Immiserizing Deindustrialization:

A Dynamic Trade Model with Credit Constraints

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Abstract

The paper develops an open economy dynamic model with bequests and credit constraints. The agricultural sector uses only labor, the industrial sector needs an indivisible investment. Under autarky, productive agriculture provides the funds needed for investment in industry and in equilibrium credit constraints are not binding. If agriculture is not sufficiently productive, the price of the industrial good must be high enough to make the industrial sector sustainable. In an open economy, if the country has the comparative advantage in agriculture, deindustrialization may occur over time. Deindustrialization is welfare-reducing when the negative wealth distributional effects swamp the gains from trade. The results are shown to extend in interesting ways for a large country and for a richer occupational structure.

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1 Introduction

It has been argued that India suffered deindustrialization during the 18th and 19th centuries. In 1750, India produced 25 percent of world manufacturing output. By 1880 its share dropped to only 2 percent\(^1\). The existing literature provides two major explanations: improved Britain’s productivity in textiles which led to a declining world textile price, and decrease in transportation costs. However, Clingingsmith and Williamson (2004) suggest an additional explanation for India’s deindustrialization, namely, the collapse of the Mughal Empire in the 18th century which caused a severe decline in agricultural productivity: ‘...the long run sources of India’s deindustrialization were both the globalization shocks due to European productivity advance in manufacturing ... plus the negative productivity shocks to Indian agriculture induced by the earlier Mughal decline’. Clingingsmith and Williamson suggest that lower agricultural productivity raised the price of non-tradable goods (such as grain) relative to tradable (such as textiles) which resulted in higher nominal wages in the textile industry. Declining textile prices and rising nominal wage resulted in rising real wage in textiles which had a negative impact on India’s competitiveness in the textile market. Moreover, they argue that ‘India .. lost ground to Britain in the world textile market during a period when most British production was still carried out using the cottage system’\(^2\).

This paper suggests another explanation of deindustrialization consistent with a low agricultural productivity and comparative disadvantage in manufactures. We show that when credit markets are imperfect, a productive agriculture generates funds needed for investment in industry. When agriculture is unproductive, then trade liberalization can profoundly affect the distribution of wealth, thereby making credit constraints much more binding and resulting in a

\(^{1}\) Source: Clingingsmith and Williamson (2004).
\(^{2}\) Clingingsmith and Williamson show that the two main epochs when India deindustrialized, i.e. 1750-1810 and 1810-1860, are characterized by very different deindustrialization causes. During the first period, the main reason was reduced agricultural productivity in India. While, during the second period improved Britain’s productivity resulting from the adoption of the factory system led to a decrease in the world price of textiles.
welfare-reducing deindustrialization. This suggests that episodes like the deindustrialization of India might have been quite damaging.

A consequence of our results is that there is reason to expect a non-monotonic relation between openness to trade and gains from such trade. Relatively productive economies tend to gain from being opened up to trade. Less productive ones, on the other hand, can suffer greater poverty and easily lose from being opened up to trade. This suggests that it may be futile to look for a monotonic relationship between these variables.

That credit constraints are relevant, especially in developing countries, is clear. Tybout (1983), Jaramillo et al. (1996), Gelos and Werner (1999), Bigsten et al. (2000), Banerjee and Duflo (2002), Harrison and McMillan (2003), Love (2003) all show that the evidence is consistent with firms being credit-constrained in a host of developing countries. Moreover, Rajan and Zingales (1998) and Levine et al. (2000) among others show that the development of financial intermediaries or a decrease in credit market imperfections seems to be positively associated with economic growth. This is consistent with less credit-constrained economies tending to gain from trade as suggested in this paper.

Recent empirical work\(^3\) finds a negative correlation between GDP growth and measures of income inequality.\(^4\) An interesting case study is that of South Korea and the Philippines discussed by Benabou (1996). In the early 1960's, South Korea and the Philippines were similar in many aspects, such as per capita GDP, population, school enrollment, etc. A significant difference was in the distribution of income. The Philippines was much more unequal than South Korea. For instance, the ratio of the income share of the top 20% to the bottom 20% was about twice as large in the Philippines. From 1960 to 1988, Korea experienced growth rate of about 6 percent per year, while for the Philippines it was only 2 percent. Benabou (1996) offers two explanations: first that credit constraints result in inefficient levels of investment by

\(^3\)See Benabou (1996) for a review of this literature.

\(^4\)Earlier authors had argued that in poor economies, inequality might promote growth by stimulating capital accumulation. See Aghion and Williamson (1998) for more on this.
poor agents and second, that higher inequality leads to rent-seeking activities by rich agents at
the expense of poor which lowers growth.

This paper offers another explanation related to trade liberalization. If an economy is suf-

ciently unequal to start with, then wages are low as labor demand from entrepreneurs cannot
keep up with the labor supply from those with too little capital to be anything but workers. In
this event, opening up to trade does not allow workers to move into more lucrative occupations
and will not lead to an industrial boom.

This paper constructs a dynamic model where credit markets are missing. There is an initial
distribution of wealth. Agents live for one period. At the beginning of the period, an agent
chooses his occupation: one of an agricultural worker or an industrial producer. In agriculture,
only labor is needed and there are constant returns to scale. In industry, an indivisible investment
in terms of the agricultural good must be made in order to employ each worker. Only those
with inherited wealth sufficient to make this investment have this option. At the end of his life,
a worker’s income is divided between his consumption and bequests.

Bequests are modeled as a “warm glow” which enter utility directly. This approach has
been commonly used in the development literature as it simplifies the analytics considerably. A
Cobb-Douglas utility function is used which results in a constant share of end of period income
being left as bequests.5

First, the steady state autarky equilibrium is analyzed. It turns out that the behavior of
the economy depends on how productive agriculture is relative to the investment needed in the
industrial sector. If agriculture is sufficiently productive, credit market imperfections are not
binding in the long run. The economy converges to a unique wealth level and consists of identical
non-credit-constrained agents who are indifferent between working in one sector and the other.

5There is some reason for concern using this setup: after all, if a small increase in ones bequest results a large
increase in an offspring’s utility, would one not choose to do so? However, the altruistic setup (where the welfare
of ones progeny is the target) as in Barro (1974), though intractable outside steady state, gives the same basic
results in steady state as outlined in Appendix B.
If agriculture is not productive enough, credit market imperfections are binding in steady state. In steady state, there are two distinct classes of agents: ‘poor’ credit-constrained agents working in agriculture and ‘rich’ non-credit-constrained agents working in industry.

Next, an open economy is considered. For a small open economy, trade is shown to raise aggregate welfare if the country has comparative advantage in industry. Basically, this is due to the standard gains from trade: agents working in industry gain as the price of their output rises, while those in agriculture lose and the gains of the former exceed those of the latter. However, if the country has comparative advantage in agriculture, trade liberalization can result in the following situation. Start from the autarky steady state. Since the price of the industrial good falls, the agents who can just afford the investment today can no longer make the bequests needed to ensure their offspring can work in industry. These offspring must now work in agriculture, even if industry is more profitable. However, this is not the end of the story. In the next period, those who were on the margin in second period after opening up to trade are in the same boat! This process results in ‘involuntary’ deindustrialization. This can be immiserizing for the economy if comparative advantage in agriculture is large, but not too large. In effect, the change in income distribution makes credit constraints more binding and may reduce aggregate welfare. If comparative advantage is small, then the distribution of income is unaffected, while if it is too large, the gains from trade swamp the income distributional effects.6

The large country case is also considered. In particular, the ‘North-South’ trade in which the two countries are distinguished only by differences in credit markets is analyzed. It is shown that differences in credit markets not only create the basis for comparative advantage7 but may also lead to different short-run and long-run trade effects. In the small country case the world

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6Matsuyama (1992) shows the possibility of deindustrialization for a small open economy when the agricultural sector is productive. In his model, the driving force for deindustrialization is an interaction between non-homothetic preferences and learning-by-doing in the manufacturing sector.

7See Beck (2002) for empirical evidence on a large causal impact of financial development on trade in manufactures.
price of the industrial good is exogenous and cannot be affected in the long run. When trading countries are large, the price evolves over time and may take various paths before converging to its steady state value. For example, the following scenario can easily occur: a non-credit-constrained country trades with a credit-constrained one, and the price of the industrial good falls at first. As a result, deindustrialization occurs in the credit-constrained country and it gets locked into agriculture. Now the price rises and the country cannot respond so that it ends up facing even higher prices than it did in autarky, but exports the agricultural good!

Finally, the model is augmented to allow another occupation, working as a laborer. For relatively high world prices and relatively equal economies, trade is shown to increase the demand for labor and drive wages up. This results in credit-constrained workers becoming unconstrained and being able to invest in more profitable occupations. The level of inequality plays an important role in determining when trade can become a real engine of growth. If economy is too unequal, opening up to trade does not allow occupational mobility and, therefore, does not lead to an industrial boom.

The rest of the paper proceeds as follows. Section 2 discusses related literature. Section 3 lays out the model. Section 4 solves for the steady state autarky equilibrium. Section 5 considers an open economy. In Section 6 the model is augmented to allow a labor market to exist. Section 7 provides some concluding thoughts.

2 Related Literature

The paper is related to two strands of literature: the literature on credit market imperfections and distribution of wealth, and the literature on the effects of trade in the presence of market imperfections. The latter has a venerable tradition, but credit market imperfections and their

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8In the basic model agricultural workers can never move to industry, only industrial workers can move to agriculture. This restricts the ability of price changes to move workers in both directions. Introduction of this richer occupational structure eliminates this weakness of the basic model.
impact on income distribution is not its prime focus.

There is a thriving field in development which looks at the evolution of wealth and the existence of poverty traps. See, for example, Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), and Piketty (1997). They employ a nonconvex investment technology and show how initial conditions determine long-run outcomes. In these papers, the economy displays multiple steady states, where inequality can be high or low. Inequality is persistent: ‘poor’ dynasties are unable to catch up with ‘rich’ ones. However, all of these papers consider a closed economy. Moreover, there is a single final good so that the determination of relative prices of final goods is not an issue. Though this paper is closest to the development literature, it analyzes a new issue: the effects of trade.

