GLOBALIZATION, INCREASING RETURNS, AND TRADE IN INTERMEDIATE INPUTS

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Abstract

Studies that analyze the effect of globalization on vertical specialization or organizational fragmentation on one hand and internalization of multinational enterprises on the other hand in general conclude that globalization facilitates either one or the other. We will argue that both can be influenced by globalization, as internal and external economies of scale interact in affecting the formation of the equilibrium vertical industrial structure.

A three-sector model of final, intermediate and non-tradable goods is developed that incorporates increasing returns to analyze the interaction between internal and external economies. We use the Dixit-Stiglitz-Ethier framework to show that with two factors of production in the monopolistically competitive intermediate good sector, gains from specialization depend only on capital, while gains from integration (internalization) face a trade-off between labor and capital as the size of the market enlarges. This is in contrast to a monopolistically competitive model that utilizes only labor input. Firms' endogenized pricing decisions however may allow taking the advantage of both internal and external economies as globalization occurs and the industry size stays small. If the size of the market augments through trade, then it will be shown that free trade in intermediate specialized inputs and a final consumption good will equalize factor prices and that the endowment basis determines the direction of trade. Compared to autarky, trade will enhance fragmentation in a relatively capital abundant country and diminish it in a relatively labor abundant country.

1   Introduction

Globalization in the world economy has brought about significant changes in the structure and location of the production of firms. Globalization, as Deardorff (2001) notes, has in economic literature been represented variously by international trade, foreign direct investment (FDI), factor mobility, and fragmentation. While earlier research in international trade examined the options of

the firms to either export or invest into a local production unit of a foreign country, the emergence of a multinational enterprise (MNE) has added many more sides to the story.

International trade in intermediate specialized inputs nowadays features as a prominent byproduct of globalization. As Grossman and Helpman (2002b) note, "this is the age of outsourcing". Firms outsource or fragment an expanding range of activities by delegating the production of intermediate inputs or offering after-sale service to outside contractors. For example, Hummels et al. (2001) try to determine the extent of vertical specialization in the world economy and report that the growth in the exports that use imported goods as inputs has accounted for 21 per cent of the OECD countries' exports in the 1990ties and that vertical specialization has grown almost 30 per cent between 1970 and 1990. Shy and Stenbacka (2003) identify that in Finland, subcontracting constituted approximately 50 percent of the sales of Finnish manufacturing (excluding energy industries) in 1996, while the magnitude of fragmentation was estimated to have increased for 30% from 1993-1996. There exist a variety of models that presently undertake to account for fragmentation and it's importance in the global economy, including Grossman and Helpman (2002a,b), Feenstra and Hanson (1996, 1998), Arndt (2001), Kohler (2001) and Kleinert (2002) among many others.

On the other spectrum from fragmentation lies the evidence and literature on how globalization enforces large-scale mergers and acquisitions (M&A), both inside and across the borders, increasing internalized transactions. For example, UNCTAD reports that since 1994, M&A have grown sixfold to dominate FDI. Already the early emergence of the theory on multinationals, such as Helpman (1984) and Markusen (1984), took the presumption that firms chose to expand their operational possibilities based on some advantage – Dunning's paradigm foresaw that the MNE would expand abroad either due to ownership advantages, locational advantages or internalization consideration. This branch of literature has however been unable to relate to a significant intra-industry interaction across borders.

It then follows that studies analyzing the effect of globalization on vertical specialization or organizational fragmentation on one hand and internalization of multinational enterprises on the other hand conclude that globalization facilitates either one or the other. We will argue, similar to Eckel (2003), that both are simultaneously influenced by globalization and that the outcome on the formation of the equilibrium vertical industrial structure may be more complicated than initially considered.

As Eckel (2003) identifies, the question of whether fragmentation or integration is enforced by globalization, amounts on a theoretical level to studying the relationship between the size of an economy and its degree of specialization. In an influential paper, Ethier (1982) developed a framework that specifically showed how scale economies resulting from an increased division of labor rather than from, for example, an increased plant size, depend at an aggregate level upon the size of the world market and not the national market. In other words, production does not need to be geographically concentrated to be able to reap gains from specialization. Ethier (1982) proceeded to develop a framework in which international returns depend on an interaction between the two types of scale economies, internal and external (to the firm), utilizing monopolistically competitive Dixit-Stiglitz "love-of-variety" setup for intermediate components.

Since the solution to the Ethier (1982) model implies a firm size that is fixed, anything that enhances the size of the market would then lead to increased specialization and fragmentation. Eckel (2003) disentangles the various advantages of specialization (external economies) and integration (internal economies), and by utilizing different cost functions, is able to determine whether an industry gets involved in a vertical specialization or integration.

