose. In monopolistic competition models, gains from trade due to variety effects accrue to all consumers. In fact, if countries are close enough in their relative factor availability, these gains swamp any losses from factor price changes. This explains why free trade with a similar country may be welcomed while free trade with a country that is very different in terms of its endowments is harder to sell.

[3] A formal proof that productivity improvements in one country do not hurt its trading partner can be found in Appendix A.

[4] This result is reminiscent of the standard variety effects in monopolistic competition.

country has more “market potential” than the smaller one and as a result, is a better export base in the trading equilibrium. Thus, more firms produce in the larger country, competition is stronger and prices are lower than in the smaller country which is why the larger country gains more from trade. In their model, an increase in the size of a country due to an increase in its labor force raises per capita welfare in the growing country leaving that in its partner unchanged.

Their results on the effects of liberalization are more striking. In standard models, unilateral liberalization is welfare improving in the absence of externalities, second best or profit shifting effects. In contrast, they show that unilateral liberalization hurts the liberalizing country while benefiting others through the market potential effect. Such liberalization makes a country a worse export base so that its market potential is reduced: firms prefer to locate behind high trade barriers and export to countries with low trade barriers. The liberalizing country suffers a reduction in productivity of domestic firms and a reduction in domestic variety which is not fully compensated for by increased import variety. In addition, they show that preferential liberalization, like a customs union, raises welfare of the union members at the expense of non union ones. The market potential of the union rises, making it a better export base, with consequent beneficial effects on productivity and variety.

In this paper yet another insight is offered for monopolistic competition models with heterogeneous firms. We identify a new effect, the technological potential effect.[5] The technological potential of a country consists of the distribution of productivities its firms draw from and the impact of this on its competitiveness in the marketplace. The technology a

[5] In the existing literature, Melitz (2003), Melitz and Ottaviano (2003), Baldwin and Forslid (2004), all firms are assumed to draw from the same distribution. As a result, this effect has been neglected.

firm has access to interacts with market conditions to determine the equilibrium distributions of productivity, the extent of competition and variety in equilibrium. We show that if countries have different technologies available to them, i.e., their firms draw from different distributions which are ordered in terms of hazard rate stochastic dominance (HRSD)[6], and there is no specialization, then productivity improvements in one country raise welfare there but hurt that of its trading partner. The intuition behind our result is the following: the improvement in the technological potential, which occurs when firms can draw from a "better" distribution of productivities, results in more entrants in the home country, and fewer abroad. Domestic entrants are drawn by the higher expected profits from being an exporter. Competition intensifies and the cutoff productivity level rises so that average domestic firm productivity rises. Though the number of foreign producers exporting to the home market falls, the surge in the entry of domestic firms overwhelms it. As a result, consumers at home face a greater variety of products and gain more from trade even though the import of the differentiated goods from abroad decreases. As for the foreign country, a fall in their domestic variety is not fully compensated by the increase in home firms exporting to it and its welfare falls. Note that our results are not coming from a terms of trade effect. If anything, a terms of trade effect should work in the opposite direction. The technological leader is a net exporter of the differentiated good. If its firms draw from an even better distribution, relative supply should shift out and its terms of trade should worsen which should raise the welfare of its partner, not reduce it!

Similarly, a fall in trade costs across the board makes it more advantageous to draw from the better productivity distribution enhancing the technological potential of the

[6] Or in the case of a Pareto distribution, ordered in terms of the usual (first order) notion of stochastic dominance.

advanced country. This results in more variety, higher productivity and lower prices in the advanced country so that its welfare rises. On the other hand, the fall in domestic entrants in the backward country may not be fully compensated for by the rise in exporters, and if this occurs, the lagging country loses![7] When both countries draw from the same distribution, as in Melitz (2003), both gain from a fall in trade cost. Thus, only when the countries draw from the distributions that are different enough, does the backward country lose.

What lies behind differences in the distributions that firms draw from and what are the policy implications of our results? One way to interpret these are just as difference in the technology available to countries. However, there is a richer interpretation that we find more useful. In developing countries, part of the reason why productivity is low is that infrastructure is inadequate. After all, if the power fails on a regular basis, either one has to invest in expensive backup generating equipment (which raises costs) or suffer from lower labor productivity. In such settings, it may also be inappropriate to use cutting edge technology if it is more sensitive to variations in voltage that are the norm in developing countries. As a result, the appropriate technology may differ depending on the infrastructure. Such an interpretation suggests that there may be a significant additional benefit from the government investing in infrastructure: namely, an increase in technological potential!

Falvey, Greenaway and Yu (2004) also use a Melitz (2003) setting and also look at the effects of differences in productivity distributions across countries. However, they consider only the Pareto distribution and a change in its support. They find that a widening of the

[7] Note that all our results still hold in the Melitz and Ottaviano (2003) setting, in which they incorporate endogenous markups using the linear demand system with horizontal product differentiation. An appendix with detailed proofs is available upon request.

gap in the supports increases the welfare gap across countries making the home country relatively better off. However, their results are about the change in relative welfare. Thus, in contrast to their work, this paper provides general results for HRSD with no functional form assumptions, as well as a complete characterization for the Pareto distribution, and provides clean results on absolute welfare changes.

The paper is organized as follows. Section 2 presents the benchmark model with heterogenous firms. Much of this is based on Melitz (2003). Section 3 describes the equilibrium in a closed economy and Section 4 studies the properties of this equilibrium. Section 5 lays out the properties of the equilibrium in the open economy and proves the main result about productivity improvement. Section 6 contains some concluding remarks.

2 The Model

The model is based on that of Melitz (2003), who extends Krugman's (1980) trade model by introducing firm level productivity differences. However, all countries in his model are symmetric in terms of the technologies available.[8] This paper allows for the difference in the countries' access to technology so that countries are no longer symmetric. Analytical results, without having to make specific distributional assumptions, are derived. Factor price equalization is achieved by introducing a homogenous good in both countries with constant return to scale production technology and zero costs of transportation. We consider an economy with two sectors and one production factor, labor. A homogenous good (the numeraire) is produced in the first sector. Firms in the second sector produce a continuum of differentiated goods indexed by z. We model this sector by taking Melitz (2003)

[8] Ghironi and Melitz (2003) and Helpman, Melitz, and Yeaple (2003) also deal with symmetric countries. Bernard, Redding and Schott (2004) develop a heterogeneous agent HOS model and so allow for asymmetries in factor endowments. However, outside the FPE region they have to resort to simulations.

as a starting point. Since all the properties of his model remain valid, we will be relatively terse in the presentation of this part.

2.1 Preferences

There are L consumers in the economy. Each supplies one unit of labor and has the a utility function given by $U = (N)^{1-\beta} (C)^{\beta}$, where $1 > \beta > 0$. N is a homogenous good and $C = \left(\int_{z\in\Omega} q(z)^{\rho} dz\right)^{1/\rho}$ can be thought of as the number of services obtained from consuming $q(z)$ unit of each variety z when there is a mass Ω of available varieties of the differentiated good. The elasticity of substitution between any two differentiated goods is $\sigma = \frac{1}{1-\rho} > 1$. Preference are Cobb Douglas over N and C so that the share of a consumer's income spent on N and C is respectively, $1-\beta$ and β. Denote the price of variety z by $p(z)$. It is easy to verify that the cost of a unit of C defines the perfect price index

$$P = \left[\int_{z\in\Omega} p(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}. \tag{1}$$

As originally shown by Dixit and Stiglitz (1977), that the demand for variety z is given by

$$q(z) = C\left[\frac{p(z)}{P}\right]^{-\sigma}. \tag{2}$$

A simple interpretation is that the demand for a variety is a derived demand, derived from the demand for C. As such, it is the product of the amount of variety z needed to make a unit of C[9] times C.

Using (2) shows that expenditure on variety z,

$$p(z)q(z) = PC\left[\frac{p(z)}{P}\right]^{1-\sigma}. \tag{3}$$

[9] By Shephard's Lemma, the unit input requirement is just the derivative of P with respect to $p(z)$.

where $PC = \int_{z \in \Omega} p(z)q(z)dz$ is the aggregate expenditure on differentiated goods. Note that the share of expenditure on a particular variety depends only on the price of that variety relative to the price index.

2.2 Production and Firm Behavior

The homogeneous good is produced under constant returns to scale and one unit of labor makes a unit of this good. Hence, we can normalize the wage rate and the price of the homogenous good in a closed economy unity. Moreover, as long as this good can be traded freely as we assume throughout, prices and nominal wages in both countries are also unity.[10] The expenditure on and (in a closed economy) the revenue earned is denoted by R^N. The labor used in the two sectors is denoted by L^N and L^C.

The differentiated good sector has a continuum of prospective entrants that are the same ex-ante. To enter, firms pay an entry cost of $f_e > 0$, which is thereafter sunk. Then they draw their productivity from a common distribution $g(\varphi)$ with positive support over $(0, \infty)$ and a continuous cumulative distribution $G(\varphi)$. At each point of time, there is a mass, M_e, of firms that make such a draw. Once a firm knows its productivity, it can choose to produce or exit. If its productivity draw is below a cutoff level, φ^*, it is best off exiting at once.[11] Any firm that stays in the market has a constant per period profit level. A firm exits (due to some unspecified catastrophic shock) with a constant probability δ in each period.[12] We assume that there is no discounting[13] and consider only stationary equilibria. Note that because exit is random, the productivity distribution for successful entrants, exiting incumbents, and hence, for active firms is the same.