Recently, there has been some work in trade in this area. In an influential contribution Findlay and Kierzkowski (1983) extend the standard Heckscher-Ohlin model by endogenizing the formation of human capital. Much of the work in trade discussed below builds on this paper. They show that trade amplifies initial differences in factor endowments through the Stolper-Samuelson effect: trade raises the reward of the abundant factor in each country. Therefore, trade leads to a decrease in the accumulation of human capital in skill-scarce countries and does the opposite in skill-abundant countries. However, there are no credit constraints.

Cartiglia (1997) incorporates credit constraints into a Findlay-Kierzkowski type model, but uses a static setting. He shows that trade leads to convergence in human capital endowments. A key element in his paper is that skilled labor is used as an input in the formation of skilled labor. Trade liberalization in a skill-scarce country reduces the cost of education and hence weakens credit constraints, resulting in a higher investment in human capital. This effect in fact dominates the Stolper-Samuelson effect of Findlay-Kierzkowski reversing their results.

Ranjan (2001) and Das (2003) are the only papers that use a dynamic framework and allow
for credit constraints. Ranjan (2001) looks at the effect of trade liberalization on skill acquisition, the skilled-unskilled wage differential, and the distribution of wealth. He points out a third effect that operates through changes in the distribution of income which influences the accumulation of human capital. However, the aggregate effect of trade is indeterminate in his model. Another prediction is that the degree of credit market imperfections can become a determinant of the pattern of comparative advantage.

Das (2003) looks for the most part at economies with perfect credit markets. Only in a final section does he look at credit market imperfections which are modelled as a fixed differential between the borrowing and lending rate. He considers a two-country general equilibrium model and shows that trade liberalization results in more human capital acquisition in the country with perfect credit markets. The opposite happens in the country where credit markets are imperfect. However, both Das (2003) and Ranjan (2001) assume incomplete specialization under trade so that neither paper actually allows for the possibility of deindustrialization.

Matsuyama (2004) analyzes the effects of financial market globalization on inequality. He incorporates credit market imperfections into overlapping-generations model and shows that stable asymmetric steady states may exist in which the world economy is endogenously divided into the rich and poor countries. However, he analyzes a model with a single final good and looks at the effects of financial market globalization rather than effects of trade liberalization.

3 The Model

There is a continuum of agents of unit mass. Each agent lives for one period and has one child. At the beginning of a period an agent makes occupational and investment choices. At the end of the period the resulting income is divided between consumption and bequests. The wealth in the economy evolves through bequests: the initial wealth of an agent in period \( t \), \( w_t \), equals the bequest from his parent, \( b_{t-1} \). Agents are assumed to be identical with respect to their abilities and preferences and differ only in their initial wealth endowments. The distribution of wealth
at the beginning of period \( t \) is represented by the distribution function, \( G_t() \). Each agent is endowed with a single unit of labor which he supplies inelastically at no disutility cost.

There are two goods in the economy. The agricultural good is treated as the numeraire. The price of the industrial good is \( p \). Goods are produced according to the following technologies. Each agent can use his unit of labor to produce \( n \) units of the agricultural good. Alternatively, an agent can invest \( k \) units of the numeraire good. Once this investment is made, the agent uses his unit of labor to produce \( a \) units of the industrial good. Note that this makes the technology in the industrial sector nonconvex.

There is no credit market, so agents cannot borrow. At the beginning of a period an agent chooses the sector in which to work. The occupational choice of each agent depends on his wealth endowment and the returns in each sector. If the agent works in the agricultural sector his wealth at the end of the period, or his disposable income, is \( Y_t = w_t + n \). If the agent works in the industrial sector his end of period income is \( Y_t = w_t + ap_t - k \). The disposable income \( Y_t \) is divided among consumption of the agricultural good, consumption of the industrial good and bequests. Bequests are made in terms of the numeraire good.

Agents have identical Cobb-Douglas preferences over consumption and bequests. Hence, optimal consumption and bequests in period \( t \) are linear functions of end of period income:

\[
    c_{At} = \alpha Y_t, \quad p_t c_{It} = \beta Y_t, \quad b_t = \gamma Y_t,
\]

where \( \alpha + \beta + \gamma = 1 \), \( c_{At} \) is the consumption of the agricultural good in period \( t \), \( c_{It} \) is the consumption of the industrial good in period \( t \), while \( b_t \) is the bequest. The resulting indirect utility is also linear in income:

\[
    W_t = \frac{A}{p_t} Y_t, \quad \text{where } A = \alpha^\alpha \beta^\beta \gamma^\gamma.
\]
4 Autarky Equilibrium

4.1 Static Equilibrium

Given the wealth distribution at the beginning of the period, the static equilibrium yields the occupational choices of the population and the price, $p_t$, of the industrial good. Agents choose where to work depending on the returns in each sector. A choice of producing the agricultural good yields a payoff of $n$, while the return from working in industry depends on the price of the industrial good, which, in turn, is endogenously determined.

The equilibrium price, $p_t$, and, hence, the return in industry, depends on the distribution of wealth in the economy. This distribution affects both supply of and demand for the industrial good. It determines the aggregate wealth in the economy, which affects demand as well as how many agents work in industry, which, in turn, affects supply. First, we analyze how the occupational choice of each agent depends on the price and then we derive the equilibrium price at the period $t$ for a given wealth distribution $G_t(w)$.

We assume that initially there is a positive proportion of relatively rich agents, i.e., agents with initial wealth more than $k$. This assumption is made to rule out degenerate equilibria where all agents are credit-constrained and the industrial good is not produced.

Since capital markets are missing, all agents with initial wealth $w_t < k$ are unable to borrow to make the investment needed in industry. Hence, all these agents work in agriculture and have end of period income of $Y_t = w_t + n$ and indirect utility

$$W_t(w_t) = \frac{A(w_t + n)}{p^3}.$$

Agents with initial wealth $w_t \geq k$ have a choice of where to work so that their indirect utility is:

$$W_t(w_t) = \max \left\{ \frac{A(w_t + n)}{p_t^3}, \frac{A(w_t + ap_t - k)}{p_t^3} \right\}.$$  \hspace{1cm} (1)

Note that all agents have the same preferences over the sector of work, the only issue is whether
they have inherited enough to make working in industry an option. From equation (1) it follows that if \( p_t \) is low enough, \( p_t < \frac{k + n}{a} \), then all agents with initial wealth \( w_t \geq k \) choose to work in the agricultural sector. If \( p_t > \frac{k}{a} \) then all these agents choose to work in the industrial sector. If \( p_t = \frac{k}{a} \) then they are indifferent between sectors.

Since both goods are essential for consumers, both goods must be produced in equilibrium ensuring that \( p_t \geq \frac{k}{a} \). Hence, the supply of the industrial good is horizontal at \( \frac{k}{a} \) and is vertical at level \( a(1 - G_t(k)) \) above \( \frac{k}{a} \). If demand intersects supply in the vertical segment, then the equilibrium price is given by the intersection of demand and supply, while if demand intersects supply in the horizontal segment, the equilibrium price is \( \frac{k}{a} \). We will first derive the price, \( p'_t \), at which demand and this vertical segment of supply intersect. Clearly, the equilibrium price is the maximum of this level and \( \frac{k}{a} \).

Demand is given by:

\[
D_{2t} = \frac{\beta Y_t^A}{p_t},
\]

where \( Y_t^A \), the aggregate disposable income, consists of the aggregate initial wealth and the returns from the production of both goods:

\[
Y_t^A = \int wdG_t(w) + nG_t(k) + (ap_t - k)(1 - G_t(k)).
\]

Recall that \( \int wdG_t(w) \) is the aggregate initial wealth, \( nG_t(k) \) is the aggregate return in the agricultural sector, and \( (ap_t - k)(1 - G_t(k)) \) is the aggregate return in the industrial sector net of investment. Let \( \theta_t = G_t(k) \) denote the measure of agents with initial wealth \( w_t < k \).

The price \( p'_t \) is equal to:

\[
p'_t = \frac{\beta}{1 - \beta} \frac{\int wdG_t(w) + n\theta_t - k(1 - \theta_t)}{a(1 - \theta_t)}.
\]

\[10\]If the price is below \( \frac{k}{a} \) then all agents in the economy work in agriculture and the industrial good is not produced. The price \( p_t \), and, as the result, the return to working in industry, will be driven by demand to infinity and agents with initial wealth \( w_t \geq k \) will switch to the industrial sector.
The equilibrium price in period \( t \), \( p_t \), is \( \max \{ p_t, p_t' \} \). Since 
\[
p_t' - p_t = \left( \frac{\beta}{1 - \beta} \right) \left[ \int wdG_t(w) + n \right] - \left( \frac{1}{1 - \beta} \right) \left( \frac{n + k}{a} \right)
\]
it follows that the equilibrium price in each period depends on two endogenously determined factors: the aggregate initial wealth, \( \int wdG_t(w) \), and the degree of poverty, \( \theta_t \). The aggregate initial wealth affects the demand side: the greater the aggregate initial wealth, the higher the demand for the industrial good. The degree of poverty, measured as a fraction of population with initial wealth less than investment needed in industry, affects the supply side: a smaller fraction of population working in agriculture leads to a higher supply of the industrial good. In period \( t \), if either the aggregate initial wealth or the degree of poverty is small enough, then the equilibrium price is equal to its minimal value \( \underline{p} \). Otherwise, the equilibrium price exceeds \( \underline{p} \).

Lemma 1 summarizes these results.

**Lemma 1** The equilibrium price in the period \( t \) is
\[
p_t = \begin{cases} 
\underline{p}, & \text{if } \beta \int wdG_t(w) + n \theta_t \leq k + (1 - \beta) n \\
\frac{\beta}{1 - \beta} \frac{\int wdG_t(w) + n \theta_t - k (1 - \theta_t)}{a(1 - \theta_t)}, & \text{otherwise}.
\end{cases}
\]

4.2 Dynamic Equilibrium

Having solved for the occupational structure and the equilibrium price given a wealth distribution, we turn to the behavior of the economy in the long run. It turns out that the behavior of the economy depends on how productive agriculture is relative to the investment needed in the industrial sector.