The purpose of this paper is twofold. *First*, we utilize a different approach than Eckel (2003) to be able to overcome the prediction of the Ethier (1982) model that the firm size is fixed. We introduce two factors of production (labor and capital) into Ethier's (1982) setup and will thereby avoid the elimination of the effect resulting from factor price differences. We develop a

three-sector model of final, intermediate and non-tradable goods that incorporates internal and external economies, to show that the impact of the interaction between the economies of scale now allows us to reach new understanding of the formation of the equilibrium vertical industrial structure. The firm size in such a framework is able to vary. We next allow the markup in the monopolistically competitive sector to be endogenized to account for the competition effects and the resulting impact on integration and fragmentation. It is shown that the number of firms active in producing the specialized components will have an impact on the equilibrium outcome of the interaction between internal and external economies. *Second*, we examine trade in specialized intermediate inputs as a source of an enlargement of the market and show how the pattern of fragmentation and trade evolves.

The rest of the paper is structured as follows. Section 2 develops the three-sector of production model and presents all assumptions. Section 3 solves for the general equilibrium outcome of the model with exogenous and endogenous markups. In section 4 we allow trade in specialized intermediate inputs and a final good to study the implications on the pattern of trade and fragmentation. Section 5 offers concluding comments.

2   The Model

Consider an economy consisting of three sectors of production: a final manufacturing good sector Q_m, an intermediate producer good sector I_m and a non-tradable good sector Q_s. The final good sector and non-tradable good sector are perfectly competitive with constant returns to scale in Cobb-Douglas fashion and have firms that are price takers at both output and input markets. The intermediate good sector on the other hand exhibits Ethier's (1982) formulation of economies of scale founded on the Dixit-Stiglitz love-of-variety approach.

Mathematically, the final manufacturing good sector's production function is given by

$$Q_m = L_m{}^\gamma I_m{}^{1-\gamma} ,$$
(1)

where L_m is the labor directly employed in producing the final good and $0 < \gamma < 1$ is the factor share of labor in output.

The CES type intermediate good sector is expressed as

$$I_m = n^{\frac{1}{\rho} - \frac{\sigma}{\sigma-1}} \left(\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} ,$$
(2)

where x_i is the output of an individual intermediate component, I_m the total output of the intermediates, n is the number of suppliers of specialized components, $\sigma > 1$ denotes the elasticity of substitution between the various components allowing for imperfect substitutability, and $0 < \rho < 1$ implies scale economies resulting from an increased division of labor, as addressed by Ethier (1982).

Given the symmetry by which individual components enter the production function of the intermediate goods and the similarity of the cost functions as will be discussed below, in the equilibrium the amount of output of each component producer will be the same or $x_i = x$. Then the intermediate good production function reduces to

$$I_m = n^{\frac{1}{\rho}-1}(nx) = n^{\frac{1}{\rho}}x \ , \tag{3}$$

such that the aggregate output quantity of all produced components in the industry is nx. In such production context n can be interpreted as successive stages in the manufacturing of the intermediate good (Ethier (1982)). As Eckel (2003) explains then, if the industry was perfectly integrated, the entire output nx would be produced by a single producer, or $n = 1$. If the industry was perfectly fragmented, nx would be spread out over an infinite number of component producers, or $n \to \infty$ and $x \to 0$. Then $n \in [1, \infty]$ would measure the industry's degree of fragmentation. However, as the industry's degree of fragmentation rises, so does the degree of specialization as the scope of activities performed by each individual producer would fall. Hence there can be two types of gains from specialization achieved from an increased division of labor across firms: first, gains from producing a variety of imperfectly substitutable components (horizontal specialization) and second, gains from fragmentation by slicing up the chain of production into various components (vertical specialization), that is the focus of this paper. This increased division of labor has an effect in this model through ρ, as $n^{\frac{1}{\rho}-1}$ represents a shift parameter in the intermediates' production implying an existence of endogenous external economies of scale to individual firms (Rivera-Batiz and Rivera-Batiz (1991)). Consequently, an increase in the number of manufactured components through an increased fragmentation will lead to more specialization by firms and yield higher marginal and average productivity in the intermediates' production. Notice that if the value of $\rho \to 1$, the exponent of the external effect approaches zero and the gains from specialization vanish. As the value of $\rho \to 0$, the external effect from the division of labor becomes more pronounced and the gains from specialization explode. Such parametric external economies are treated as constant by each firm in the intermediate good industry (Chipman (1970)) as in the market equilibrium each firm maximizes its profit subject to internal economies of scale and zero profit constraint. Finally, we assume that the division of labor is limited by the extent of the market as otherwise the gains from specialization would lead to the production of an infitesimal amount of an infinite number of components (Ethier (1982)).