[10] Even if unit labor requirements differ, factor price equalization in efficiency units is achieved.

[11] The existence and uniqueness of φ^* will be shown in Section 5.

[12] It would be more plausible to make the probability of exit depend on the firm's productivity. For example, Hopenhayan (1992) models exit caused by series of bad shocks affecting the firm's productivity.

[13] Again, this assumption is made for simplicity.

The productivity distribution of successful entrants in the economy is proportional to the initial productivity distribution with the factor of proportionality being the mass of firms that are alive in the stationary equilibrium denoted by M. In a stationary equilibrium, in every period the mass of new successful entrants should exactly replace the firms who face the bad shock and exit. As a result, we have the aggregate stability condition: $p_{in}M_e = \delta M$, where $p_{in} = 1 - G(\varphi^*)$ is the probability of successful entry. In this manner, M_e and φ^* determine M and φ^* is endogenously determined.

The labor needed to produce q units of a variety is $l(\varphi) = f + q/\varphi$. $f > 0$ is a fixed overhead cost in terms of labor while and $\frac{1}{\varphi}$ is the unit labor requirement of a firm with productivity $\varphi > 0$. All firms have the same fixed costs, but differ in their productivity levels. Due to symmetry, the constant elasticity of substitution form assumed and the fact that their are a continuum of firms, each firm faces a downward sloping demand function with a constant demand elasticity of σ. And as expected in the CED case, it chooses its price so that its marginal revenue, $p(1-\frac{1}{\sigma})$, equals it marginal costs, $\frac{1}{\varphi}$. From this it follows that price is

$$p(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{\varphi}\right) = \frac{1}{\rho\varphi}. \tag{4}$$

Hence, profits are

$$\begin{aligned}
\pi(\varphi) &= r(\varphi) - \frac{p(\varphi)q}{p(\varphi)\varphi} - f && (5)\\
&= \frac{r(\varphi)}{\sigma} - f. && (6)
\end{aligned}$$

Variable profits are thus a constant share of revenue and this share is greater the less the substitutability between varieties. Also note that

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}; \qquad \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}. \tag{7}$$

so that a more productive firm has larger output and revenues, charges a lower price and earns higher profits compared to a firm with the low productivity level.

Only a firm with $\pi(\varphi) \geq 0$ will find it profitable to produce once it has entered. A firm's value function is given by $\max\left\{0, \sum_{t=0}^{\infty} (1-\delta)^t \pi(\varphi)\right\} = \max\left\{0, \frac{1}{\delta}\pi(\varphi)\right\}$. Since $\pi(0) = -f$ is negative, and $\pi(\varphi)$ is increasing in φ, we can determine the lowest productivity level at which a firm will produce (the cutoff level φ^*) by $\pi(\varphi^*) = 0$. Any entering firm drawing a productivity level $\varphi < \varphi^*$ will immediately exit. Therefore, the distribution of productivity in equilibrium, $\mu(\varphi)$, is:

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)}, & \text{if } \varphi \geq \varphi^*, \ \textit{and} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Since each firm produces a unique variety z and draws a productivity φ, with a mass M of firms, the price index is given by

$$P = \left[\int_{\varphi^*}^{\infty} \left[\int_0^M p(z,\varphi)^{1-\sigma} dz\right] \mu(\varphi)\, d\varphi\right]^{\frac{1}{1-\sigma}}.$$

As firms are symmetric ex-ante, p does not depend on z so that $\int_0^M p(z,\varphi)^{1-\sigma} dz = Mp(\varphi)^{1-\sigma}$. Hence

$$P = M^{\frac{1}{1-\sigma}} \left[\int_{\varphi^*}^{\infty} p(\varphi)^{1-\sigma} \mu(\varphi)\, d\varphi\right]^{\frac{1}{1-\sigma}}.$$

Recall that $p(\varphi) = \frac{1}{\rho\varphi}$ and define $\tilde{\varphi}$[14] as:

$$\tilde{\varphi}(\varphi^*) \equiv \left[\int_0^{\infty} \varphi^{\sigma-1} \mu(\varphi)\, d\varphi\right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{1-G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\sigma-1}}, \tag{9}$$

so that

$$P = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi}). \tag{10}$$

[14] The assumption of a finite $\tilde{\varphi}$ requires the $(\sigma-1)^{th}$ un-centered moment of $g(\varphi)$ be finite.

As in Melitz (2003), all aggregate variables can similarly be written in terms of a representative firm, $\tilde{\varphi}$, and M.

$$Q = M^{1/\rho} q\left(\tilde{\varphi}\right), \quad R^C = PQ = Mr\left(\tilde{\varphi}\right) \equiv M\bar{r}, \quad \Pi^C = M\pi\left(\tilde{\varphi}\right) \equiv M\bar{\pi}. \tag{11}$$

where $Q = C = \left(\int_{z\in\Omega} q\left(z\right)^{\rho} dz\right)^{1/\rho}$, $R^C = \int_0^{\infty} r(\varphi)M\mu\left(\varphi\right) d\varphi$ and $\Pi^C = \int_0^{\infty} \pi(\varphi)M\mu\left(\varphi\right) d\varphi$ represent aggregate revenue and profits in the differentiated good sector, $\bar{r}$ and $\bar{\pi}$ represent the average revenue and profit as well as the revenue and profit of the firm with productivity $\tilde{\varphi}$. Note that this allows a heterogeneous firm setting to be transformed to a homogenous firm one where all firms have productivity $\tilde{\varphi}$.

3 Equilibrium in a Closed Economy

To derive the productivity cutoff level φ^* in the equilibrium, we use the free entry (FE) condition:

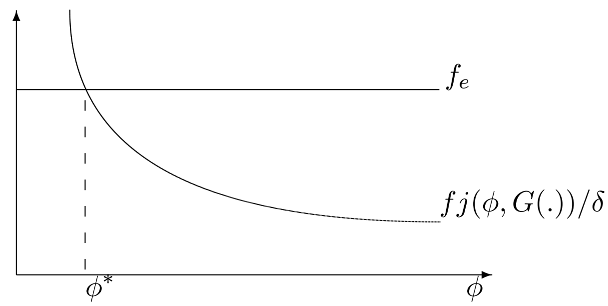
$$\left(1 - G\left(\varphi^*\right)\right) \frac{\bar{\pi}}{\delta} = f_e. \tag{12}$$

The average profit level $\bar{\pi}$ is a function of φ^*: $\bar{\pi} = fk\left(\varphi^*\right)$, where $k\left(\varphi^*\right) = \left[\tilde{\varphi}\left(\varphi^*\right)/\varphi^*\right]^{\sigma-1} - 1$ (see appendix B).Using this formula in (12) and denoting $(1-G\left(\varphi^*\right))k\left(\varphi^*\right)$ by $j(\varphi^*, G\left(\cdot\right))$, we obtain a final equation for φ^*:

$$\frac{f}{\delta} j(\varphi^*, G\left(\cdot\right)) = f_e, \tag{13}$$

where $\frac{f}{\delta} j(\varphi^*, G\left(\cdot\right))$ is the present discounted value of the expected profits upon entering. As shown in Melitz (2003), $\frac{f}{\delta} j(\varphi^*, G\left(\cdot\right))$ is decreasing in φ^* and intersects the f_e line only once. This ensures the existence and uniqueness of φ^*. The solution of (13) does not depend on the labor stock in the economy. Moreover, a graphical representation of (13)

Figure 1: The Closed Economy Equilibrium



in Figure 1 provides a simple way to analyze the changes in φ^* due to changes in the parameters of the model.

Since there are zero profits ex-ante and only one factor, labor, the value added in a sector, or a revenue in this case, equals the value of payments to factors. As a result, the aggregate revenues in both sectors are exogenously fixed by the country size L: $L^N = (1-\beta)L$ and $L^C = \beta L$.

In any period, the mass of firms which produce differentiated goods, is given by $M = R^C/\bar{r} = \beta L/(\sigma(\bar{\pi}+f))$. Note that the larger the country size L, the more firms enter the market. As a result, the price index falls and welfare per worker[15] rises due to an increase in product variety.

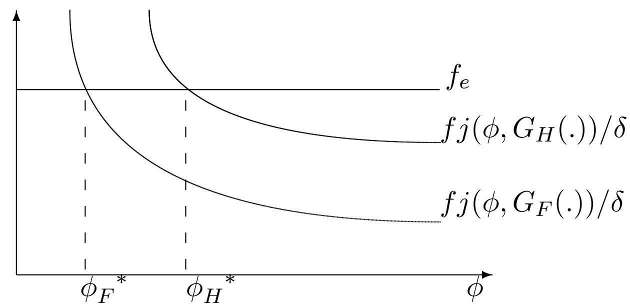
4 Analysis of the Equilibrium

Now we turn to the effect of a better productivity distribution.

Definition 1 *The productivity distribution $G_H(\varphi)$ dominates the productivity distribution $G_F(\varphi)$ in terms of the hazard rate order, $G_H(\cdot) \succ_{hr} G_F(\cdot)$, if for any given productivity level φ, $g_h(\varphi)/(1-G_H(\varphi)) < g_F/(1-G_F(\varphi))$.*

[15] It is determined by the indirect utility function: $W = \left(\frac{(1-\beta)w}{1}\right)^{1-\beta}\left(\frac{\beta w}{P}\right)^{\beta} = \frac{(1-\beta)^{1-\beta}\beta^{\beta}}{P^{\beta}}$.