**Evolution of bequests.** Since each agent leaves a share \( \gamma \) of his end of period income as bequests, wealth dynamics are described by:
\[
w_{t+1} = b_t = \begin{cases} 
\gamma (w_t + n), & \text{if } w_t < k; \\
\gamma (w_t + a p_t - k), & \text{if } w_t \geq k.
\end{cases}
\]
This is shown in Figure 1 which depicts bequests at every inherited wealth level. For credit-constrained agents, who work in agriculture, bequests are independent of price. Therefore, there is a single bequest line \( w_{t+1} = \gamma (w_t + n) \) when \( w_t < k \). For agents with initial wealth more than \( k \) bequests do depend on price, therefore, there is a bequest line for every \( p_t \). Recall that \( p_t \geq p \).

When \( p_t = p \) agents earn the same income in the two sectors. As a result, the bequest line at \( p_t = p \) coincides with \( w_{t+1} = \gamma (w_t + n) \). When \( p_t > p \) agents with inherited wealth more than \( k \) earn more and hence leave higher bequests than if they had worked in agriculture. As a result, the function relating bequests to inherited wealth is discontinuous at \( k \) as depicted.

The intersection of the bequest line \( w_{t+1} = \gamma (w_t + n) \) with the 45° line occurs at \( w = \gamma 1 - \gamma n \). There are two possibilities: either \( w \geq k \) as in Figure 1(a), or \( w < k \) as in Figure 1(b). Note that a more productive agriculture shifts the bequest line \( w_{t+1} = \gamma (w_t + n) \) upwards. If agriculture is productive enough, i.e., \( \gamma n \geq (1 - \gamma) k \), then even the agents with inherited wealth less than \( k \) are able to earn enough, over time, to leave bequests more than \( k \) and their offspring become non-credit-constrained. Thus, Figure 1(a) corresponds to having a sufficiently productive agriculture.

From Figure 1(a) it is clear that the credit-constrained class (with initial wealth \( w_t < k \)) must shrink in each period: an agent with inherited wealth \( k \) will leave more than \( k \) to his offspring who will therefore not be credit-constrained. Moreover, since all credit-constrained agents leave more than the amount they themselves inherited, the credit-constrained class is eliminated in the long run. Hence, credit constraints are not binding in the long run.

Let \( \bar{w}(p) \) denote the intersection of the bequest line for the non-credit-constrained agents at price \( p \) with the 45 degree line. Note that in Figure 1(a), for all prices, agents who are not credit-constrained, and who inherit less than \( \bar{w}(p) \), will leave more than what they themselves inherited, while agents who are not credit-constrained, and who inherit more than \( \bar{w}(p) \), will leave less than what they themselves inherited. As a result, the wealth distribution converges to a single point, \( \bar{w}(p) \).

If agriculture is not adequately productive, i.e., \( \gamma n < (1 - \gamma) k \), the situation is as depicted
in Figure 1(b). It is clear from Figure 1(b) that all agents with wealth less than \( k \) leave less than \( k \) as bequests. Hence, the credit-constrained group does not shrink over time and credit market imperfections matter for the long-run behavior of the economy.

For agents with initial wealth \( w_t \geq k \), bequests depend on the equilibrium price. Let the price \( \hat{p} \) be the minimal price at which the offspring of agents with initial wealth \( w_t = k \) are not credit-constrained. Solving \( \gamma(k + ap - k) = k \) gives \( \hat{p} = \frac{k}{\gamma a} \).

If \( p_t \geq \hat{p} \), then all agents who are not credit-constrained leave more than \( k \) as bequests. Therefore, the group of non-credit-constrained agents also does not shrink. If the price is low, \( p_t \in [p, \hat{p}) \), then agents with initial wealth \( w_t \in [k, k + a(\hat{p} - p_t)] \) leave bequests less than \( k \) and agents with initial wealth \( w_t \geq k + a(\hat{p} - p_t) \) leave bequests more than \( k \). In this event, the group of non-credit-constrained agents shrinks. However, when it shrinks, supply of the good falls. Demand also falls, but, as shown below in Proposition 2, the supply effect dominates. As a result, prices rise. Hence, prices below \( \hat{p} \) cannot be sustained in steady state.

**Price and limiting wealth distribution.** Proposition 1 characterizes the steady state equilibrium when we have a productive agriculture, i.e. \( \gamma n \geq (1 - \gamma)k \).

**Proposition 1** If \( \gamma n \geq (1 - \gamma)k \), the economy converges to unique autarky wealth level

\[
x^A = \frac{\gamma}{1 - \gamma} n \geq k.
\]

The equilibrium autarky price converges to \( p^A = p = \frac{n + k}{a} \). All agents are indifferent between sectors and have the same level of income and utility.

The aggregate welfare is

\[
W^A = \frac{An}{(1 - \gamma)(p^A)\beta}.
\]

**Proof.** We have already shown that the equilibrium price cannot lie below \( p \) and that the credit-constrained class is eliminated in the long run. All that remains to be shown is that the equilibrium price cannot exceed \( p \).
If the equilibrium price is \( p^A > p \) then in the steady state all agents choose to work in the industrial sector. Since both goods are essential, the agricultural good must be produced in equilibrium, hence, the price \( p^A > p \) cannot be an equilibrium price. Therefore, the equilibrium price is \( p^A = p \). At this price all agents are indifferent between sectors. The wealth dynamics for each agent follows \( w_{t+1} = \gamma(w_t + n) \). Hence the initial wealth and end of period income for each agent converge to

\[
w^A = w = \frac{\gamma}{1 - \gamma} n, \quad Y^A = \frac{w^A}{\gamma},
\]

The aggregate welfare is

\[
W^A = \frac{AY^A}{(p^A)^\beta} = \frac{An}{(1 - \gamma) (p^A)^\beta}.
\]

If the agricultural sector is productive enough, then credit constraints are not binding in the long run and the credit-constrained class is eliminated. There is perfect equality: all agents are identical in terms of their wealth endowments and, therefore, have the same potential occupational choices. This suggests a reason for greater income equality in early industrialization where productive agriculture was a precondition for takeoff.

What if the agricultural sector is not productive enough?

**Proposition 2** If \( \gamma n < (1 - \gamma) k \), the economy converges to two distinct wealth levels \( w^A \) and \( \overline{w}^A \), where

\[
w^A = \frac{\gamma}{1 - \gamma} n < k, \quad \overline{w}^A = \frac{\gamma}{1 - \gamma} (ap^A - k) \geq k.
\]

Agents with wealth level \( w^A \) work in agriculture and agents with wealth level \( \overline{w}^A \) work in industry. Agents with higher wealth level enjoy higher utility. The equilibrium price converges to

\[
p^A = \frac{\beta}{\alpha} \left[ \frac{n\theta^A - k(1 - \theta^A)}{(1 - \theta^A)a} \right] \geq \hat{p},
\]
where $\theta^A$ is the measure of agents with wealth $w^A$ in steady state, which satisfies

$$
\theta^A \geq \hat{\theta} = \frac{(1 - \gamma)(1 - \beta)k}{(1 - \gamma)(1 - \beta)k + \beta \gamma n}.
$$

The aggregate welfare is

$$
W^A = A \left( \theta^A n - (1 - \theta^A)k \right).
$$

**Proof.** Recall that $\theta_t = G_t(k)$ denotes the measure of agents in period $t$ with initial wealth $w_t < k$. Consider Figure 1(b). Recall that the group of credit-constrained agents does not shrink over time. Hence, $\theta_t$ does not decrease over time. In particular, the following relationship between $\theta_{t+1}$ and $\theta_t$ holds:

$$
\theta_{t+1} \begin{cases} 
\geq \theta_t, & \text{if } p_t < \hat{p}; \\
= \theta_t, & \text{if } p_t \geq \hat{p}.
\end{cases}
$$

Consider Figure 2 which depicts demand and supply of the industrial good. Supply of the industrial good, $S$, is horizontal at $p$ up to the level $a(1 - \theta)$ and vertical for prices above that. If the equilibrium price is $p < \hat{p}$, then the fraction of the population that is credit-constrained must rise. If it rises by $\Delta$, the supply of the industrial good at a given $p$ shifts in by $a\Delta$ and is depicted by $S'$. Demand also shifts in (depicted by $D'$) as the credit-constrained agents earn less than the non-credit-constrained ones. The shift in demand at a given $p$ is

$$
\Delta \beta \left( \frac{ap - k - n}{p} \right).
$$

But only a part of income is spent on the industrial good, $ap - \beta(ap - k - n) > 0$, and the shift in demand is less than that of supply, so that the price rises to $p'$. This process continues till $p_t$ reaches $\hat{p}$. Note that the fraction of credit-constrained agents, $\theta_t$, also rises till it gets above $\hat{\theta}$, which is determined below.

Thus, the price in the steady state cannot be less than $\hat{p}$ and we can focus on $p \geq \hat{p}$.

It is clear from Figure 1(b) that for any given price $p \geq \hat{p}$ the initial wealth of agents working in agriculture converges to $w$ and the initial wealth of agents working in industry converges to $\overline{w}(p) = \gamma \left( \frac{ap - k}{1 - \gamma} \right)$. Note that this depends on $p$. Hence we need to see what level of $p$ is consistent, in the steady state, with all the unconstrained agents inheriting wealth $\overline{w}$. Recall
that the only effect of a change in \( \pi \) is a shift of demand and thereby an increase in price. The measure of agents with \( w \) in steady state, denoted by \( \theta^A \), is determined by the initial distribution of wealth, so that supply is fixed.