The production of an individual component x requires both capital and labor input in this monopolistically competitive industry. In particular, capital is assumed to enter as a fixed input and labor as a variable input, such that the cost function of each representative firm is

$$c_i = r\theta + w\lambda x_i \ , \tag{4}$$

where θ denotes the fixed capital requirement and λx_i is labor demanded by each component producer, r is the interest rate and w is the wage rate. Following the standard Chamberlinian framework, and for simplicity, we assume that the technology used by all individual firms is identical. Production with such input requirements represents an increasing internal economies of scale as the average cost function derived from (4), $AC = \dfrac{r\theta}{x_i} + w\lambda$, is decreasing in an individual firm's size. Notice that in our production context the internal economies of scale relate to the firm, and not to the plant level (Eckel (2003)) as x can represent both traditional internal economies of scale resulting from producing the same good in large quantities as well as economies of scope in vertically related production taking place within the boundaries of a firm. Due to the presence of a fixed cost no two firms will produce the exact same product in the equilibrium as products can be differentiated or fragmented costlessly.

The production of the non-tradable good takes a form

$$Q_s = AL_s^{\beta}K_s^{1-\beta} \quad , \tag{5}$$

where L_s is the amount of labor and K_s the amount of capital employed in producing non-tradables. $A = \dfrac{1}{\beta^{\beta}(1-\beta)^{1-\beta}}$ and $0 < \beta < 1$ is the factor share of labor in output.

We assume that all individuals in the economy have the same Cobb-Douglas utility function $U_i = Q_{m,i}^{\alpha}Q_{s,i}^{1-\alpha}$ and assuming homogeneity and strict quasi-concavity, the aggregate utility function takes a form $U = Q_m^{\alpha}Q_s^{1-\alpha}$. Maximizing this utility function subject to a budget constraint allows us to derive demand for the manufacturing good Q_m and the non-tradable good Q_s. National income consists of wage and interest rate payments. We denote by p_m the price of a manufacturing good and p_i the price of an individual component, $P_I = n^{(\frac{\sigma}{\sigma-1}-\frac{1}{\rho})}[\sum_{i=1}^{n}p_i^{1-\sigma}]^{\frac{1}{1-\sigma}}$ is then the composite price index for the intermediate good, which reduces in the symmetric equilibrium to $P_I = n^{1-\frac{1}{\rho}}p$. Non-tradable good is the *numeraire*.

The full-employment of labor and capital in the economy imply

$$\lambda nx + L_m + L_s = \overline{L} \tag{6}$$

and

$$\theta n + K_s = \overline{K} \quad , \tag{7}$$

where λnx is the amount of labor and θn the amount of capital demanded in the intermediate good production, $\overline{L}$ denotes labor endowment and $\overline{K}$ capital endowment in the economy.

Labor and capital are assumed to be perfectly mobile across three production sectors. This completes the setup of the model.

3 General Equilibrium

3.1 Exogenous markup

The producers of the non-tradable good sector maximize their profits in the perfectly competitive environment by choosing the optimal input mix of labor and capital, taking the prices of inputs and that of the output as given. The first order conditions from the profit maximization result in (as $p_s = 1$)

$$\beta\frac{Q_s}{L_s} = w \text{ and } (1-\beta)\frac{Q_s}{K_s} = r \Rightarrow \frac{1-\beta}{\beta}\frac{L_s}{K_s} = \frac{r}{w} \quad . \tag{8}$$

Zero profit condition equates the price of the output to the unit cost (for $Q_s > 0$), and since $c_s = \frac{1}{A}\left(\frac{w}{\beta}\right)^{\beta}\left(\frac{r}{1-\beta}\right)^{1-\beta}$, we obtain

$$w^{\beta} r^{1-\beta} = 1 \implies w = r^{\frac{\beta-1}{\beta}} \; . \tag{9}$$

Firms that produce various components take the composite price index for the intermediate good as well as the national income as given and maximize their profits by setting their marginal revenue equal to marginal cost. Hence we assume in the standard Chamberlinian fashion that each producer conjectures in the Cournot-Nash way that the other firms in the sector will not change their output in response to that firm's price change and that there is a large enough number of firms producing components, unable to influence the total output of the intermediate good sector. Then the demand for each component by manufacturing producers faces a constant price elasticity of σ that is exogenously given by the elasticity of substitution. This price elasticity in turn determines the markup that the firms charge. Hence the price of each component is a constant markup over the marginal cost

$$p_i\left(1 - \frac{1}{\sigma}\right) = w\lambda \implies p_i = \frac{\sigma}{\sigma-1} w\lambda \; . \tag{10}$$

It can be seen immediately that with identical technology all firms charge the same price or $p_i = p$. Equation (10) then states that the markup is inversely related to the elasticity of substitution between the various components. As $\sigma \to 1$, specialized components become less substitutable, permitting the firms operating at the market charge a higher markup for their products.