Figure 2: Two Closed Economies, Home and Foreign



Hazard rate stochastic dominance (HRSD) allows us to compare the expectations of an increasing function above some cutoff level, i.e., if $y(x)$ is increasing in x and $G_H(\cdot) \succ_{hr} G_F(\cdot)$, then for any given level φ, $E_H[y(x) \mid x > \varphi] > E_F[y(x) \mid x > \varphi]$.[16] In terms of our model, this means that for any given level φ, entrants in the home country with the productivity distribution $G_H(\cdot)$ have a better chance of obtaining a productivity draw above this level than do entrants in the foreign country with the productivity distribution $G(\cdot)$. Given this difference, we obtain

Lemma 1 *For any given level* φ*,* $j(\varphi, G_H(\cdot)) > j(\varphi, G_F(\cdot))$

Proof. *See appendix C.* ■

Using Lemma 1 in Figure 2, we conclude that $\varphi_H^* > \varphi_F^*$. Intuitively, since home firms have a better chance of obtaining a productivity above any cutoff level, only more productive firms can survive. As a result, the home country has a lower price index and a higher welfare per worker than the foreign country.

[16] Note that the usual (first-order) stochastic dominance allows us to compare only the unconditional expectations, i.e., if $G_H(\cdot) \succ_{st} G_F(\cdot)$, then $E_H[y(x)] > E_F[y(x)]$. For more detail see Shaked and Shanthikumar (1994).

5 The Open Economy

Trade has two basic effects in an economy: on the one hand, it provides an opportunity to sell in the new market; on the other hand, it brings new competitors from abroad. We consider trade with costs: when firms become exporters, they face new costs, such as transport costs, tariffs, etc. As in Melitz (2003), we assume that both countries have the same size and in each country, after the firm's productivity is revealed, a firm who wishes to export must pay a per-period fixed cost, $f_x > 0$. Per-unit trade costs are modeled in the standard iceberg formulation: $\tau > 1$ units shipped result in 1 unit arriving. Regardless of export status, a firm still incurs the same overhead production cost of f per period.

In order to ensure factor price equalization across countries and to focus our analysis on firm selection effects, we assume that the homogenous good is produced using the same technology in both countries after trade[17], and that its export does not incur transport costs.[18] In the next two sections we consider trade with no specialization.

5.1 Equilibrium in the Open Economy

In each country under trade, the aggregate revenue earned by domestic firms in the differentiated good sector, R_i^C, can differ from the aggregate expenditure on the differentiated goods, E_i^C. (By construction, $R_i^C = \gamma_i L$, where γ_i is the fraction of labor employed in the differentiated good sector in country i, and $E_i^C = \beta L$.[19]).

Since consumers in each country spend a share β of their incomes on the differentiated goods, and as the world expenditure on the differentiated goods equals the revenues earned in this sector, $\gamma_H + \gamma_F = 2\beta$. The export price is $p_x(\varphi) = \tau p(\varphi)$. Using (3), we can write

[17] This requires $2\beta < 1$.

[18] $\tau = 1$ for the homogenous good.

[19] As in the autarky, the aggregate revenue R_i^C in the differentiated good sector equals to the total payment to the labor, i.e., $R_i^C = L_i^C = \gamma_i L$. The total revenue is $R_i = R_i^N + R_i^C = L$, $i = H, F$.

the revenues earned by a firm in country i from domestic sales as $r_i(\varphi) = E_i^C (P_i \rho \varphi)^{\sigma-1}$, $i = H, F$, where P_i denotes the price index in the differentiated good sector. The revenue of a firm in country i is $r_i(\varphi)$, if the firm does not export, and $r_i(\varphi) + r_j(\tau^{-1}\varphi)$, $i \neq j$, if the firm exports. The actual bundle of goods available can differ across countries as not every firm in each country decides to export.

We assume that $G_H \succ_{hr} G_F$ and consider stationary equilibria only. Then, in country i, the profits earned by a firm from sales in the domestic and foreign markets are, respectively,

$$\pi_{di}(\varphi) = \frac{r_i(\varphi)}{\sigma} - f, \qquad \pi_{xi}(\varphi) = \frac{r_j(\tau^{-1}\varphi)}{\sigma} - f_x, \quad i = H, F. \tag{14}$$

Total profits can be written as $\pi_i(\varphi) = \max\{0, \pi_{di}(\varphi)\} + \max\{0, \pi_{xi}(\varphi)\}$. As in autarky, the productivity cutoff levels must satisfy $\pi_{di}(\varphi_i^*) = 0$ and $\pi_{xi}(\varphi_{xi}^*) = 0$.

Assumption 1 *Only firms producing in the domestic market can export, i.e., $\varphi_{xi}^* > \varphi_i^*$.*[20]

Lemma 2 *The productivity cutoff levels in both countries are linked: $\varphi_{xH}^* = A\varphi_F^*$ and $\varphi_{xF}^* = A\varphi_H^*$, where $A = \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$.*

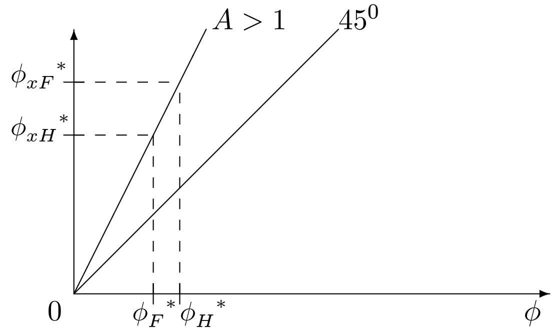
Proof. *See appendix D.* ■

Note that from Assumption 1 and Lemma 2, A should be more than 1. We depict the results of Lemma 2 in the Figure 3.[21] The productivity cutoff level for exporting firms depends on the price index and the mass of domestic firms in the country they export to, which, in turn, depend on the productivity cutoff level for domestic firms in this country.

The ex-ante probabilities of successful entry and being an exporter conditional on successful entry are, respectively, $p_{in,i} = 1 - G(\varphi_i^*)$ and $p_{xi} = [1 - G_i(\varphi_{xi}^*)] / [1 - G_i(\varphi_i^*)]$. The

[20] *See appendix F for the restriction this assumption requires.*
[21] That $\varphi_F^* < \varphi_H^*$ is proved in Lemma 3 below.

Figure 3: The Open Economy Productivity Cutoff Levels



productivity distribution for incumbent firms in country i is $\mu_i(\varphi) = g_i(\varphi) / [1 - G_i(\varphi_i^*)]$ $\forall \varphi \geq \varphi_i^*$ and zero otherwise. Let M_i denote the mass of firms in country i that are alive in the equilibrium. Then the mass of exporting firms and the total mass of varieties available in country i are $M_{xi} = p_{xi} M_i$ and $M_{ti} = M_i + p_{xj} M_j$.

Using (9), we define a representative domestic firm by $\tilde{\varphi}_i \equiv \tilde{\varphi}(\varphi_i^*, G_i(\cdot))$ and a representative exporting firm by $\tilde{\varphi}_{xi} \equiv \tilde{\varphi}(\varphi_{xi}^*, G_i(\cdot))$. The average revenue and profit in country i are

$$\bar{r}_i = r_i(\tilde{\varphi}_i) + p_{xi} r_j\left(\tau^{-1}\tilde{\varphi}_{xi}\right), \qquad \text{and} \qquad \bar{\pi}_i = \pi_{di}(\tilde{\varphi}_i) + p_{xi}\pi_{xi}(\tilde{\varphi}_{xi}). \tag{15}$$

For each country we can write all aggregate variables in terms of $\tilde{\varphi}_{ti}$[22], where:

$$\tilde{\varphi}_{ti} \equiv \left\{ \frac{1}{M_{ti}} \left[M_i \tilde{\varphi}_i^{\sigma-1} + M_{xj}\left(\tau^{-1}\tilde{\varphi}_{xj}\right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}}, \; i = H, F, \; i \neq j. \tag{16}$$

$$\text{Then,} \quad P_i = M_{ti}^{\frac{1}{1-\sigma}} p(\tilde{\varphi}_{ti}), \quad \text{and} \quad E_i^C = M_{ti} r_i(\tilde{\varphi}_{ti}), \quad i = H, F. \tag{17}$$

As in autarky , the FE condition for country i is

$$(1 - G_i(\varphi_i^*)) \frac{\bar{\pi}_i}{\delta} = f_e \tag{18}$$

[22] $\tilde{\varphi}_{ti}$ is a productivity level of the representative firm in country i. Note that in contrast to Melitz (2003), $\bar{r}_i \neq r_i(\tilde{\varphi}_{ti})$ and $\bar{\pi}_i \neq \pi_i(\tilde{\varphi}_{ti})$ because of asymmetric countries.

Using the same technique as before, we can show that

$$\pi_{di}(\varphi_i^*) = 0 \iff \pi_{di}(\tilde{\varphi}_i) = fk_i(\varphi_i^*), \tag{19}$$

$$\pi_{xi}(\varphi_{xi}^*) = 0 \iff \pi_{xi}(\tilde{\varphi}_{xi}) = f_x k_i(\varphi_{xi}^*), \tag{20}$$

where $k_i(\varphi) = [\tilde{\varphi}_i(\varphi)/\varphi]^{\sigma-1} - 1$. Thus, $\bar{\pi}_i$ in an open economy is:

$$\bar{\pi}_i = fk_i(\varphi_i^*) + p_{xi} f_x k_i(\varphi_{xi}^*). \tag{21}$$

For the time being, denote $j(\varphi, G_i(\cdot))$ by $j_i(\varphi)$. Substituting (21) into (18) leads to a system of equations with two unknown variables (see appendix E):

$$\frac{f}{\delta} j_H(\varphi_H^*) + \frac{f_x}{\delta} j_H(A\varphi_F^*) = f_e, \tag{22}$$

$$\frac{f}{\delta} j_F(\varphi_F^*) + \frac{f_x}{\delta} j_F(A\varphi_H^*) = f_e, \tag{23}$$

where $j_i(\cdot)$ is a decreasing function. The left-hand side of equation (22) (equation (23)) is the present discounted values of the expected profits earned by a firm in the home (foreign) country considering entry into the market.