Using Lemma 1 to solve for the equilibrium \( p \), given \( \pi \), gives

\[
p(\pi) = \beta \int \frac{wdG_t(w) + n\theta^A - k(1 - \theta^A)}{a(1 - \theta^A)} \, dw = \frac{\beta \theta^A \pi + (1 - \theta^A) \pi + n\theta^A - k(1 - \theta^A)}{a(1 - \theta^A)}.
\]

The system

\[
\begin{align*}
\pi(p) &= \frac{\gamma}{1 - \gamma} (ap - k) \\
p(\pi) &= \frac{\beta}{(1 - \beta) a} \pi + \frac{\beta \theta^A (w + n + k) - k}{a(1 - \theta^A)}
\end{align*}
\]

is depicted in Figure 3. Note that \( p(\pi) \) line is flatter than \( \pi(p) \) line and \( p(\pi) \) exceeds \( \hat{p} \) at \( \pi = k \):

\[
\text{slope } (p(\pi)) = \frac{\beta}{(1 - \beta) a} > \frac{1 - \gamma}{\gamma a} = \text{slope } (\pi(p)^{-1});
\]

\[
p(k) = \frac{\beta}{(1 - \beta) (1 - \gamma) a} \left[ \frac{\theta^A n}{(1 - \theta^A)} \right] > \frac{k}{\gamma a} = \hat{p}
\]

Hence, the system (4) is stable and converges to \((\pi^A, p^A)\).

Solving \( p^A \geq \hat{p} \) gives

\[
\theta^A \geq \hat{\theta} \equiv \frac{(1 - \gamma) (1 - \beta) k}{(1 - \gamma) (1 - \beta) k + \beta \gamma n}.
\]

The aggregate welfare is

\[
W^A = \frac{A \theta^A n + (1 - \theta^A) (ap^A - k)}{(p^A)^\beta} = \frac{A \left( \theta^A n - (1 - \theta^A) k \right)}{\alpha (p^A)^\beta}
\]

In the case when agriculture is not productive enough, credit constraints are binding in the long run. As a result, the economy in the steady state exhibits inequality and consists of two different classes: credit-constrained agents working in agriculture and non-credit-constrained
ones working in industry. Note that the autarky price of the industrial good must be high enough to guarantee the existence of the industrial sector, i.e., price must be above \( \hat{p} \). This in turn implies that the proportion of credit-constrained agents is relatively high, i.e. \( \theta^A \geq \hat{\theta} \), or, in other words, the supply of the industrial good is sufficiently small.

5 The Effects of Trade

In the previous section we have derived steady state equilibrium for a closed economy. Now we want to know how opening up the economy to trade affects occupational choices and wealth distribution and what are the associated welfare effects. First, the effects of trade are analyzed for a small country, and second, large country case is considered.

5.1 Small Country Case

This section analyzes effects of trade for the case when the country is small and cannot affect the world price of the industrial good, denoted by \( p^W \). If the autarky price of the industrial good is higher (lower) than the world price, the industrial (agricultural) good is imported.

The welfare effects of trade differ greatly depending on whether the credit constraints are binding or not in the autarky steady state. Figures 4 (a) and (b) help understand these welfare effects.

When agriculture is productive enough, i.e., \( \gamma n > (1 - \gamma) k \), all agents have the same wealth level and credit constraints are not binding. Hence, there is nothing to stop agents (who are identical in the autarky steady state) from moving to the more profitable sector when the economy opens up. If the world price lies below \( p^A \), then agriculture is more profitable than industry and all agents work in the former and are net buyers of the industrial good. Hence their welfare rises as price falls. If the world price exceeds the autarky price, then the industrial sector is more profitable and, since credit constraints are not binding, all agents will be both willing and able to work in industry. Since all agents are net sellers of the industrial good, welfare rises with
its price. Consequently, trade has the same positive effect on everyone and is Pareto superior to autarky. Note that as a result, welfare is at a minimum at the autarky price as depicted in Figure 4(a).

If the agricultural sector is not productive enough, i.e., \( \gamma n < (1 - \gamma) k \), then we know from Proposition 2 that the autarky price in steady state exceeds \( \hat{p} \). As a result, credit constraints are binding in steady state and there are two groups of agents: credit-constrained ones with wealth endowment \( w^A < k \) who work in agriculture, and non-credit-constrained agents with wealth endowment \( w^A \geq k \) who work in industry.

To understand the welfare effects of trade it is useful to look at the effect trade has on the welfare of the two groups of agents. Consider Figure 4(b), which depicts the welfare under trade for each group as well as aggregate welfare. Agents who are credit-constrained in the autarky steady state always remain credit-constrained and work in agriculture no matter what the world price is. Therefore, they are affected by the world price only via their consumption and their welfare under trade, denoted by \( W^{T}_{cc} \), falls as price rises. Non-credit-constrained agents could, in addition, be affected through the supply side. If the world price exceeds \( \hat{p} \), they remain net suppliers of the industrial good and gain from an increase in its price. Hence, their welfare, denoted by \( W^{T}_{ncc} \), rises with price, for \( p^W \geq \hat{p} \). However, if price is below \( \hat{p} \), but above \( \underline{p} \), non-credit-constrained agents want to produce the industrial good but their bequests are not large enough for their progeny to be able to do so. Of course, if price is below \( \underline{p} \), they choose to work in agriculture. In either event, in steady state, the non-credit-constrained agents become credit-constrained and end up working in agriculture. This is reflected in \( W^{T}_{ncc} \) jumping down to \( W^{T}_{cc} \) when the world price equals \( \hat{p} \). Aggregate welfare is just a convex combination of \( W^{T}_{ncc} \) and \( W^{T}_{cc} \) curves and, hence, lies between these two curves for prices above \( \hat{p} \) and coincides with \( W^{T}_{cc} \) for prices below \( \hat{p} \), as depicted.

We can say one more thing about this aggregate welfare curve. At \( p^A \) it must be increasing
in price. Aggregate welfare in the trade equilibrium, as a function of the world price, is

\[ W_T(p^W) = \begin{cases} 
W_{cc}^T(p^W), & \text{if } p^W < \hat{p}; \\
\theta A W_{cc}^T(p^W) + \left(1 - \theta A\right) W_{ncc}^T(p^W), & \text{if } p^W \geq \hat{p}.
\end{cases} \]

Let \( x_{cc} \) and \( x_{ncc} \) denote the demand for industrial goods from the credit-constrained and unconstrained agents respectively. Due to the homotheticity of preferences, the indirect utility or welfare of an agent is linear in his income and can be written as \( \varphi(p) Y \), where \( \varphi(p) = \frac{A}{p^A} \).

Moreover, recall that disposable income, \( Y \), equals earnings as well as inherited wealth, so that in the steady state \( Y \) equals earnings scaled up by the factor \( \frac{1}{1 - \gamma} \). Using the above and Roy’s identity we see that

\[
\left. \frac{dW}{dp} \right|_{p^A} = \theta A \frac{dW_{cc}}{dp} + (1 - \theta A) \frac{dW_{ncc}}{dp} = \varphi(p) \left[ -\left( \theta A x_{cc} + (1 - \theta A) x_{ncc} \right) + (1 - \theta A) \frac{a}{1 - \gamma} \right] = \varphi(p) \left[ -\bar{x} + (1 - \theta A) a + (1 - \theta A) \frac{\gamma a}{1 - \gamma} \right] = \varphi(p)(1 - \theta A) \frac{\gamma a}{1 - \gamma} > 0
\]

The last equality follows from the observation that \( \bar{x} = \left( \theta A x_{cc} + (1 - \theta A) x_{ncc} \right) \) is the aggregate consumption of the industrial good, while \( (1 - \theta A) a \) is the aggregate production. In autarky, these two are equal.

In other words, when the country has a comparative advantage in the industrial good, gains of non-credit-constrained agents exceed losses of credit-constrained ones, and the economy as a whole benefits from trade. However, when the country has a comparative advantage in the agricultural good, there is a net loss in aggregate welfare for world prices close to the autarky level, even though the occupational structure is not affected. The intuition behind these results is as follows. At autarky, the economy is neither a net buyer nor a net seller of the industrial good, so these direct effects vanish. However, an increase in the price of the industrial good raises the bequests of agents working there, which in turn raises steady state level of income of non-credit-constrained agents by \( \frac{\gamma}{1 - \gamma} a \). Therefore welfare is not at its minimum at the autarky
price, as depicted in Figure 4(b).

More importantly, when the country has a comparative advantage in agriculture and the world price lies below $\hat{p}$, trade affects the occupational structure and deindustrialization results. Aggregate welfare curve jumps at $\hat{p}$, as depicted in Figure 4(b). If the comparative advantage in agriculture is not too large, i.e., the difference between world price of the industrial good and its autarky price is not too large, negative income distributional effects exceed the gains from trade and aggregate welfare falls relative to autarky. In this event, opening the economy up to trade results in immiserizing deindustrialization! If the comparative advantage is too large, gains from trade swamp negative income distributional effects, and the deindustrialization is not welfare reducing. As a result, the country benefits from trade.

Proposition 3 summarizes our results on the steady state equilibrium under trade.

**Proposition 3** If agriculture is productive enough, i.e., $\gamma n \geq (1 - \gamma) k$, then the opening up to trade results in complete specialization: if the world price of the industrial good is higher (lower) than its autarky price then agricultural (industrial) sector disappears. All agents benefit from trade.

If agriculture is not productive enough, i.e., $\gamma n < (1 - \gamma) k$, and the world price satisfies $p^W \geq \hat{p}$, then the opening up to trade does not change the occupational structure. When the country has a comparative advantage in the industrial good, gains of unconstrained agents exceed losses of credit-constrained ones and aggregate welfare increases under trade. When the country has a comparative advantage in the agricultural good, there is a net loss if world prices are close to $p^A$. If $p^W < \hat{p}$, then the opening up to trade results in deindustrialization; moreover, if $p^W \in (\underline{p}, \hat{p})$ this deindustrialization is ‘involuntary’. Deindustrialization is immiserizing if the comparative advantage in agriculture is small.
5.2 Large Country Case

In this section we examine the ‘North-South’ trade in which the two countries are identical with respect to technologies and distinguished only by differences in credit markets. The South is a developing country with missing credit markets. The North is a developed country with perfect capital markets: agents can costlessly lend and borrow at the same interest rate. We show that not only do differences in credit markets create comparative advantage in the North for the industrial good, but that the short-run and long-run implications of trade can be very different.