Free entry on the other hand does not allow the firms to charge a price higher than the average cost, driving profits to zero and making it unprofitable to share the demand for any given component with any other firm. This is expressed as

$$p = \frac{r\theta}{x_i} + w\lambda \implies p = \frac{r\theta}{x_i}\sigma \implies x_i = \frac{r\theta}{w\lambda}(\sigma-1) \; . \tag{11}$$

Since firms have the same technology and face the same elasticity of substitution between components, each firm operating in this monopolistically competitive sector produces the same level of output in the equilibrium or $x_i = x$.

With the same prices and output levels for the components in the intermediate good production, total output $I_m = n^{\frac{1}{\rho}-\frac{\sigma}{\sigma-1}}\left(\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ reduces to $I_m = n^{\frac{1}{\rho}} x$ and the composite price

index $P_I = n^{\left(\frac{\sigma}{\sigma-1}-\frac{1}{\rho}\right)}\left[\sum_{i=1}^{n} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ reduces to $P_I = n^{1-\frac{1}{\rho}} p$.

The above implies that in the symmetric equilibrium

$$P_I I_m = npx \tag{12}$$

or the total revenue in the intermediate good sector equals the total revenue from manufacturing all the components.

The producers of the final manufacturing good sector maximize their profits in the perfectly competitive environment by choosing the optimal input mix of labor and intermediate goods, taking the prices of inputs and that of the output as given. The first order conditions from the profit maximization result in

$$p_m\gamma\frac{Q_m}{L_m} = w \quad \text{and} \quad p_m(1-\gamma)\frac{Q_m}{I_m} = P_I \quad . \tag{13}$$

The consumers in the economy maximize their utility $U = Q_m{}^{\alpha}Q_s{}^{1-\alpha}$ subject to the budget constraint $p_mQ_m + Q_s = I$, where $I = w\overline{L} + r\overline{K}$ stands for national income. Then a share α of the income will be spent on final manufacturing goods and a share $(1-\alpha)$ on non-tradables. Mathematically, $p_mQ_m = \alpha I$ and $Q_s = (1-\alpha)I$.

We solve for the equilibrium in this autarkic economy by utilizing equation (12) and noticing that zero profit condition expressed by equation (11) equates total revenue with total cost in each component producing firm. The first order conditions of manufacturing producers' profit maximization in (13) imply that the total spending of the manufacturing good sector on intermediate inputs equals an exogenous share of its revenue. Since the share of income spent on manufacturing goods must equal the sales in that industry,

$$npx = nc = n(r\theta + w\lambda x) = P_I I_m = (1-\gamma)\alpha I \quad . \tag{14}$$

Equation (14) allows us to express n as the function of exogenous parameters, capital and labor endowments in the economy, and the wage rate. In particular,

$$n = \frac{(1-\gamma)\alpha(\overline{K} + w^{\frac{1}{1-\beta}}\overline{L})}{\sigma\theta} \quad . \tag{15}$$

First order conditions from the profit maximization in the non-tradable production in (8) imply $L_s = \beta\frac{Q_s}{w} = \beta(1-\alpha)\frac{I}{w} = \beta(1-\alpha)(\overline{L} + w^{\frac{1}{\beta-1}}\overline{K})$ and $L_s = \left(\frac{\beta}{1-\beta}\right)K_s w^{\frac{1}{\beta-1}}$. The full employment of capital in the economy in (7) on the other hand allows us to express $K_s = \overline{K} - n\theta$. Then

$$\left(\frac{\beta}{1-\beta}\right)(\overline{K} - n\theta)w^{\frac{1}{\beta-1}} = \beta(1-\alpha)(\overline{L} + w^{\frac{1}{\beta-1}}\overline{K}) \tag{16}$$

and after we substitute for n from equation (15) we can solve for the wage rate as follows

$$w = \left(\frac{\overline{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))}{\overline{L}((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)}\right)^{1-\beta} \quad . \tag{17}$$

Equation (17) shows that the wage rate in the economy is influenced by capital and labor endowments, parameters α, β, γ and the elasticity of substitution σ. Increase in the capital stock/labor force *ceteris paribus* would lead to an increase/decrease in the wage rate. Notice that compared to Krugman (1980), it is the country that is *relatively more capital abundant* that has a higher wage.