Assumption 2 *Trade results in no specialization.*

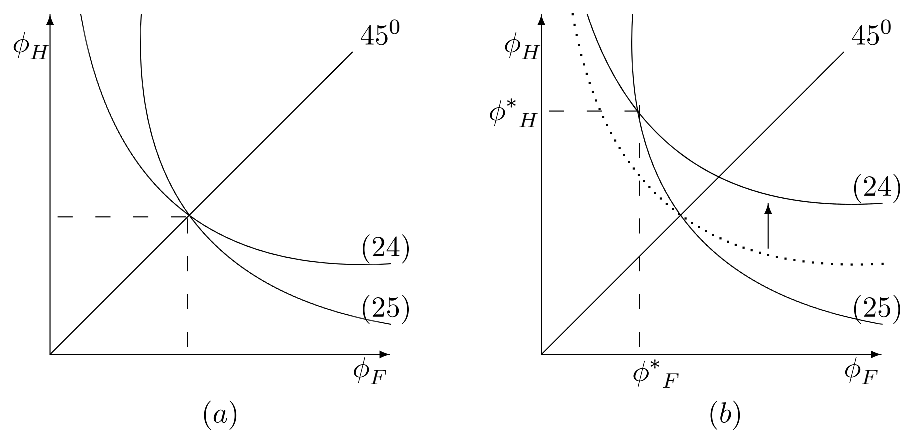
Assumption 2 requires $f_e < \frac{f}{\delta} j_F\left(\frac{1}{A} j_H^{-1}\left(\frac{\delta f_e}{f_x+f}\right)\right) + \frac{f_x}{\delta} j_F\left(A j_H^{-1}\left(\frac{\delta f_e}{f_x+f}\right)\right)$, otherwise only one country produces the differentiated goods and the other one specializes in the production of the homogenous good. (See appendix F for the proof.)

Lemma 3 *If Assumption 1 and Assumption 2 hold, there exists a unique solution* $(\varphi_H^*, \varphi_F^*)$ *of (22) and (23). Moreover,* $\varphi_F^* < \varphi_H^* < \varphi_{xH}^* < \varphi_{xF}^*$.

Proof. *The sketch of a proof is following.*[23] *First, for any productivity levels* φ_H *and* φ_F,

[23] *See appendix F for the complete proof.*

Figure 4: Proof of Lemma 3



we express φ_H as a function of φ_F, using (22) and (23):

$$(22) \implies \varphi_H = j_H^{-1}\left(\frac{\delta f_e}{f} - \frac{f_x}{f} j_H(A\varphi_F)\right), \tag{24}$$

$$(23) \implies \varphi_H = \frac{1}{A} j_F^{-1}\left(\frac{\delta f_e}{f_x} - \frac{f}{f_x} j_F(\varphi_F)\right). \tag{25}$$

Then, we can plot both functions in the same figure and find the equilibrium pair $(\varphi_H^*, \varphi_F^*)$ as an intersection of two curves. Note that both curves are decreasing in φ_F and for any pair of distributions $G_H(\cdot)$ and $G_F(\cdot)$, $G_H(\cdot) = G_F(\cdot)$, the curve corresponding to equation (24) is flatter than the curve corresponding to equation (25). Moreover, the intersection of two curves lies on the 45^0 line as shown in Figure 4(a).

Finally, we can show that if the productivity distribution in a country improves (worsens) in terms of HRSD, the curve corresponding to the equation for this country becomes flatter (steeper) and shifts up (down). In particular, if the home country has a better distribution in terms HRSD ($G_H(\cdot) \succ_{hr} G_F(\cdot)$), the curve corresponding to equation (24) shifts up as shown in Figure 4(b) and in the equilibrium, $\varphi_F^* < \varphi_H^*$. From Lemma 2, $\varphi_{xi}^* = A\varphi_j^*$, $i \neq j$, which leads us to $\varphi_F^* < \varphi_H^* < \varphi_{xH}^* < \varphi_{xF}^*$. ■

The resulting productivity cutoff levels are depicted in Figure 3. Ex-ante, home firms receive productivity draws from a better distribution. As a result, the home productivity cutoff level for surviving firms, φ_H^*, is higher than φ_F^*. However, while making an export decision, home firms face less severe competition abroad compared to that faced by foreign firms in the home country. Thus, $\varphi_{xH}^* < \varphi_{xF}^*$.

Given φ_H^* and φ_F^*, we can write the trade balance equation and derive γ_H and γ_F, the shares of labor in the differentiated good sectors in both countries, as the functions of φ_H^* and φ_F^*. (See appendix H for details.)

Lemma 4 *If Assumption 1 and Assumption 2 hold, then the home country imports the homogenous good and exports the differentiated goods. The foreign country also exports the differentiated goods, but unlike the home country, it exports the homogenous good as well.*

Proof. *See appendix H.* ■

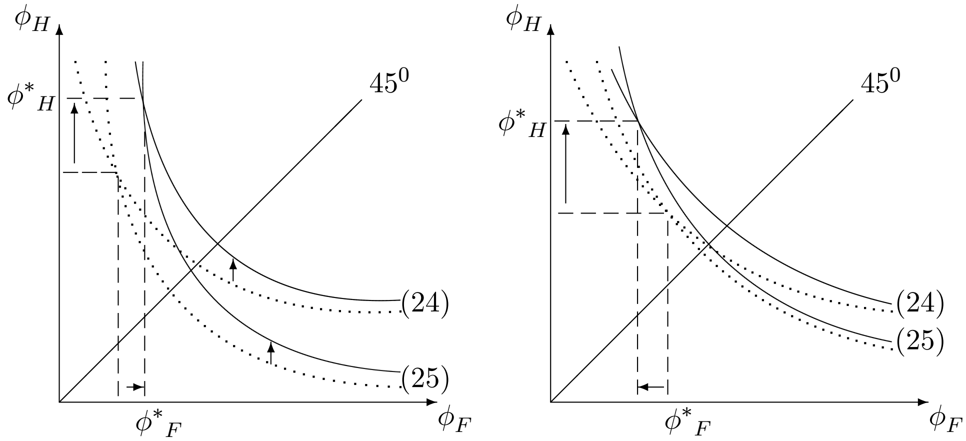
Having φ_H^* and φ_F^*, we obtain $\bar{\pi}_i$ and $M_i = \frac{R_i^C}{\bar{r}_i} = \frac{\gamma_i L}{\sigma(\bar{\pi}_i + f + p_{xi} f_x)}$. In turn, this determines the price index and the mass of variety available in each country. Note that from (17), the price index in country i depends on the average productivity there, $\tilde{\varphi}_{ti}$, and the mass of variety available, M_{ti}. In turn, M_{ti} depends on $\tilde{\varphi}_{ti}$ and the productivity cutoff level φ_i^*. This allows us to write P_i as a function of φ_i^* (see equation (40)) and the welfare per worker as:

$$W_i = \frac{(1-\beta)^{1-\beta}\,\beta^\beta}{P_i^\beta} = (1-\beta)^{1-\beta}\,\beta^\beta \left(\frac{\beta L}{\sigma f}\right)^{\beta/(\sigma-1)} (\rho\varphi_i^*)^\beta . \quad (26)$$

Thus, comparative advantage in the differentiated good sector at home (a better distribution in terms of HRSD) leads to a greater technological potential and a higher welfare per worker at home than abroad.[24]

[24] Note that both countries gain from trade compared to autarky.

Figure 5: A Fall in Trade Cost.



(a) Both Home and Foreign Countries Gain.

(b) Home Country Gains, Foreign Country Loses.

Note that a fall in the per-unit trade cost τ as a consequence of globalization shifts both curves corresponding to equations (24) and (25) up and makes them steeper. As a result, φ_H^* (and, consequently, φ_{xF}^*) increases.[25] However, as shown in Figure 5, φ_F^* (and, consequently, φ_{xH}^*) may increase or decrease. In other words, there is a possibility of welfare loss in the less developed country. Intuitively, when identical countries draw from the same distribution, as in Melitz (2003), we know that a fall in trade costs raises both countries welfare. A fall in transport costs creates more export opportunities, which intensifies competition, and this raises the cutoff level and hence welfare. However, this result is crucially dependent on symmetry all around. As everything is continuous, when countries draw from similar distributions, Melitz (2003) result must go through. However, when countries draw from very different distributions, the backward one can lose. All firms lose a part of their domestic market, but exporting firms more than make up for this loss. However, when home firms are more advanced, the home market is a tougher one for

[25] An increase in φ_H^* can be shown mathematically using equation (38) from appendix F. ($\psi(\varphi_H^*)$ decreases as A falls.)

foreign firms than vice versa. Hence, home firms expand at the expense of foreign ones. As not all firms export, the productivity cutoff level (and hence welfare) at home rises while that abroad falls. Now, we obtain our first result:

Proposition 1 *In the absence of specialization, falling trade costs raise welfare in the advanced country. The laggard country may gain or lose: it must gain if it is not too different from its trade partner and can lose if it is very backward.*[26]

Proposition 1 offers an explanation of why globalization may adversely affect developing countries whose technology is likely to be dominated by that of the developed world.