**Autarky Equilibrium in the North.** Since there are perfect credit markets and both goods must be produced, in the steady state equilibrium all agents have the same initial wealth $w^N$ and are indifferent between occupations. Denote by $p^N$ the price of industrial good and by $R^N$ the gross interest rate. Lemma 2 describes the North autarky equilibrium.

**Lemma 2** If agriculture is productive enough, i.e., $\gamma_n \geq (1 - \gamma) k$, then there is no borrowing/lending in the steady state equilibrium: all agents have initial wealth more than $k$. The equilibrium price is $p^N = \underline{p}$.

If agriculture is not productive enough, i.e., $\gamma_n < (1 - \gamma) k$, then in the steady state equilibrium the loan market is active. The interest rate and price are

$$R^N = \max \left\{ 1, \frac{\beta}{\gamma} - \frac{n}{k} \right\}, \quad p^N = \max \left\{ \frac{p}{\gamma a}, \frac{\beta k}{\gamma} \right\}.$$

The formal proof is relegated to Appendix A, but the intuition is as follows.

If the return in agriculture is high enough, then over time the wealth endowment for all agents exceeds the level of investment needed in the industrial sector. Therefore, there is no demand for loans in steady state.

If agriculture is not sufficiently productive, then agents working in industry do not have enough to invest. Hence, they need to borrow from agents working in agriculture. If the demand for the industrial good is quite low, i.e., $\beta$ is relatively small, then the level of production of the
industrial good is low as well. Therefore, demand for loans which comes from the producers of the industrial good lies below supply of loans, and, as a result, the equilibrium gross interest rate equals 1. Note that at this interest rate the price at which agents are indifferent between occupations is exactly \( p \). If the demand for industrial good is high enough, i.e., \( \beta \) is relatively large, then at the interest rate \( R_N = 1 \) demand for loans exceeds its supply and, as a result, the equilibrium gross interest rate is more than 1, and the price at which agents are indifferent between sectors exceeds \( p \).

**Trade Equilibrium.** If the agricultural sector is productive enough relative to the investment needed in the industrial sector, i.e., \( \gamma n \geq (1 - \gamma) k \), the steady state equilibrium in the South is the same as that in the North. Therefore, the two countries have identical autarky prices and opening up the economies to trade has no effect.

The imperfections in the South’s credit markets matter only when \( \gamma n < (1 - \gamma) k \). In this case, let \( \theta^S \) denote the proportion of agents working in agricultural sector in the South in the autarky steady state equilibrium. Recall that the South’s autarky price must be high enough: \( p^S \geq \hat{p} \), which in turn implies that the proportion of agents working in agriculture must be also relatively high: \( \theta^S \geq \hat{\theta} \), where \( \hat{\theta} \) denotes the minimal proportion of credit-constrained agents compatible with autarky steady state equilibrium. Note that the price of the industrial good in the South exceeds its price in the North: \( p^N \leq \beta \frac{k}{\gamma a} < \frac{k}{\gamma a} = \hat{p} \leq p^S \). Therefore, better credit markets create comparative advantage in the North for the industrial good. Lemma 3 summarizes this result.\(^{11}\)

**Lemma 3** Differences in credit markets create comparative advantage in the North for the industrial good.

The trade equilibrium in this case is described in Proposition 4.

\(^{11}\)Ranjan (2001) and Das (2003) also find that the degree of credit market imperfections can become a determinant of the pattern of comparative advantage.
Proposition 4 If agriculture is not productive enough, i.e., $\gamma n < (1 - \gamma)k$, the trade equilibrium is as follows.

I. If $\gamma n \geq (2\beta - \gamma)k$ then the equilibrium price is $p^T = \underline{p}$ and trade results in deindustrialization in the South.

II. If the following conditions are satisfied: $\beta \leq \frac{1}{2}$ and $\gamma n < (2\beta - \gamma)k$, or, $\beta > \frac{1}{2}$ and $\gamma n \leq \frac{(1 - \beta)(1 - \gamma)}{\beta} k$, then the equilibrium price satisfies $p^T \in (\underline{p}, \hat{p})$ and trade results in ‘involuntary’ deindustrialization in the South.

III. If $\beta > \frac{1}{2}$, $\gamma n > \frac{(1 - \beta)(1 - \gamma)}{\beta} k$, and $\theta^S < 2\hat{\theta}$, then the equilibrium price exceeds $\hat{p}$ and trade results in ‘involuntary’ deindustrialization in the South.

IV. If $\beta > \frac{1}{2}$, $\gamma n > \frac{(1 - \beta)(1 - \gamma)}{\beta} k$, and $\theta^S \geq 2\hat{\theta}$, then the equilibrium price exceeds $\hat{p}$ and the autarky occupational structure in the South does not change.

The formal proof is relegated to Appendix A, but the intuition behind these results is as follows.

Consider Figure 5 which depicts the aggregate supply of the industrial good in the steady state. The aggregate supply curve consists of three segments. When the price is $\underline{p}$, the industrial good is produced only by the North. At this price the gross interest rate in the North equals 1 and all agents are indifferent between sectors. At $\underline{p}$ the inherited wealth of each agent is less than $k$, hence, some agents work in agriculture and lend the needed funds to industrial workers. Therefore, the maximal supply of the industrial good is less than $a$, which is the amount produced when all agents work in industry. Thus, the aggregate supply curve is horizontal at $\underline{p}$ as represented by segment $I$. When the price is above $\underline{p}$ but below $\hat{p}$, the industrial good is again produced only by the North. At this price the gross interest rate exceeds unity, in fact it is such that agents are indifferent between sectors. A higher price of the industrial good not only increases returns in the industrial sector, but also raises the bequests, reducing the amount of loan needed for investment. This in turn increases the proportion of agents working in industry and the supply curve is upward sloping, as represented by segment II. For prices
above $\hat{p}$ there are two possible cases. If the price falls below $\hat{p}$ along the convergence path to the steady state and deindustrialization occurs in the South, then even though the steady state price is above $\hat{p}$, the deindustrialization in the South is irreversible and the North is the only producer of the industrial good. At prices above $\hat{p}$ the inherited wealth for agents in the North exceeds $k$, therefore, all agents work in industry. Thus, the supply curve is vertical at level $a$, which is represented by segment III. In the second case, such deindustrialization does not take place, and the industrial good is produced by both countries. In this event, the aggregate supply curve is vertical at level $(2 - \theta^S)a$ and represented by segment IV.

Now we need to relate the type of trade equilibrium, i.e., which segment of the aggregate supply curve intersects demand in the steady state, to the parameters of the model. If the propensity to consume the industrial good is relatively low, i.e., $\beta \leq \frac{1}{2}$, then the aggregate demand for the industrial good is relatively low, and even without specializing in the industrial good, the North can make what is needed by the South. As a result, trade results in a significant fall in the price of industrial good in the South and deindustrialization occurs. In this case, aggregate demand for the industrial good intersects supply at segments I or II. As before, welfare effects of opening up to trade for the South depend on whether negative income distributional effects exceed the gains from trade or not.

If $\beta > \frac{1}{2}$, demand from the South cannot be met at North’s autarky price. There are three possible scenarios. The first scenario involves ‘involuntary’ deindustrialization in the South with the steady state price less than $\hat{p}$ and this takes place if the agricultural sector is significantly unproductive, i.e. $\gamma_n \leq \frac{(1 - \beta)(1 - \gamma)}{\beta} k$. In this case, the low return in agriculture results in low income for credit-constrained agents in the South. Even though these agents spend a large share of their income on the industrial good, the quantity demanded is low because their disposable income is low. As a result, at prices above $\hat{p}$ the aggregate demand is relatively low and the price falls below $\hat{p}$. In this event, demand for the industrial good intersects supply at segment II.
The second scenario occurs when the agricultural sector is not that unproductive, i.e., $\gamma_n > \frac{(1 - \beta)(1 - \gamma)}{\beta} k$, and the supply of the industrial good in the South is relatively low, i.e., $\theta^S \geq 2\hat{\theta}$. In this case the aggregate demand for industrial good is large enough and is accommodated by the aggregate production in the North and the South at the price above $\hat{p}$. As a result, in the steady state the South continues to produce the industrial good, and the intersection of demand and supply occurs at segment $IV$.

The third scenario, which takes place when $\gamma_n > \frac{(1 - \beta)(1 - \gamma)}{\beta} k$ and $\theta^S < 2\hat{\theta}$, is the most interesting. In this event the aggregate demand for the industrial good is large, but the supply in the South is also relatively large. Initially, the price falls below $\hat{p}$ in the South and deindustrialization results. But with deindustrialization in the South the aggregate supply of the industrial good falls significantly, and, as a result, the price starts to rise until it exceeds $\hat{p}$. But even though the price in the steady state is higher than $\hat{p}$, the process of deindustrialization has occurred and now is irreversible. Therefore, in the steady state the industrial good is produced only by the North, and the intersection of demand and supply occurs at segment $III$ in Figure 5.

This example points to the differences in short-run and long-run price effects of trade. In the small country case the world price of the industrial good is exogenous and cannot be affected in the long run. When trading countries are large, the price evolves over time, and may take various paths before converging to its steady state value. Moreover, trade can result in ‘involuntary’ deindustrialization with the steady state price above $\hat{p}$, which cannot occur in the small country case.

6 Enriching the Occupational Structure

The model is augmented to allow for additional occupations which create an active labor market. In the model outlined in the previous sections a world price below $\hat{p}$ forces industrial producers to move to the agricultural sector. However, agricultural workers are unable to move to industry
when the world price of the industrial goods rises! This makes the effect of price changes asymmetric. A richer occupational structure eliminates this weakness of the basic model.