Next we can solve for the interest rate by using equation (9), such that

$$r = \left(\frac{\overline{L}((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}{\overline{K}(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))} \right)^{\beta}, \tag{18}$$

which depends on the same variables as the wage rate. Here an increase in the capital stock/labor force *ceteris paribus* would lead to a decrease/increase in the rental rate, identifying the factor price effect of the change in endowment. Now the country that is *relatively more capital abundant* has a lower interest rate.

Utilizing equations (15) and (17) allows us to derive the solution for n in the economy that has a monopolistically competitive sector with exogenous markup. Specifically (ignoring the integer constraint),

$$n = \frac{(1-\gamma)\alpha}{\theta} \frac{1}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha} \overline{K}. \tag{19}$$

Notice here that the number of firms in the sector producing components (the equilibrium degree of fragmentation) depends only on the stock of capital in the economy and parameter values, being independent of the labor endowment. This parallels the solution reached by Rivera-Batiz and Rivera-Batiz (1991) for a two-sector economy with a slightly different setup. On the other hand, this solution diverges from a more commonly used result that is derived from using only one factor of production in the monopolistically competitive sector, labor. In our model, labor endowment in the economy (the size of $\overline{L}$) has absolutely no impact on the number of firms in the equilibrium.

The solution for the price and output in the component producing sector can be derived from (10) and (11) and results in

$$p = \frac{\sigma}{\sigma-1}\lambda \left(\frac{\overline{K}(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))}{\overline{L}((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)} \right)^{1-\beta} \tag{20}$$

and

$$x = \frac{\theta}{\lambda}(\sigma-1) \left(\frac{\overline{L}((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}{\overline{K}(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))} \right). \tag{21}$$

Equation (21) implies that the output level of a component producer or the size of a firm is not fixed as in the Ethier-type models with one factor of production, but varies depending on the labor and capital endowment in the economy. An increase in the capital stock/labor force *ceteris paribus* would lead to a decrease/increase in the output of each component. The aggregate output quantity of all produced components in the industry nx is however independent of the capital stock $\overline{K}$ and directly related to the labor force $\overline{L}$. The reason is that as the size of the market

expands, increasing labor force would lead to a decrease in the wage-rental ratio and thereby to an increase in the ratio of fixed to variable costs in the component producing firms. There would also be an increased demand for final manufactures and therefore the intermediates. Then an expansion in the intermediate production would force each component producer to increase the quantity supplied, as specialization is kept unaffected and total quantity supplied expands.

Capital used in the non-tradable sector in this economy follows from the full-employment condition expressed in (7). Labor employed in manufacturing final good sector can be derived from the f.o.c. of the manufacturing production in (13). In particular,

$$L_m = nx\lambda \left(\frac{\gamma}{1-\gamma} \right) \left(\frac{\sigma}{\sigma-1} \right) \tag{22}$$

and since the amount of labor employed in the intermediate good sector $L_I = nx\lambda$, it follows that the allocation of labor between manufacturing and intermediate's production is solely determined by the parameters of the model. Labor used in the non-tradable sector can then be found from the full-employment condition in (6).

This completes the autarkic equilibrium of this model, as all endogenous variables, $n, p, x, r, w, p_m, P_I, I_m, Q_m, Q_s, L_I, L_m, L_s, K_s, U, I$, can be solved by expressing them through exogenous variables (capital and labor endowment) and the parameters of the model.

To determine how external and internal economies of scale interact in this model and how this can affect firm behavior, we utilize the framework developed by Eckel (2003). As we already addressed in the setup of the model, there are external economies of scale present in the intermediate goods' sector production as long as $0 < \rho < 1$, reflecting gains from an increased division of labor and determining the level of fragmentation. Since in a symmetric equilibrium intermediate goods are produced according to $I_m = n^{\frac{1}{\rho}-1}(nx) = n^{\frac{1}{\rho}}x$, we define an index $\mu(n) = \frac{I_m}{nx}$ to capture gains from vertical specialization explicitly. As it follows, in the model with exogenously determined markup, gains from specialization are

$$\mu(n) = n^{\frac{1}{\rho}-1} = \left(\frac{(1-\gamma)\alpha}{\theta} \frac{1}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha} \overline{K} \right)^{\frac{1}{\rho}-1}. \tag{23}$$