5.2 Trade and Productivity Improvement

How does technological progress in a country affect its trading partner? What is the effect of productivity improvement in a trading partner on welfare in each country? To answer this question, we use the same technique as in the proof of Lemma 4: productivity improvement in terms of HRSD in the foreign country flattens the curve corresponding to equation (25) and shifts it up as shown in Figure 6(a). Thus, we proved Lemma 5:

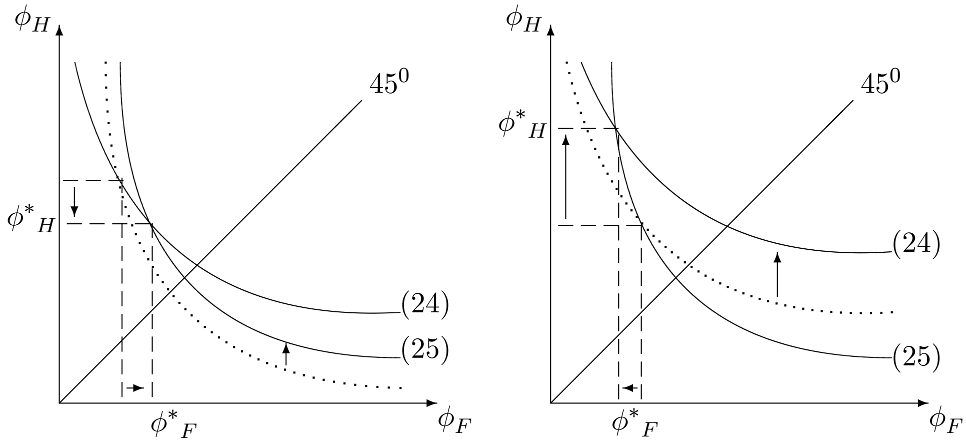
Lemma 5 *In the absence of specialization, the productivity improvement in terms of HRSD in the foreign country raises* φ_F^* *and* $\varphi_{xH}^* = A\varphi_F^*$*, and reduces* φ_H^* *and* $\varphi_{xF}^* = A\varphi_H^*$*.*[27]

The interpretation of this result is that when the foreign country faces the productivity improvement, firms there have a better chance of receiving a high productivity draw.

[26] *An example of the decrease of* φ_F^* *in the case of the Pareto distribution is shown in appendix L.*

[27] *Productivity improvements may result in Assumption 1 and/or Assumption 2 being violated. Note that we exclude this case from our analysis as we assume both Assumption 1 and Assumption 2 hold true after the productivity improvement. However, there exist parameter values where the entire range depicted in Figure 7 occurs. (See appendix I for an example.)*

Figure 6: Proof of Lemma 5



(a) Productivity Improvement in the Foreign Country

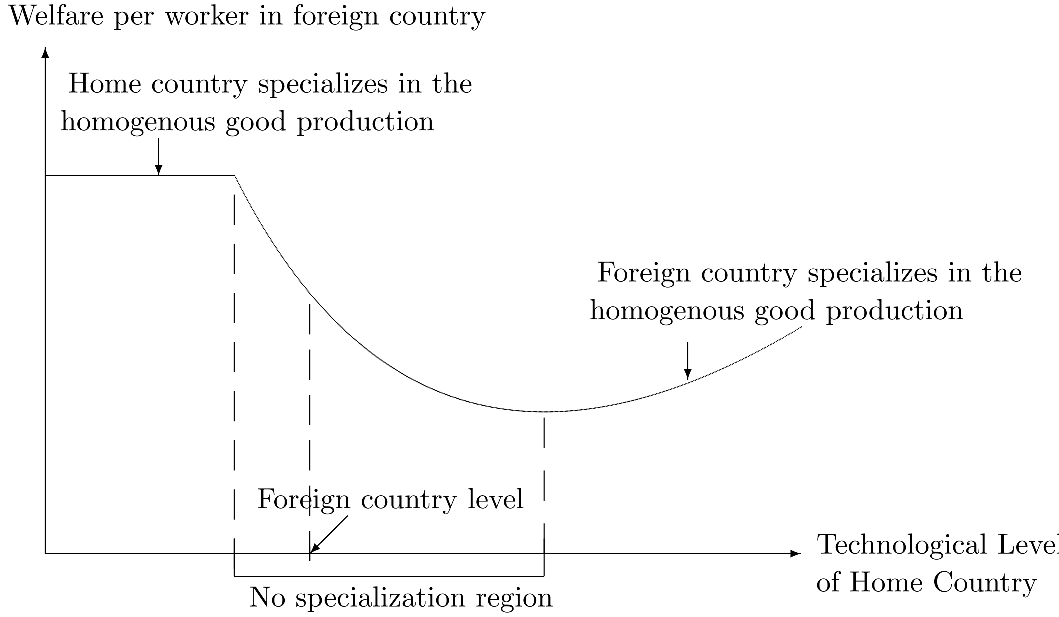
(b) Productivity Improvement in the Home Country

Therefore, some foreign firms with low productivity levels, which survive before, exit and φ_F^* rises. As for the home country, a more competitive foreign market decreases the present discounted value of the expected profits of firms at home. Thus, fewer firms enter the market and the productivity cutoff level φ_H^* falls.

In the absence of specialization, in both cases trade occurs according to Lemma 4. Productivity improvement in the foreign country leads to the fall in the volume of trade. In particular, the home country produces and exports fewer differentiated goods and the foreign country produces and exports less homogenous good. (See appendix J.)

The productivity improvement in the foreign country raises the technological potential there while reducing it at home. Hence, the foreign country gains from its technological progress, and the home country loses. Note that using the same technique, we can show that technology improvement in the home country makes the gap between the countries larger. (See Figure 6(b).) Thus, the foreign country loses. An explanation of why we have this result, which, as pointed out earlier, is at odds with usual intuition, is that productivity improvement at home increases welfare there because more firms enter the market and

Figure 7: Welfare per Worker in Foreign Country



the variety of products at home rises. However, in the foreign country, consumers face a fall in the variety available.[28] Proposition 2 summarizes our main results.

Proposition 2 *In the absence of specialization, productivity improvement in one country raises the productivity cutoff level there while reducing it in the other country. As a result, consumers in the country, which makes the progress and raises its technological potential, gain, while consumers in the other country lose.*

Figure 7 depicts our conclusion about the relationship between welfare per worker in the foreign country and the technological level of its trading partner. We show that in the absence of specialization, productivity improvement in the home country decreases welfare per worker in the foreign country: a fall in the domestic variety in the foreign country is not fully compensated for by the increase in the importing variety from abroad. Thus, while the home country gains from its productivity improvement, the foreign country loses.

The next section presents the results of the trade with specialization.

[28] An increase in M_{tH} and a decrease in M_{tF} can be shown analytically or by using simulations.

5.3 Open Economy: Specialization

As was shown in Section 5.2, productivity improvement in the leading home country or productivity deterioration in the less developed foreign country raises the share of the home country in the production of the differentiated goods. At some point, the gap between the two countries becomes large enough to make the foreign country specialize in the homogenous good, while the home country produces and exports the differentiated goods[29], and the productivity cutoff levels for domestic producers and exporters there, φ_H^* and φ_{xH}^*, determine the price indices, volume of trade, and welfare in both countries. (For a complete description of the equilibrium see appendix K.) A difference between trade with no specialization and the case in this section is that now welfare at home does not depend on the productivity distribution in the foreign country and the foreign country gains from productivity improvement at home. This increase in welfare in the foreign country is shown in the right part of Figure 7. The horizontal part in Figure 7 corresponds to the case of the home country specialization in the homogenous good, in which the welfare in the foreign country depends only on its own productivity distribution.

5.4 Results for Pareto Productivity Distribution.

In this section we will show that in the case of the Pareto productivity distribution the assumption of HRSD can be relaxed by using the usual (first order) stochastic dominance (USD) instead.[30] Assume that the Pareto productivity distribution is given by $G_i(\varphi) = 1 - \left(\frac{\varphi_{\min,i}}{\varphi}\right)^{k_i}$, where $\varphi > \varphi_{\min,i}$, $k_i > \sigma - 1$, $i = H, F$. The hazard rate for the Pareto distribution is $\frac{g_i(\varphi)}{1-G_i(\varphi)} = \frac{k_i}{\varphi}$. Therefore, if $k_H < k_F$ (or $k_H > k_F$), i.e., the productivity

[29] In terms of our model, this means that $\gamma_H = 2\beta$ and $\gamma_F = 0$.

[30] Note that HRSD implies USD, however, the opposite is not always true.

distribution in the home country dominates that in the foreign country in terms of HRSD, $G_H(\cdot) \succ_{hr} G_F(\cdot)$ (or $G_H(\cdot) \prec_{hr} G_F(\cdot)$), then lemmas 3 and 4 and propositions 1 and 2 can be used to describe the equilibrium in the economy and the effects of productivity improvements and a fall in trade cost on welfare in both countries.