In addition to the two existing occupational choices: agricultural worker and industrial producer (which is now called ‘small-scale entrepreneur’) two new occupational options are introduced. A new technology that allows production of the industrial good by a ‘large-scale entrepreneur’ is posited. An agent can invest $lk$ units of the numeraire good. Once this investment is made, the agent can hire and use his unit of labor to monitor $l \geq 2$ industrial workers, where each worker produces $a$ units of the industrial good. Let $b$ denote the market wage rate. Hence, the payoff from being a large-scale entrepreneur is $(ap - k - b)l$. Therefore, this technology introduces two additional occupations: large-scale entrepreneur and industrial worker (with the return equal to the market wage rate).

Since the objective of this section is to look at the situation when opening up to trade allows credit-constrained agents to move to occupations with higher payoffs, we focus on the case where credit constraints are binding in the autarky steady state: $\gamma n < (1 - \gamma) k$.

### 6.1 Autarky Equilibrium

To derive the autarky equilibrium we analyze how the occupational choice of each agent depends on the price and then derive long-run equilibrium price and the wealth distribution.

Consider Figure 6 which depicts return to each occupation as a function of price. As before, all agents with initial wealth $w_t < k$ are credit-constrained but now have two choices: to work in agriculture or to become an industrial worker. What they choose depends on the wage rate. Hence, if the labor market is active, the wage rate must be equal to the return in agricultural sector: $b = n$. Therefore, horizontal line at the level $n$ represents the payoff from being agricultural or industrial worker.

The payoff from working as a small-scale (large-scale) entrepreneur is represented by a straight line SSE (LSE). Note that SSE is flatter than LSE and intercepts the horizontal axis closer to
the origin.\footnote{Recall that SSE is given by the equation $(ap - k)$, while for LSE the equation is $(ap - k - b)l$, where $b = n$. Therefore, SSE is flatter since $a < al$, and its horizontal intercept is less than that for LSE: $\frac{k}{a} < \frac{k + n}{a}$. Note that the horizontal intercept for LSE equals $p$.}

Agents with initial wealth $w_t \in [k, lk)$ have two options: either to become small-scale entrepreneurs or to work in agriculture. The intersection of SSE and the horizontal line at the level $n$ occurs at $p_L \equiv \frac{k + n}{a}$. Hence, if the price is above $p_L$, then all these agents choose to become small-scale entrepreneurs.

Agents with initial wealth $w_t \geq lk$ have all possible options. Similarly, the intersection of LSE and SSE lines occurs at $p_H \equiv \frac{k + n}{a}l - 1$. Therefore, if the price is above $p_H$, then all agents with initial wealth more than $lk$ choose to become large-scale entrepreneurs. Note that the labor market is active only when the price is above $p_H$.

Next, we turn to the analysis of the economy in the long run.

Recall that in previous sections $\hat{p}$ denotes the minimal price at which the offspring of non-credit-constrained agents are also unconstrained and able to invest. Introduction of two levels of investment results in three price thresholds.

Let $\hat{p}_L = \frac{k}{\gamma a}$ (which corresponds to $\hat{p}$ in the model from previous sections) be the minimal price at which the offspring of small-scale entrepreneurs are able invest $k$ and become small-scale entrepreneurs. Similarly, the price $\hat{p}_H$ is the minimal price at which the offspring of large-scale entrepreneurs are not credit-constrained and able to become large-scale entrepreneurs. Solving $\gamma(lk + (ap_H - k) l) = lk$ gives $\hat{p}_H = \frac{k + \gamma n}{\gamma a} > \hat{p}_L$. If the price is above $\hat{p}_H$, then all large-scale entrepreneurs leave more than $lk$ as bequests, and, as a result, the group of potential large-scale entrepreneurs does not shrink.

Finally, let the price $\hat{p}_{HL}$ be the minimal price at which the offspring of small-scale entrepreneurs are able to become large-scale entrepreneurs. Solving $\frac{\gamma(ap_{HL} - k)}{1 - \gamma} = lk$ gives $\hat{p}_{HL} = \frac{((1 - \gamma)l + \gamma)k}{\gamma a} > \hat{p}_H$. If the price is above $\hat{p}_{HL}$, then all small-scale entrepreneurs leave
more than \(lk\) as bequests, therefore, the group of small-scale entrepreneurs shrinks and the group of large-scale ones grow.

Since both goods are essential for consumers, in equilibrium the industrial good must be produced. Hence, it is produced either by small-scale entrepreneurs, or by large-scale ones, or by both. We characterize the possible steady state equilibria according to the types of entrepreneurs producing the industrial good.

Type 1. There are only small-scale entrepreneurs in the steady state. Note that in this case the price in the steady state must be in the range \(p^A \in [\hat{p}_L, \hat{p}_H]\). If the price is below \(\hat{p}_L\) then the class of small-scale entrepreneurs shrinks over time. If the price is above \(\hat{p}_H\) then the class of large-scale entrepreneurs emerges.

Type 2. There are small-scale entrepreneurs and large-scale entrepreneurs, hence, the equilibrium price in the steady state satisfies \(p^A \in [\hat{p}_H, \hat{p}_{LH}]\). Similarly, for the prices below \(\hat{p}_H\) the class of large-scale entrepreneurs disappears in the long run and for prices above \(\hat{p}_{LH}\) the class of large-scale entrepreneurs grows over time.

Type 3. There are only large-scale entrepreneurs. In this case the equilibrium price is above \(\hat{p}_{LH}\). Small-scale entrepreneurs become large-scale ones and remain there.

The type of equilibrium that emerges in the long run depends on the relationship between \(\underline{p}_H\) and \(\hat{p}_L\). Note that inequalities \(\underline{p}_L < \underline{p}_H < \hat{p}_H < \hat{p}_{LH}\) and \(\underline{p}_L < \hat{p}_L\) are always satisfied for \(\gamma n < (1 - \gamma)k\) and \(l \geq 2\). Moreover, from formulas above it follows that \(\underline{p}_H\) is increasing with \(n\), while \(\hat{p}_L\) does not depend on \(n\). When agriculture is quite unproductive, i.e., \(\gamma n < \frac{l - 1}{l} (1 - \gamma)k\), then \(\underline{p}_H\) is less than \(\hat{p}_L\). When the return in agriculture increases, \(\underline{p}_H\) increases as well and for medium levels of productivity in the agricultural sector, i.e., \(\gamma n \in \left[\frac{l - 1}{l} (1 - \gamma)k, (1 - \gamma)k\right]\), \(\underline{p}_H\) exceeds \(\hat{p}_L\).

Proposition 5 describes steady state equilibrium in autarky.

**Proposition 5** If agriculture is unproductive, i.e., \(\gamma n < \frac{l - 1}{l} (1 - \gamma)k\), then there must exist large-scale entrepreneurs in the autarky steady state equilibrium, i.e., depending on the initial
wealth distribution the autarky steady state equilibrium is either of Type 2 or of Type 3.

If the agricultural sector is of medium productivity, i.e., \( \gamma_n \in \left[ \frac{l - 1}{l} (1 - \gamma) k, (1 - \gamma) k \right] \), then, in addition, there may exist small-scale entrepreneurs only.

**Proof.** The logic behind these results is similar to the one behind Proposition 2. First, by the same reasoning as before, the price in the steady state cannot be less than \( \hat{p}_L \). Hence, we can focus on \( p \geq \hat{p}_L \).

If agriculture is very unproductive, then the following inequality is satisfied: \( p < \hat{p}_H < \hat{p}_L \). In this case the price in the steady state must be above \( \hat{p}_H \). If \( p < \hat{p}_H \) then the fraction of the population that works as large-scale entrepreneurs\(^{13}\) must shrink. As a result, supply of the industrial good shifts in. Demand also shifts in as the small-scale entrepreneurs earn less than the large-scale ones, but since only part of their income is spent on the industrial good, the shift in demand is less than that of supply, so that the price rises. This will continue till \( p \) reaches \( \hat{p}_H \). Therefore, in the steady state large-scale entrepreneurs must exist. The equilibrium is of Type 2 or Type 3 depending on how high the equilibrium price is. If the price is above \( \hat{p}_{LH} \) then there are only large-scale entrepreneurs, otherwise small-scale entrepreneurs and large-scale ones coexist in the steady state.

In the case when the agricultural sector is of medium productivity, the following is satisfied: \( \hat{p}_L < p < \hat{p}_H \). In this case the price in the steady state can be below \( \hat{p}_H \). At this price all agents with wealth more than \( k \) choose to work as small-scale entrepreneurs. Note also that at this price the class of small-scale entrepreneurs does not shrink and does not grow. Hence, the price \( p \in \left[ \hat{p}_L, \hat{p}_H \right] \) is sustainable for some initial wealth distributions, and in this case the steady state equilibrium is of Type 1. If the price is above \( \hat{p}_H \) but below \( \hat{p}_{LH} \) then the equilibrium is of Type 2, and if it is above \( \hat{p}_{LH} \) then of Type 3. \( \blacksquare \)

Unproductive agriculture results in low wage rate, since \( b = n \). Therefore, low labor costs lead to large profits for large-scale entrepreneurs, and they always exist in the long-run equilibrium.

\(^{13}\)Note that such class of agents potentially exists since the price is above \( \hat{p}_L \).
If the agricultural sector is of medium productivity, then for some economies the industrial good can be produced only by small-scale entrepreneurs, since high wages and relatively low price make large-scale entrepreneurship unsustainable.

6.2 Effects of Trade

Having described the closed economy, we turn to the analysis of the effects of opening up to trade on labor mobility for a small country case. Let $p^W$ be the world price of the industrial good. Note that the introduction of a labor market does not preclude the possibility of immiserizing deindustrialization. For example, if the world price is below $\hat{p}_L$ then complete deindustrialization occurs independent of type of autarky equilibrium: over time all entrepreneurs leave bequests less than $k$ and their progeny work in the agricultural sector. As before, welfare effects of such deindustrialization depend on whether negative income distributional effects exceed gains from trade or not. Similarly, if the world price is $p^W \in (\hat{p}_L, \hat{p}_H)$, then in the trade equilibrium only small-size entrepreneurs survive: over time large-scale entrepreneurs leave bequests less than $lk$ and their offspring become small-scale entrepreneurs.