Internal economies of scale are reflected by decreasing average costs in the production of components. Following Eckel (2003), we define the inverse of average costs as our measure of internal economies, so that this measure rises when the economies of scale increase. Since the presence of internal economies would support a larger firm size, we use it to evaluate gains from integration (internalization). Mathematically, $v(x_i) = \left(\frac{x_i}{c_i(x_i, r, w)} \right) = \left(\frac{1}{p_i} \right)$ and in our case, from (20),

$$\upsilon(x) = \frac{\sigma-1}{\sigma} \frac{1}{\lambda} \left(\frac{\overline{L}((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}{\overline{K}(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))} \right)^{1-\beta}. \tag{24}$$

Equation (23) discloses that the external economies of scale in the Dixit-Stiglitz-Ethier model with exogenous markup are affected by capital endowment only and that the size of the labor force ($\overline{L}$) has no impact on it (i.e. $\dfrac{\partial \mu}{\partial K} > 0$ and $\dfrac{\partial \mu}{\partial L} = 0$). As the stock of capital is augmented, *ceteris paribus*, the ratio of fixed to variable costs declines, inducing entry and enhancing specialization and fragmentation.

Equation (24) on the other hand shows that the adjustment in both capital and labor endowments has an impact on internal economies, and that they have a divergent effect on the final outcome. Specifically, it clearly follows from (24) that $\dfrac{\partial \upsilon}{\partial K} < 0$ and $\dfrac{\partial \upsilon}{\partial L} > 0$. As the size of the capital stock increases, the firms are scaled back, *ceteris paribus*, and as the size of available labor increases, *ceteris paribus*, integration would be encouraged by an increased demand and by raising the ratio of fixed to variable costs. In our model, these effects offset each other, if the endowments of the capital stock and labor force turn out to be equal and if they both grow at the same pace. This special outcome compares to the one factor of production Ethier-type models, in which the internal economies of scale are constant (as firm size is constant), such that an increase in the size of the market through globalization always enhances specialization and has no effect on integration. This enhancement in specialization occurs through an increase in population in those models (and population reflects market size), not through an increase in capital stock as here. In our model an increase in population would not encourage the entry of new firms, but enhance internalization by expanding the existing ones.

Hence the inclusion of capital into the Dixit-Stiglitz-Ethier model makes an important adjustment: specialization depends only on the size of capital stock and integration faces a trade-off, except in a special case as explained above, as the market enlarges due to globalization (letting the size of the market to be determined by total income). Internal economies do not stay constant as long as globalization can take a form of only augmenting the amount of existing capital (foreign direct investment, which, as shown by Rivera-Batiz and Rivera-Batiz (1991) can improve national welfare under increasing returns) or only increasing the size of population (immigration) or if one of these variables grows faster than the other. While an increase in population would enhance integration, a possible simultaneous increase in capital stock would work against it, instead encouraging the entry of new firms.

We consequently assume that as globalization occurs, the market can be enlarged on the account of both capital and labor inflow. Proposition 1 summarizes the above discussion.

Proposition 1 *If the component producing firms' markup is exogenous and two factors of production are employed, globalization would always increase specialization and could either increase, decrease or not affect integration, depending on how much capital augmentation is able to offset that of labor as the market enlarges. The internalization motive of the firms is always supported if globalization enlarges the market through significant population increase.*

3.2   Endogenous markup

Models of monopolistic competition in which individual producers' markups do not depend on the number of producers abstract from interdependence among firms, since the number of firms in operation is assumed to be large as an approximation and hence a firm takes the composite price index for the intermediate good as well as the national income as given. Competition in prices implied earlier that producers set their marginal revenue to marginal cost,

allowing us to express (10) in the Chamberlinian fashion where the price elasticity of demand is a constant.

In this section we are interested in endogenizing the markup (through endogenizing the price elasticity of demand) that component producing firms charge, so that we would not completely eliminate expansion effects that can have an impact on the equilibrium vertical industrial structure. We proceed in the lines of Eckel (2003), whose Ethier-type model with one factor of production permits the firm size to be endogenously determined once the expansion effects are taken into account. Analogously to Eckel (2003), we wish to pursue the inclusion of expansion effects to see how globalization changes firm behavior, albeit a constant firm size is not an issue here.