We need to consider the case when $k_H = k_F = k$, but $\varphi_{\min,H} > \varphi_{\min,F}$, i.e., the productivity distribution in the home country dominates that in the foreign country in terms of USD, however, the productivity distributions in both countries are equivalent in terms of HRSD. In this case the system of equilibrium equations can be written as

$$\frac{\sigma-1}{k-(\sigma-1)}\left(\varphi_{\min,H}\right)^k\left[\frac{f}{\delta}(\varphi_H^*)^{-k}+\frac{f_x}{\delta}(A\varphi_F^*)^{-k}\right] = f_e, \tag{27}$$

$$\frac{\sigma-1}{k-(\sigma-1)}\left(\varphi_{\min,F}\right)^k\left[\frac{f}{\delta}(\varphi_F^*)^{-k}+\frac{f_x}{\delta}(A\varphi_H^*)^{-k}\right] = f_e. \tag{28}$$

Using similar techniques as before, it can be shown straightforwardly that the properties of the system of equations (27) and (28) are the same as those of the system of equations (22) and (23) under the HRSD assumption. (See appendix L for the proof.) Thus, in the case of the Pareto productivity distribution, the assumption of HRSD can be replaced by the assumption of USD and the results remain the same.

6 Conclusion

We develop a stochastic, general equilibrium model of international trade between two asymmetric countries, one of which has a comparative advantage over another in terms of the productivity distribution. We derive our results without resorting to simulations or imposing strong restrictions on the model. We show that in the absence of specialization, falling trade costs may hurt the laggard country while helping the advanced one. Moreover, productivity improvement in one country increases its technological potential and welfare but hurts its trading partner. In contrast, if a country is the only producer of the differentiated goods (the other one specializes in the homogenous good), then its welfare does not depend on the productivity distribution in the differentiated good sector abroad and the laggard country gains from productivity improvement in the advanced country.

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Appendix A

Let's consider the same model as one described in this paper, but now assume that in each country, firms are homogeneous and the only difference between two countries is in the technology used to produce the differentiated goods: in country i, all firms have the same productivity level φ_i and cost function is given by $l(\varphi_i) = f + \frac{q}{\varphi_i}$, $i = H, F$. Assume that firms at home are more productive than those in the foreign country: $\varphi_H > \varphi_F$. We assume that there is a free entry and only the firms, which produce in the domestic market, can export. Trade with no specialization is possible if $\tau^{\sigma-1} f_x = f$. By using the same technique as in this paper, we can show that welfare in country i is given by $W_i = \frac{(1-\beta)^{1-\beta}\beta^{\beta}L}{P_i}$, where $P_i = \left(\frac{\beta L}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\varphi_i}$. Thus, in the absence of specialization, productivity improvement in country i (φ_i increases) raises the welfare there but does not change the welfare of its trading partner. Moreover, in the case of specialization, productivity growth in country i is beneficial to both countries.

Appendix B

By definition, $\pi(\varphi^*) = \frac{r(\varphi^*)}{\sigma} - f = 0$ or $r(\varphi^*) = \sigma f$. From (7), $r(\tilde{\varphi}) = r(\varphi^*)\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} = f\sigma\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1}$. Thus, $\bar{\pi} = \pi(\tilde{\varphi}) = \frac{r(\tilde{\varphi})}{\sigma} - f = fk(\varphi^*)$, where $k(\varphi^*) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1$.

Appendix C

Using (9), we can write $j(\varphi^*, G_i(\cdot))$ as

$$j_i(\varphi^*) \equiv j(\varphi^*, G_i(\cdot)) = \frac{1}{(\varphi^*)^{\sigma-1}} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g_i(\varphi)\, d\varphi - [1 - G_i(\varphi^*)] \tag{29}$$

$$= [1 - G_i(\varphi^*)]\left(E_i\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \mid \varphi > \varphi^*\right] - 1\right), \quad i = H, F. \tag{30}$$

Thus, for any given level φ^*,

$$\begin{aligned} j_H - j_F &= [1 - G_H(\varphi^*)] \left(E_H \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} \mid \varphi > \varphi^* \right] - 1 \right) - \\ & [1 - G_F(\varphi^*)] \left(E_F \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} \mid \varphi > \varphi^* \right] - 1 \right). \end{aligned}$$

$G_H(\cdot) \succ_{hr} G_F(\cdot)$, then, for any given level φ^*, $1 - G_H(\varphi^*) > 1 - G_F(\varphi^*)$. Moreover, since $\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1}$ is increasing in φ, $E_H\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \mid \varphi > \varphi^*\right] > E_F\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \mid \varphi > \varphi^*\right]$. Note that $E_i\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} \mid \varphi > \varphi^*\right] > 1$, $i = H, F$. Therefore, $j_H - j_F > 0$.

Appendix D

Recall that $r_i(\varphi) = \beta L (P_i \rho \varphi)^{\sigma-1}$, $r_{xi}(\varphi) = \tau^{1-\sigma} r_j(\varphi)$, $i \neq j$, $r_i(\varphi_i^*) = \sigma f$, and $r_{xi}(\varphi_{xi}^*) = \sigma f_x$. Define $A \equiv \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$. Then we have:

$$\frac{r_H(\varphi_H^*)}{r_F(\varphi_F^*)} = 1, \Longrightarrow \frac{\varphi_H^*}{\varphi_F^*} = \frac{P_F}{P_H}; \quad \frac{r_{xH}(\varphi_{xH}^*)}{r_{xF}(\varphi_{xF}^*)} = 1, \Longrightarrow \frac{\varphi_{xH}^*}{\varphi_{xF}^*} = \frac{P_H}{P_F}, \tag{31}$$

$$\frac{r_{xi}(\varphi_{xi}^*)}{r_i(\varphi_i^*)} = \frac{f_x}{f}, \Longrightarrow \frac{\varphi_{xH}^*}{\varphi_H^*} = A\frac{P_H}{P_F}; \quad \text{and} \quad \frac{\varphi_{xF}^*}{\varphi_F^*} = A\frac{P_F}{P_H}. \tag{32}$$

$$\text{Thus,} \quad \varphi_{xH}^* = A\varphi_F^*, \quad \varphi_{xF}^* = A\varphi_H^*. \tag{33}$$

Appendix E

As in autarky, substituting (21) in (18) for each country leads to the system:

$$\frac{f}{\delta}(1 - G_H(\varphi_H^*))k_H(\varphi_H^*) + \frac{f_x}{\delta}(1 - G_H(\varphi_{xH}^*))k_H(\varphi_{xH}^*) = f_e, \tag{34}$$

$$\frac{f}{\delta}(1 - G_F(\varphi_F^*))k_F(\varphi_F^*) + \frac{f_x}{\delta}(1 - G_F(\varphi_{xF}^*))k_F(\varphi_{xF}^*) = f_e. \tag{35}$$

Using the definition of $j_i(\varphi)$ and Lemma 2, we obtain (22) and (23) from the system above.

Appendix F

First, let's assume that $G_H(\cdot) \succcurlyeq_{hr} G_F(\cdot)$. (The productivity distribution at home is the same as that in the foreign country or dominates it in terms of HRSD.) First, note that the function $j_i(\varphi)$, $i = H, F$, is a decreasing function of φ.[31] Thus, both curves corresponding to equations (24) and (25) are decreasing in φ_F. We need to compare the slopes of these curves at each point:

$$\left| -\frac{f_x}{f} A \frac{j_H'(A\varphi_F^*)}{j_H'\left(j_H^{-1}\left(\frac{\delta f_e}{f} - \frac{f_x}{f} j_H(A\varphi_F)\right)\right)} \right| \gtrless \left| -\frac{f}{f_x} \frac{1}{A} \frac{j_F'(\varphi_F^*)}{j_F'\left(j_F^{-1}\left(\frac{\delta f_e}{f_x} - \frac{f}{f_x} j_H(\varphi_F)\right)\right)} \right|$$

$$\text{or} \quad \left(\frac{f_x}{f}\right)^2 \gtrless \frac{|j_H'(\varphi_H^*)|}{A\left|j_H'(A\varphi_F^*)\right|} \cdot \frac{|j_F'(\varphi_F^*)|}{A\left|j_F'(A\varphi_H^*)\right|}. \tag{36}$$

Using the formula for $j_i'(\varphi)$, $j_i'(\varphi) = -\frac{1}{\varphi}(\sigma - 1)\varphi^{1-\sigma}\int_\varphi^\infty x^{\sigma-1} g_i(x)\,dx$, and Assumption 1, we obtain

$$\frac{|j_i'(\varphi_i^*)|}{A\left|j_i'\left(A\varphi_j^*\right)\right|} = \frac{(\varphi_i^*)^{-\sigma}\int_{\varphi_i^*}^\infty x^{\sigma-1} g_i(x)\,dx}{A^{1-\sigma}\left(\varphi_j^*\right)^{-\sigma}\int_{A\varphi_j^*}^\infty x^{\sigma-1} g_i(x)\,dx} \equiv A^{\sigma-1}\frac{(\varphi_i^*)^{-\sigma}}{\left(\varphi_j^*\right)^{-\sigma}} Q_i, \tag{37}$$

where $Q_i > 1$. We can rewrite (36) as $\left(\frac{f_x}{f}A^{1-\sigma}\right)^2 \gtrless Q_1 Q_2$. By definition, $A = \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$. Thus, $\left(\frac{f_x}{f}A^{1-\sigma}\right)^2 = \left(\tau^{1-\sigma}\right)^2 < 1 < Q_1 Q_2$. Now we proved that the curve corresponding to equation (24) is flatter than the curve corresponding to equation (25). (See Figure 4(a).)