In the case when the autarky steady state equilibrium is of Type 2 or Type 3 and the world price is $p^W \in (\hat{p}_L, \hat{p}_{LH})$ the opening up to trade does not change the occupational structure relative to the autarky. The same occupational structure remains for world prices above $\hat{p}_{LH}$ when the autarky equilibrium is of Type 3. Note that in all these cases the opening up to trade does not move workers to occupation with higher payoffs: with trade they earn the same return $n$.

The only case when the opening up to trade results in the change of occupation for workers is as follows. First, in order to increase wages, opening up to trade must allow small-scale entrepreneurs to become large-scale ones, therefore, the autarky equilibrium has to be of Type 1 or 2, so that small-scale entrepreneurs exist in autarky. Second, the world price must be above $\hat{p}_{LH}$ to allow small-scale entrepreneurs to become large-scale ones. Third, the potential labor
supply, which consists of credit-constrained agents who are not able to invest into the production of the industrial good, cannot satisfy the demand for labor, which increases with trade.

**Proposition 6** If the initial wealth distribution is such that there exist small-scale entrepreneurs in the autarky equilibrium and the proportion of agents working in the steady state either as industrial or agricultural workers is relatively small, i.e., \( \theta < \frac{l}{l+1} \), then if the world price of the industrial good is high enough, i.e., \( p^W \geq \hat{p}_{LH} \), opening up to trade leads to perfect equality: all agents in the economy work as small-scale entrepreneurs.

The formal proof is relegated to Appendix A, but the intuition is as follows.

When the world price of the industrial good is high enough, i.e., \( p^W \geq \hat{p}_{LH} \), the income of small-scale entrepreneurs increases and over time they are able to leave more than \( lk \) to their offspring. Therefore, with trade the class of large-scale entrepreneurs grows and, as a result, the demand for labor increases. If the proportion of credit-constrained agents is relatively small, then the increased labor demand exceeds labor supply at the current wage equal to \( n \). Wage rate starts to increase, until it rises above \( \frac{1 - \gamma}{\gamma} k \). At this wage industrial workers leave more than \( k \) as bequests, and their offspring are able to become small-scale entrepreneurs, and since the price is high enough, they will eventually be able to invest \( lk \) and become large-scale entrepreneurs. Therefore, the opening up to trade makes industrial workers non-credit-constrained. The wage rate which makes agents indifferent between being worker and large-scale entrepreneur is very high and makes large-scale entrepreneurship unprofitable, and, as a result, all agents become small-scale entrepreneurs.

This case points to the role of inequality in determining when trade can become a real engine of growth. If economy is too unequal, i.e., the proportion of ‘poor’ workers is relatively large, opening up to trade does not allow occupational mobility and, therefore, does not lead to an industrial boom. This may help explain the dissimilarities in the economic performances of Korea and the Philippines.
7 Conclusions

This paper constructs a simple model where trade liberalization may have adverse wealth distributional effects when credit markets are imperfect. If the world price of the industrial good is below the bequest-sustaining level, opening up to trade results in deindustrialization. Moreover, we show that deindustrialization is welfare reducing if the comparative advantage is small, so that the negative wealth distributional effects swamp the gains from trade. ‘North-South’ case points to the differences in short-run and long-run price effects of trade. In the small country case the world price of the industrial good is exogenous and cannot be affected in the long run. When trading countries are large, the price evolves over time, and may take various paths before converging to its steady state value. Moreover, trade can result in ‘involuntary’ deindustrialization with the steady state price above the bequest-sustaining level, which cannot occur in the small country case. Finally, the setting with labor market shows that the level of inequality plays an important role in determining when trade can become a real engine of growth.

Appendix A

Proof of Lemma 2. Since there are perfect credit markets and both good must be produced, in the steady state equilibrium all agents must have the same wealth endowment \( w^N = \frac{\gamma n}{(1 - \gamma R)} \) and must be indifferent between occupations. Hence, the following condition holds:

\[ ap - Rk = n \]  

(5)

Agents borrow and lend non-zero amounts only if \( w^N < k \). In the case of \( \gamma n \geq (1 - \gamma) k \), the initial wealth \( w^N \geq k \) for all \( R \geq 1 \), therefore, there is no borrowing/lending in this case.

Consider now the case when \( \gamma n < (1 - \gamma) k \). Note that \( p > \frac{1}{2} \) if \( R > 1 \). Denote by \( \delta \) the proportion of agents working in industry. The supply of loans comes from agents working in
agriculture and the demand for loans is from agents working in industry:

\[ S^L = (1 - \delta) w^N, \quad D^L = \delta (k - w^N) \]  

(6)

Then equilibrium in the loans market is given by

\[ R = 1, \text{ if } S^L|_{R=1} > D^L|_{R=1} \]  

(7)

\[ R : S^L = D^L, \text{ otherwise} \]

From (5) the price of industrial good equals

\[ p = \frac{n + Rk}{a} \]  

(8)

The supply and the demand for industrial good are

\[ S^2 = \begin{cases} \delta a, & \text{if } p > \underline{p} \\ \in [0, \delta a], & \text{if } p = \underline{p} \end{cases}, \quad D^2 = \frac{\beta Y}{p} = \frac{\beta n}{p(1 - \gamma R)} \]

Then equilibrium in the market for industrial good is given by intersection of supply and demand:

\[ S^2 = D^2 \]  

(9)

The solution to (7), (8), and (9) gives the equilibrium interest rate and price of industrial good:

\[ R^N = \frac{\beta}{\gamma} \frac{n}{k} > 1, \quad p^N = \frac{\beta k}{\gamma a} > \underline{p} \text{ if } n < \frac{\beta - \gamma}{\gamma k}, \]

\[ R^N = 1, \quad p^N = \underline{p}, \text{ otherwise} \]

\begin{flushright}
\[ \blacksquare \]
\end{flushright}

**Proof of Proposition 4.** Note that the price in the steady state trade equilibrium must satisfy \( p^T \geq \underline{p} \) since at prices below \( \underline{p} \) all agents in both countries choose to work in agriculture and industrial good is not produced. Hence, there are two possible cases for equilibrium price: either \( p^T \in [\underline{p}, \hat{p}) \), or \( p^T \geq \hat{p} \).

Suppose that \( p^T \in [\underline{p}, \hat{p}) \). In this case trade results in deindustrialization in the South: the South produces only agricultural good and imports industrial good. Since \( p^T \leq \hat{p} \), some
proportion of agents in the North must work in agricultural sector in order to lend \((k - w^N)\) to agents working in industrial sector, as \(w^N = \frac{\gamma}{1 - \gamma} (ap - k) < k\). Interest rate is linked to the price by (5). Denote by \(\delta\) the proportion of agents working in industry in the North. Using (5) and (6) the supply of industrial good is

\[
S = \delta a = \begin{cases} 
\frac{\gamma n}{k - \gamma (ap - n)} a, & \text{if } p^T > \hat{p}; \\
\left[0, \frac{\gamma n}{(1 - \gamma) k}\right], & \text{if } p^T = \hat{p}.
\end{cases}
\]

Excess demand in the South is

\[
ED^S = \frac{\beta}{1 - \gamma} \frac{n}{p}
\]

and excess supply in the North is

\[
ES^N = \frac{n}{1 - \gamma k} \left\{ \begin{array}{ll}
\frac{n}{k - \gamma (ap - n)} \left(\frac{\gamma a - \frac{\beta k}{p}}{p}\right), & \text{if } p^T > \hat{p} \\
\left[0, \frac{na}{(1 - \gamma) k} \frac{\gamma (n + k) - \beta k}{n + k}\right], & \text{if } p^T = \hat{p}
\end{array} \right.
\]

(10) and (11) intersect at the price

\[
p^T = \left\{ \begin{array}{ll}
\frac{(2 - \gamma) k + \gamma \beta n}{\gamma (1 + \beta - \gamma)}, & \text{if } n < \frac{2\beta - \gamma}{k} \\
\hat{p}, & \text{otherwise.}
\end{array} \right.
\]

Since we are in case \(\gamma n < (1 - \gamma)\), the inequality \(n < \frac{2\beta - \gamma}{k}\) holds for all \(\beta > \frac{1}{2}\). The price satisfies condition \(p^T < \hat{p}\) if \(\gamma n < \frac{1 - \beta}{\beta} (1 - \gamma) k\).\(^{14}\)

Suppose now that \(p^T \geq \hat{p}\). There are two possible scenarios: the occupational structure in the South does not change with trade, or there is deindustrialization. Consider the first scenario first. At this price the interest rate in the North is \(R^N = 1\) and all agents work in the industrial sector. Excess demand in the South is

\[
ED^S = \frac{\beta}{1 - \gamma} \frac{\theta^S n + \left(1 - \theta^S\right) (ap - k)}{p \left(1 - \theta^S\right) a} = \frac{\beta}{1 - \gamma} \frac{\theta^S n - \left(1 - \theta^S\right) k}{p} = \frac{\alpha}{1 - \gamma} \frac{\left(1 - \theta^S\right) a}{a}
\]

\(^{14}\)This inequality is always satisfied if \(\beta \leq \frac{1}{2}\).
and excess supply in the North is

\[ ES^N = a - \frac{\beta}{1 - \gamma} \frac{ap - k}{p} = \frac{\alpha}{1 - \gamma}a + \frac{\beta k}{(1 - \gamma)p} \]  

(13)

(12) and (13) intersect at the price

\[ p^T = \frac{\beta \theta^S n - (2 - \theta^S k)}{\alpha 2 - \theta^S} \]

Simple calculations show that \( p^T \) is below \( \hat{p} \) for all \( \theta^S \in \left[ \hat{\theta}, 1 \right] \) if \( \beta \leq \frac{1}{2} \). In the case when \( \beta > \frac{1}{2} \) the price is below \( \hat{p} \) if \( \gamma_n < \frac{1 - \beta}{\beta} (1 - \gamma) k \). If the last inequality is not satisfied, then \( p^T \geq \hat{p} \) for \( \theta^S \geq 2\hat{\theta} \).