Demand for each component in a more general format can be derived from a cost function, corresponding to (2) and making use of the Shepard's lemma. Then $C_I = \left(\sum_{i=1}^{n} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} n^{\frac{\sigma}{\sigma-1} - \frac{1}{\rho}} I_m$

is the respective cost function and $x_j = n^{\left(\frac{\sigma}{\sigma-1} - \frac{1}{\rho} \right)(1-\sigma)} \left(\frac{P_I}{p_j} \right)^{\sigma} I_m$ the derived demand for a component

j. The price elasticity of this demand can be shown to equal

$$\frac{\partial x_j}{\partial p_j} \frac{p_j}{x_j} = -\sigma + \left(\sigma + \frac{\partial (1-\gamma)}{\partial P_I} \frac{P_I}{(1-\gamma)} + \frac{\partial \alpha}{\partial P_I} \frac{P_I}{\alpha} - 1 \right) \frac{\partial P_I}{\partial p_j} \frac{p_j}{P_I} + \frac{\partial I}{\partial p_j} \frac{p_j}{I} \quad . \tag{25}$$

In addition to the substitution effect as expressed by the first term of (25), price elasticity can be influenced by expansion effects, the impact of pricing behavior on the industry price index and it's impact on national income (so-called "Ford effect"). The importance of those expansion effects depends on whether a single firm in the intermediate good industry is large enough to manipulate price index and income. In the Chamberlinian tradition of atomistic firms with no perceived interdependence, a firm is too small to have any influence, eliminating expansion effects from consideration.

We next relax this Chamberlinian assumption by assuming that a firm in the intermediate good industry can be large enough to influence the industry level price index, but small enough not to have significant impact on national income (Neary, 2003). Then the "Ford effect" continues

not to apply ($\frac{\partial I}{\partial p_j} \frac{p_j}{I} = 0$). We still assume that firms do not engage in any type of strategic

behavior, such that expenditures on fixed and variable costs are incurred simultaneously. If each component producing firm takes the pricing behavior of all competitors as given, then

$\frac{\partial P_I}{\partial p_j} = n^{\frac{\sigma}{\sigma-1} - \frac{1}{\rho}} \left(\sum_{i=1}^{n} p_i^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} p_j^{-\sigma}$ and therefore $\frac{\partial P_I}{\partial p_j} \frac{p_j}{P_I} = \frac{p_j^{1-\sigma}}{\sum_{i=1}^{n} p_i^{1-\sigma}}$. In addition, since Cobb-

Douglas type of utility function implies that the income shares spent on goods are exogenously

given, $\frac{\partial (1-\gamma)}{\partial P_I} = \frac{\partial \alpha}{\partial P_I} = 0$. Finally then, the price elasticity of demand in (25) reduces to

$$\frac{\partial x_j}{\partial p_j} \frac{p_j}{x_j} = -\sigma + \frac{1}{n}(\sigma - 1) \tag{26}$$

in a symmetric equilibrium, where the technology of all firms is identical. Then it is straightforward to show that the price of each component is no longer a constant markup over the marginal cost, but equals

$$p = \left(\frac{1}{\sigma-1} \left(\sigma + \frac{1}{n-1} \right) \right) w\lambda \quad . \tag{27}$$

The output of each component on the other hand, unlike (11), is now expressed by

$$p = \frac{r\theta}{x} + w\lambda \Rightarrow p = \frac{r\theta}{x} \frac{1+\sigma(n-1)}{n} \Rightarrow x = \frac{r\theta}{w\lambda}(\sigma-1)\left(\frac{n-1}{n} \right) \quad , \tag{28}$$

disclosing how the quantity produced by a single component manufacturer is made dependent on the number of active firms in the intermediate good industry.

We again solve for the equilibrium in this autarkic economy by utilizing equation (12) and the first order conditions of manufacturing producers' profit maximization in (13). Zero profit condition expressed in this endogenous markup model by equation (28) equates total revenue with total cost in each component producing firm. As the share of income spent on manufacturing goods must equal the sales in that industry, we can express (14) as before, which after some manipulation yields a new expression for the number of firms

$$n = \frac{(1-\gamma)\alpha(\overline{K} + w^{\frac{1}{1-\beta}}\overline{L}) + \theta(\sigma-1)}{\sigma\theta} \quad . \tag{29}$$

Compared to the exogenous markup case as expressed by (15), there is an additional term present in (29), which will affect the equilibrium outcome as will be shown below. By utilizing (16) and substituting for n allows us to solve for the wage rate as follows

$$w = \left(\frac{\overline{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1)}{\overline{L}((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)} \right)^{1-\beta} \quad . \tag{30}$$

Equation (30) reveals that the wage rate in the economy is lower when the markup in the intermediate good sector is endogenously determined. In particular, the fixed input requirement as expressed by θ has a negative impact on the equilibrium wage level, having absolutely no impact when exogenous markup was under consideration. Hence, when capital gets augmented, there will still be an increase in the wage rate, but not as significant as implied by (17). A relatively more capital abundant country continues to have a higher wage.

The expression for the interest rate follows directly from (9), which this time leads to

$$r = \left(\frac{\overline{L}((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)}{\overline{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1)} \right)^{\beta} \tag{31}$$

and demonstrates that the presence of the fixed input requirement θ increases the interest rate in the equilibrium. Augmentation in the capital stock is not able to decrease the interest as much as shown by (18).