Second, if the home country faces the productivity improvement, i.e., $G_{H,A}(\cdot) \succ_{hr} G_{H,B}(\cdot)$, then from Lemma 1, $j_{H,A}(\varphi) > j_{H,B}(\varphi)$ for any φ. Using this result and recalling that $j_{H,n}(\varphi)$, $n = A, B$, is decreasing in φ, we obtain

$$j_{H,A}^{-1}\left(\frac{\delta f_e}{f} - \frac{f_x}{f} j_{H,A}(A\varphi_F)\right) > j_{H,B}^{-1}\left(\frac{\delta f_e}{f} - \frac{f_x}{f} j_{H,B}(A\varphi_F)\right).$$

[31] $j'(\varphi) = -\frac{1}{\varphi}(\sigma - 1)[1 - G(\varphi)][k(\varphi) + 1] < 0$. (See Melitz (2003).)

Thus, the curve corresponding to equation (24) shifts up and in the equilibrium, $\varphi_F^* < \varphi_H^* < \varphi_{xH}^* < \varphi_{xF}^*$. (See Figure 4(b).) The similar result can be proved in the case when the foreign country faces the productivity improvement, i.e., $G_{F,A}(\cdot) \succ_{hr} G_{F,B}(\cdot)$.

Finally, we discuss the restrictions imposed on parameters to ensure the existence of the equilibrium. We start with Assumption 2, which means that in the equilibrium with no specialization, both countries produce the differentiated goods, thus, both (22) and (23) should hold. Therefore, from (22), we derive $\varphi_F^* \equiv s(\varphi_H^*) = \frac{1}{A} j_H^{-1}\left(\frac{\delta f_e - f j_H(\varphi_H^*)}{f_x}\right)$ and substitute it in (23) to obtain an equation just for φ_H^* :

$$\psi(\varphi_H^*) \equiv \frac{f}{\delta} j_F(s(\varphi_H^*)) + \frac{f_x}{\delta} j_F(A\varphi_H^*) = f_e. \tag{38}$$

Note that $\psi'(\varphi_H^*) > 0$. (We can use the same technique as we used to compare the slopes of curves corresponding to (24) and (25) to prove it.) From Assumption 1, $\varphi_H^* < \varphi_{xH}^* = A\varphi_F^*$. Moreover, from (24), $\varphi_H^* < j_H^{-1}\left(\frac{\delta f_e}{f_x + f}\right)$. Therefore, we can derive the necessary condition for Assumption 2: the solution of (38) exists only if $f_e < \psi\left(j_H^{-1}\left(\frac{\delta f_e}{f_x+f}\right)\right)$ or $f_e < \frac{f}{\delta} j_F\left(\frac{1}{A} j_H^{-1}\left(\frac{\delta f_e}{f_x+f}\right)\right) + \frac{f_x}{\delta} j_F\left(A j_H^{-1}\left(\frac{\delta f_e}{f_x+f}\right)\right)$.

Assumption 1 implies that for any i and j, $i \neq j$, $\frac{\varphi_i^*}{\varphi_j^*} < A$. We proved that in the equilibrium, $\varphi_H^* > \varphi_F^*$. Thus, Assumption 1 requires $\frac{\varphi_H^*}{\varphi_F^*} < A$. ($\frac{\varphi_F^*}{\varphi_H^*} < A$ follows from it.) From (38), $\varphi_H^* = \psi^{-1}(f_e)$. Recalling that $\varphi_F^* = s(\varphi_H^*)$, we derive the necessary condition for Assumption 1:

$$\frac{\psi^{-1}(f_e)}{s\left(\psi^{-1}(f_e)\right)} < A. \tag{39}$$

Appendix G

By definition, $M_{ti} = \beta L / r_i\left(\tilde{\varphi}_{ti}\right)$, where $r_i\left(\tilde{\varphi}_{ti}\right) = r_i\left(\varphi_i^*\right)\left(\frac{\tilde{\varphi}_{ti}}{\varphi_i^*}\right)^{\sigma-1} = \sigma f\left(\frac{\tilde{\varphi}_{ti}}{\varphi_i^*}\right)^{\sigma-1}$. As a result, formula (17) can be written as

$$P_i = \left(\frac{\beta L}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho \varphi_i^*}. \tag{40}$$

Appendix H

Given φ_H^* and φ_F^*, we can write the trade balance equation

$$p_{xH} M_H r_F\left(\tau^{-1}\tilde{\varphi}_{xH}\right) + \left(1-\gamma_H\right) L - \left(1-\beta\right) L = p_{xF} M_F r_H\left(\tau^{-1}\tilde{\varphi}_{xF}\right). \tag{41}$$

By using $M_i = \frac{R_i^C}{\bar{r}_i} = \gamma_i L / \bar{r}_i$, $i = H, F$, in the trade balance equation (41) and denoting $\frac{r_i(\tilde{\varphi}_i)}{p_{xi} r_j\left(\tau^{-1}\tilde{\varphi}_{xi}\right)}$ by b_i, we obtain the following expression for γ_H:

$$\gamma_H = \beta \frac{\left(b_F - 1\right)\left(b_H + 1\right)}{b_H b_F - 1} = \beta\left(1 + \frac{b_F - b_H}{b_H b_F - 1}\right). \tag{42}$$

By construction, $\gamma_F = 2\beta - \gamma_H$.

To prove that $\gamma_H > \beta$ (home country exports the differentiated goods), we need to show that $b_H b_F > 1$ and $b_F > b_H$. Using formula (40) and given that $r_i\left(\varphi\right) = E_i^C\left(P_i \rho \varphi\right)^{\sigma-1}$, we obtain:

$$b_i = \tau^{\sigma-1}\frac{1}{p_{xi}}\left(\frac{\tilde{\varphi}_i}{\varphi_i^*} * \frac{\varphi_j^*}{\tilde{\varphi}_{xi}}\right)^{\sigma-1} = \tau^{\sigma-1}\frac{\left(\varphi_i^*\right)^{1-\sigma}\int_{\varphi_i^*}^{\infty} x^{\sigma-1} g_i\left(x\right) dx}{\left(\varphi_j^*\right)^{1-\sigma}\int_{A\varphi_j^*}^{\infty} x^{\sigma-1} g_i\left(x\right) dx}.$$

Thus, $b_F b_H > \tau^{2\sigma-2} > 1$. To prove that $b_F > b_H$, we rewrite b_i as $b_i = \tau^{\sigma-1} A^{1-\sigma}\frac{a_i\left(\varphi_i^*\right)}{a_i\left(A\varphi_j^*\right)} = \frac{f}{f_x}\frac{a_i\left(\varphi_i^*\right)}{a_i\left(A\varphi_j^*\right)}$, $i \neq j$, where $a_i\left(\varphi\right) \equiv \varphi^{1-\sigma}\int_{\varphi}^{\infty} x^{\sigma-1} g_i\left(x\right) dx$ is decreasing in φ.[32] Using Lemma 2, we find that $b_F = \frac{f}{f_x}\frac{a_F\left(\varphi_F^*\right)}{a_F\left(A\varphi_H^*\right)} > \frac{f}{f_x}\frac{a_F\left(\varphi_H^*\right)}{a_F\left(A\varphi_F^*\right)}$. We want to show that $\frac{f}{f_x}\frac{a_F\left(\varphi_H^*\right)}{a_F\left(A\varphi_F^*\right)} > b_H = \frac{f}{f_x}\frac{a_H\left(\varphi_H^*\right)}{a_H\left(A\varphi_F^*\right)}$. To do this, we compare the elasticities of the decreasing functions $a_F\left(\cdot\right)$ and

[32] $a_i'\left(\varphi\right) = \left(1-\sigma\right)\varphi^{-\sigma}\int_{\varphi}^{\infty} x^{\sigma-1} g_i\left(x\right) dx - g_i\left(\varphi\right) < 0.$

$a_H(\cdot)$, or, respectively, ε_F and ε_H, and prove that $\varepsilon_F > \varepsilon_H$.

$$a_i'(\varphi) = \frac{1-\sigma}{\varphi} a_i(\varphi) - g_i(\varphi) \Longrightarrow \varepsilon_i(\varphi) = -\frac{a_i'(\varphi)}{a_i(\varphi)}\varphi = (\sigma-1) + \varphi\frac{g_i(\varphi)}{a_i(\varphi)}.$$

$$\frac{g_i(\varphi)}{a_i(\varphi)} = \frac{g_i(\varphi)}{1-G_i(\varphi)}\left(\frac{\varphi^{1-\sigma}}{1-G_i(\varphi)}\int_{\varphi}^{\infty} x^{\sigma-1} g_i(x)\,dx\right)^{-1}.$$

HRSD implies that $\frac{1}{1-G_H(\varphi)}\int_{\varphi}^{\infty} x^{\sigma-1} g_H(x)\,dx > \frac{1}{1-G_F(\varphi)}\int_{\varphi}^{\infty} x^{\sigma-1} g_F(x)\,dx$ and $\frac{g_F(\varphi)}{1-G_F(\varphi)} > \frac{g_H(\varphi)}{1-G_H(\varphi)}$. Thus, $g_F(\varphi)/a_F(\varphi) > g_H(\varphi)/a_H(\varphi)$, $\varepsilon_F > \varepsilon_H$, $b_F > b_H$, and $\gamma_H > \beta$. Thus, $\gamma_F < \beta$. This proves Lemma 4.