The second scenario occurs when trade equilibrium involves deindustrialization in the South and \( p^T \geq \hat{p} \). This situation happens for \( \theta^S \in \left[ \hat{\theta}, 2\hat{\theta} \right] \) when \( \beta > \frac{1}{2} \) and \( \gamma_n \geq \frac{1 - \beta}{\beta} (1 - \gamma) k \).

**Proof of Proposition 6.** The industrial workers remain credit-constrained in the steady state if the wage rate is low enough: \( \frac{\gamma b}{1 - \gamma} < k \), or \( b < \frac{1 - \gamma}{\gamma} k \). If the wage is above \( \frac{1 - \gamma}{\gamma} k \) then the industrial workers are able to become small-scale entrepreneurs, and since the world price is high enough, \( p^W \geq \hat{p}_{LH} \), they will eventually be able to invest \( lk \) and become large-scale entrepreneurs.

The demand for labor exists when the return from working as a large-scale entrepreneur exceeds that of a small-scale one, i.e.,

\[ (ap^W - k - b) l > ap^W - k. \]

Thus, the maximum wage when the demand for labor is positive equals \( \frac{l - 1}{l} (ap^W - k) \). Note that this wage exceeds \( \frac{1 - \gamma}{\gamma} k \). At this wage agents prefer to become entrepreneurs rather than workers:

\[ (ap^W - k - b) l > b \]

Therefore, if the wage is above \( \frac{1 - \gamma}{\gamma} k \) and below \( \frac{l - 1}{l} (ap^W - k) \) then all agents prefer to be large-scale entrepreneurs, and the supply of labor is zero. Thus, the wage in this interval cannot be an equilibrium wage.
If the wage rate is less than $\frac{1 - \gamma}{\gamma} k$, then small-scale entrepreneurs choose to become large-scale ones. However, the supply of labor is not sufficiently large to satisfy the increased demand for labor: $\theta < \frac{l}{1 + l}$. Thus, the wage $b < \frac{1 - \gamma}{\gamma} k$ cannot be an equilibrium wage either.

This implies that there is no wage at which labor demand and supply intersect, hence, the labor market is not active. Therefore, in the state trade equilibrium all agents work as small-scale entrepreneurs. ■

Appendix B

**Altruistic Preferences.** Consider the model described in Section 2. Suppose now that dynasties are linked by fully altruistic preferences as in Barro (1974). Then generation $t$ payoff is given by

$$\sum_{\tau=1}^{\infty} \beta^{T-t} u(c_{A,\tau}, c_{I,\tau}),$$

where $u(c_{A,\tau}, c_{I,\tau})$ is one-period utility function, which depends on $c_{A,\tau}$ (consumption of the agricultural good) and $c_{I,\tau}$ (consumption of the industrial good), $\beta < 1$ is discount rate.

The objective of this analysis is to show that the main result of the paper, namely, that opening up to trade may result in involuntary deindustrialization, is robust to the specification of preferences. In order to show that we need to identify the range of world prices that result in involuntary deindustrialization for altruistic preferences.

We restrict our analysis to the particular Cobb-Douglas specification of the one-period utility function:

$$u(c_{A}, c_{I}) = c_{A}^{\alpha} c_{I}^{1-\alpha}$$

This results in indirect utility being linear in income spent on consumption and precludes the transfer of wealth across generations for consumption purposes. The only motive for bequests is to provide future generations with investment opportunities.
The agent’s problem can be rewritten in the following form:

$$V_t(w_t, p_t) = \max_{i_t \in \{\text{agriculture, industry}\}, w_{t+1}} \left\{ \frac{A}{p_t^{1-\alpha}}(w_t - w_{t+1} + R_{i_t}) + \beta V_{t+1}(w_{t+1}, p_{t+1}) \right\},$$

where $i_t \in \{\text{agriculture, industry}\}$ is the occupational choice, $R_{i_t}$ is the earned income:

$$R_{i_t} = \begin{cases} n, & \text{if } i_t = \text{agriculture} \\ ap_t - k, & \text{if } i_t = \text{industry} \end{cases},$$

and $A = \alpha^\alpha (1 - \alpha)^{1-\alpha}$.

This specification of preferences makes analytics intractable, especially outside steady state. Thus, we restrict our analysis only to steady state outcomes.

**Autarky.** We construct steady state autarky equilibrium for the following mutually exhaustive cases: $n \geq k$ and $k - in \leq n < k - (i-1)n$, $i = 1, 2, ...$, in turns.

Consider the case $n \geq k$. In such a case an agent working in agriculture is able to leave $k$ as a bequest to his offspring. Therefore, at most one generation is needed to switch from agriculture to industry.

**Lemma 4** If $n \geq k$ there are two groups of agents in the steady state autarky equilibrium: constrained agents with initial wealth $w = 0$, who work in agriculture and leave bequests of 0, and unconstrained agents with initial wealth $\bar{w} = k$, who work in industry and leave bequests of $k$. The price is

$$p^A = \frac{k + \beta n}{\beta a}.$$  

**Proof.** Denote by $V_1(0)$ the indirect utility of an agent with initial wealth $w_t = 0$, who works in agriculture and leaves bequest $w_{t+1} = 0$. Similarly, $V_2(k)$ is the indirect utility of an agent with initial wealth $w_t = k$, who works in industry and leaves bequest $w_{t+1} = k$. Price $p$ is compatible with steady state if and only if

$$V_1(0) = \frac{An}{p^{1-\alpha}} + \beta V_1(0) \geq \frac{A(n - k)}{p^{1-\alpha}} + \beta V_2(k); \quad (14)$$

$$V_2(k) = \frac{A(ap - k)}{p^{1-\alpha}} + \beta V_2(k) \geq \frac{A(ap)}{p^{1-\alpha}} + \beta V_1(0). \quad (15)$$

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Inequality (14) represents the non-deviation condition for agents with initial wealth \( w_t = 0 \). The left hand side is the payoff from consuming all income and leaving bequest of \( w_{t+1} = 0 \), so that the next generation continues to work in agriculture. The right hand side is the payoff from sacrificing consumption by \( k \), so that the next generation is able to switch to more profitable occupation: working in the industrial sector.

Inequality (15) represents the non-deviation condition for agents with initial wealth \( w_t = k \). The left hand side is the payoff from leaving bequest of \( w_{t+1} = k \), so that the next generation continue to work in industry. The right hand side is the payoff from increasing consumption by \( k \), so that the next generation do not have sufficient initial wealth and have to switch to less profitable occupation: working in agriculture.

Inequalities (14) and (15) are equivalent to:

\[
\frac{Ak}{p^{1-\alpha}} = \beta (V_2(k) - V_1(0)) = \beta \frac{A (ap - k - n)}{1 - \beta} \frac{p^{1-\alpha}}{p^{1-\alpha}}
\]

or,

\[
p = \frac{k + \beta n}{\beta a}
\]

Now consider the case when \( i \geq 1 \) generations of agricultural workers are needed to accumulate enough wealth, so that \((i + 1)^{th}\) generation is able to switch from agriculture to industry.

**Lemma 5** If \( n \in \left[ \frac{k}{i + 1}, \frac{k}{i} \right] \) there are two groups of agents in the steady state autarky equilibrium: constrained agents with initial wealth \( w = 0 \), who work in agriculture and leave bequests of 0, and unconstrained agents with initial wealth \( w = k \), who work in industry and leave bequests of \( k \). The price satisfies

\[
p^A \in \left[ \frac{k + \beta a}{\beta a}, \frac{(1 - \beta + \beta^{i+1}) k + (\beta + \beta i - i) n}{\beta^{i+1} a} \right]
\]
Proof. Price \( p \) is compatible with steady state if and only if

\[
\begin{align*}
V_1(0) &= \frac{An}{p^{1-\alpha}} + \beta V_1(0) \geq \frac{A(n - (k - in))}{p^{1-\alpha}} + \beta^{i+1} V_2(k); \\
V_2(k) &= \frac{A(ap - k)}{p^{1-\alpha}} + \beta V_2(k) \geq \frac{A(ap)}{p^{1-\alpha}} + \beta V_1(0)
\end{align*}
\] (16) (17)

Inequality (16) is similar to (14) The left hand side is the payoff from consuming all income and leaving bequest of \( w_{t+1} = 0 \). The right hand side is the payoff from sacrificing consumption of \((k - in)\), so that the \((i + 1)^{th}\) generation is able to switch to more profitable occupation: working in industry. Inequality (17) represents the non-deviation condition for agents with initial wealth \( w_t = k \) and is the same as (15) because industrial workers are always able to switch to agriculture in the next generation.

From (16) and (17) it follows that the price must satisfy

\[
\frac{k + \beta n}{\beta a} \leq p^d \leq \frac{(1 - \beta + \beta^{i+1}) k + (\beta + \beta i - i) n}{\beta^{i+1} a}
\]

\[
\square
\]

Small Open Economy. If the world price is \( p^W < \frac{k + \beta n}{\beta a} \) then conditions (15) and (17) are violated. Therefore, agents with initial wealth \( w_t = k \) choose bequests of \( w_{t+1} = 0 \) and deindustrialization results. If the world price satisfies \( p^W \in \left[ \frac{k + \beta n}{\beta a} \right] \) then deindustrialization is ‘involuntary’ in the sense that even though the price is high enough, that working in the industrial sector yields a higher one-period income than working in the agricultural sector, it is not high enough to provide incentives to sacrifice today’s consumption for future investment.
References


$\gamma(w_t + \alpha p - k) = \gamma(w_t + \alpha p - k)$

$k$

$w_t$

$w_t + 1$

Figure 1 (a).
Figure 1 (b).
Figure 2.
Figure 3.
Figure 4 (a).
Figure 4 (b).
Figure 5.
Figure 6.