Appendix I

Assume that both home and foreign countries have Pareto productivity distributions: $G_i(\varphi) = 1 - \left(\frac{0.1}{\varphi}\right)^{k_i}$, where $\varphi > 0.1$ and $k_H = k_F = 6$. Let $f = 40$, $f_x = 70$, $f_e = 1000$, $\tau = 1.3$, $\sigma = 3.8$, $\delta = 0.025$, $\beta = \frac{1}{3}$, and $L = 1$. A decrease (an increase) in k_F results in $G_F(\cdot) \succ_{hr} G_H(\cdot)$ $(G_H(\cdot) \succ_{hr} G_F(\cdot))$. It can be shown that for these parameters, varying k_F yields the entire range depicted in Figure 7 occurs.

Appendix J

The homogenous and differentiated good exports from the foreign country are, respectively, $[\gamma_H - \beta] L = \beta L \frac{b_F - b_H}{b_H b_F - 1}$ and $(2\beta - \gamma_H) L \frac{1}{1+b_F} = \beta L \frac{b_H - 1}{b_H b_F - 1}$. The export of differentiated goods from the home country and the volume of trade are $\gamma_H L \frac{1}{1+b_H} = \beta L \frac{b_F - 1}{b_H b_F - 1}$. By construction, $b_H(\cdot)$ is decreasing in φ_H^*, whereas b_F is increasing in φ_H^*. The trade comparison is straightforward, if we take the derivatives of γ_H and export functions with respect to φ_H^* and recall that $b_F > b_H > 1$, $b_H b_F > 1$, and φ_H^* falls when the foreign country faces the productivity improvement.

Appendix K

If $\gamma_H = 2\beta$ and $\gamma_F = 0$, then $P_H = (M_H)^{\frac{1}{\sigma-1}} p(\tilde{\varphi}_H)$ and $P_F = \tau (p_{xH} M_H)^{\frac{1}{\sigma-1}} p(\tilde{\varphi}_{xH})$.

$$\frac{\varphi_{xH}^*}{\varphi_H^*} = A\frac{P_H}{P_F} = \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \frac{\left[\int_{\varphi_H^*}^{\infty} \varphi^{\sigma-1} g_H(\varphi)\, d\varphi\right]^{\frac{1}{1-\sigma}}}{\left[\int_{\varphi_{xH}^*}^{\infty} \varphi^{\sigma-1} g_H(\varphi)\, d\varphi\right]^{\frac{1}{1-\sigma}}} \quad or \quad f a(\varphi_H^*) = f_x a(\varphi_{xH}^*),$$

where $a(\varphi) \equiv \varphi^{1-\sigma} \int_{\varphi}^{\infty} x^{\sigma-1} g_H(x)\, dx$ is decreasing in φ. Thus, $\varphi_{xH}^* = a^{-1}\left(\frac{f}{f_x} a(\varphi_H^*)\right)$.

Two additional restrictions in the case of specialization are $\frac{f_x}{f} > 1$ (then $\varphi_{xH}^* > \varphi_H^*$) and $\beta < \frac{1}{2}$ (then both countries produce the homogenous good). The FE condition is

$$\frac{f}{\delta} j_H(\varphi_H^*) + \frac{f_x}{\delta} j_H(\varphi_{xH}^*) = f_e \;\text{ or }\; \frac{f}{\delta} j_H(\varphi_H^*) + \frac{f_x}{\delta} j_H\left(a^{-1}\left(\frac{f}{f_x} a(\varphi_H^*)\right)\right) = f_e. \tag{43}$$

Thus, φ_H^* does not depend on $G_F(\cdot)$. There exists a unique solution of (43), since its left-hand side is decreasing in φ_H^* from zero to infinity. The average profit $\bar{\pi}_H$ is $\delta f_e / (1 - G_H(\varphi_H^*))$. The equilibrium mass of firms is $M_H = R_H^C / \bar{r}_H$. Given M_H, we can derive M_e, P_H, and P_F and complete the description of the equilibrium.

Appendix L

In the case of the Pareto productivity distribution, we can write $j(\varphi^*, G_i(\cdot))$ as

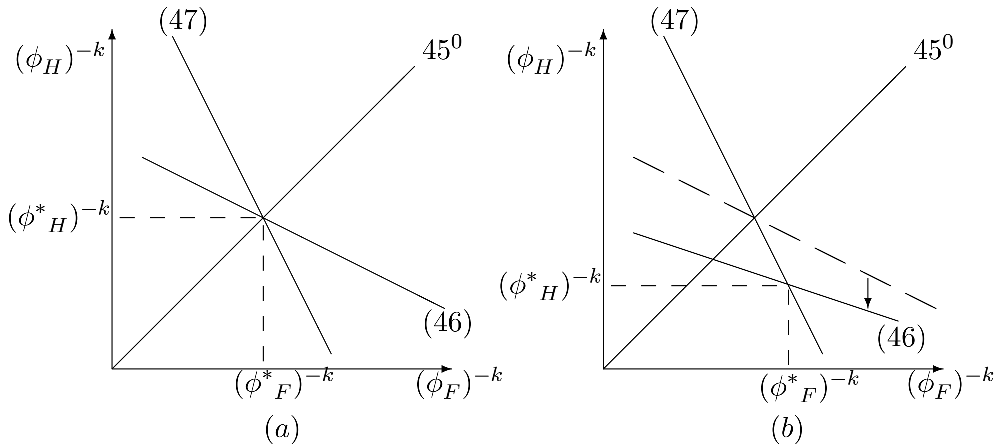
$$j(\varphi^*, G_i(\cdot)) = \frac{\sigma - 1}{k - (\sigma - 1)} \left(\frac{\varphi_{\min,i}}{\varphi^*}\right)^k, \quad i = H, F. \tag{44}$$

By using (44) in the system of equations (22) and (23), we obtain the system of equations (27) and (28). From (27) and (28) we can obtain $(\varphi_H)^{-k}$ as a linear function of $(\varphi_F)^{-k}$:

$$(27) \implies (\varphi_H)^{-k} = \frac{\delta f_e}{f} \frac{(k - (\sigma - 1))}{(\sigma - 1)(\varphi_{\min,H})^k} - \frac{f_x}{f} A^{-k} (\varphi_F)^{-k}, \tag{45}$$

$$(28) \implies (\varphi_H)^{-k} = A^k \left(\frac{\delta f_e}{f_x} \frac{(k - (\sigma - 1))}{(\sigma - 1)(\varphi_{\min,F})^k} - \frac{f}{f_x} (\varphi_F)^{-k}\right). \tag{46}$$

Figure 8: The case of Pareto Productivity Distribution



Using similar techniques as before, it is easy to see that in both equations (45) and (46) $(\varphi_H)^{-k}$ is decreasing in $(\varphi_F)^{-k}$. Thus, the lines corresponding to these equations are decreasing in $(\varphi_F)^{-k}$. Moreover, the line corresponding to equation (45) is flatter than the line corresponding to equation (46). Note that if $\varphi_{\min,H} = \varphi_{\min,F}$, then $(\varphi_H^*)^{-k} = (\varphi_F^*)^{-k}$ as shown in Figure 8(a)[33]. By increasing $\varphi_{\min,H}$, we shift the line corresponding to equation (45) down as shown in Figure 8(b). Thus, $(\varphi_H^*)^{-k} < (\varphi_F^*)^{-k}$ or $\varphi_H^* > \varphi_F^*$. Moreover, an increase in the gap between $\varphi_{\min,H}$ and $\varphi_{\min,F}$ increases φ_H^* and decrease φ_F^*. Thus, propositions 2 remains the same under the assumption of USD.

Note that a fall in the trade cost τ decreases $A = \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$ and increases φ_H^*. From equations (45) and (46), $(\varphi_F^*)^{-k}$ can be written as

$$(\varphi_F^*)^{-k} = \frac{(k-(\sigma-1))\,\delta f_e}{(\sigma-1)\,f\left(\varphi_{\min,F}\right)^k}\left[\frac{A^k - \frac{f_x}{f}\left(\frac{\varphi_{\min,F}}{\varphi_{\min,H}}\right)^k}{A^k - \left(\frac{f_x}{f}\right)^2 A^{-k}}\right]. \tag{47}$$

Note that the right-hand side of equation (47) is a decreasing in A (and τ), if

$$\left(\frac{\varphi_{\min,F}}{\varphi_{\min,H}}\right)^k < \frac{2}{\frac{f}{f_x}A^k + \frac{f_x}{f}A^{-k}}.$$

33 Note that unlike the previous figures in this paper, Figure 8 is drawn in the $\left((\varphi_F)^{-k}, (\varphi_H)^{-k}\right)$ space, not in the (φ_F, φ_H) space.

Therefore, if $\frac{\varphi_{\min,F}}{\varphi_{\min,H}}$ is small enough, i.e., if the technological difference between the two countries is large enough, then the foreign country loses from the falling trade costs. Otherwise, it gains. It remains to check that this need not violate (1) the implicit assumption being made that some firms exit, i.e., $\varphi_i^* \geq \varphi_{\min,i}$, and (2) the assumption of non specialization. (1) implies

$$\left(\frac{\varphi_{\min,F}}{\varphi_{\min,H}}\right)^k > \frac{f}{f_x}\left[A^k - \frac{f(\sigma-1)}{\delta f_e(k-(\sigma-1))}\left(A^k - \left(\frac{f_x}{f}\right)^2 A^{-k}\right)\right]$$

Note that making f_e large enough prevents specialization from occurring. When $f = 2000$, $f_x = 2500$, $f_e = 2000$, $\delta = 0.025$, $\sigma = 3$, $k = 2.2$, $\varphi_{\min,H} = 100$, $\varphi_{\min,F} = 71$, τ decreases from 1.5 to 1.45, it can be verified that both hold.