Utilizing equations (29) and (30), the number of firms operating in the equilibrium with endogenous markup (ignoring the integer constraint) is expressed by

$$n = \frac{(\sigma-1)(1-\beta)(1-\alpha)}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha} + \frac{(1-\gamma)\alpha}{\theta} \frac{1}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha}\overline{K} \, , \qquad (32)$$

where the first term of (32) characterizes the increase in specialization brought about by an endogenized pricing rule. The number of component suppliers in the economy still depend only on the existing capital stock and parameters of the model, while labor endowment has no effect on this outcome.

Finally, equations (27) and (28) allow us to solve for the price and output in the component producing sector under endogenous markup, resulting in

$$p = \lambda \frac{\left(\frac{\sigma}{\sigma-1}\right)\overline{K}-\theta}{\overline{K}-\theta\left(\frac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha}+1\right)} \left(\frac{\overline{K}(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))-\theta(\sigma-1)}{\overline{L}((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}\right)^{1-\beta} \qquad (33)$$

and

$$x = \frac{\theta}{\lambda}(\sigma-1) \frac{\overline{K}-\theta\left(\frac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha}+1\right)}{\overline{K}+\theta\left(\frac{(\sigma-1)(1-\beta)(1-\alpha)}{(1-\gamma)\alpha}\right)} \left(\frac{\overline{L}((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}{\overline{K}(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))-\theta(\sigma-1)}\right) \, .$$

$$(34)$$

Equations (33) and (34) reveal that capital endowment has a more complicated impact on the price and output solution compared to the exogenous markup case as an equal increase of both $\overline{K}$ and $\overline{L}$ will no longer offset each other under any circumstances. The implication of this will be examined in terms of how internal economies of scale in this model are affected and how this in turn can impact firm behavior. The aggregate output quantity nx is also no longer independent of the capital stock $\overline{K}$.

Labor used in manufacturing final good sector can be derived utilizing the f.o.c. of the manufacturing production, such that

$$L_m = nx\lambda\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{1+\sigma(n-1)}{(\sigma-1)(n-1)}\right) \qquad (35)$$

and since the amount of labor employed in the intermediate good sector $L_I = nx\lambda$, equation (35) implies that the allocation of labor between manufacturing and intermediate's production is no longer solely determined by the parameters of the model, but is also made endogenous. Finally, the remaining variables of the model can be computed analogous to the exogenous markup framework.

Thereby the autarkic equilibrium of the model with endogenous markup can be solved for all endogenous variables, $n, p, x, r, w, p_m, P_I, I_m, Q_m, Q_s, L_I, L_m, L_s, K_s, U, I$.

Implications of external and internal economies for the firm behavior in this model evolve as follows. Gains from specialization in the model with endogenous markup are expressed by substituting from (32) and result in

$$\mu(n) = n^{\frac{1}{\rho}-1} = \left(\frac{(\sigma-1)(1-\beta)(1-\alpha)}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha} + \frac{(1-\gamma)\alpha}{\theta} \frac{1}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha} \overline{K} \right)^{\frac{1}{\rho}-1},$$

(36)

showing that the external economies of scale in the Dixit-Stiglitz-Ethier model with endogenous markup also depend solely on the capital availability and stay independent of population (i.e. $\frac{\partial \mu}{\partial \overline{K}} > 0$ and $\frac{\partial \mu}{\partial \overline{L}} = 0$). Again, as the stock of capital is augmented, *ceteris paribus*, the ratio of fixed to variable costs declines, inducing entry and enhancing specialization and fragmentation. This effect is equivalent to that under an exogenous markup, except that the positive impact on specialization brought about by an increase in the capital stock is more profound.

The gains from integration in this model can be found utilizing (33) and equal

$$\upsilon(x) = \frac{\overline{K} - \theta \left(\frac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} + 1 \right)}{\left(\frac{\sigma}{\sigma-1} \right) \overline{K} - \theta} \frac{1}{\lambda} \left(\frac{\overline{L}((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}{\overline{K}(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1)} \right)^{1-\beta}.$$

(37)

Equation (37) discloses that the internal economies are affected by both capital and labor endowments as in the model with exogenous markup, while they no longer always have a divergent effect. As new firms enter due to the market enlargement induced by an additional capital flow and demand for components gets more elastic, the existing firms lower their prices to capture a larger market share. In Eckel (2003), this effect works through an increase in population and an increase in the size of the market always enhances both specialization and firm size under endogenous markup. As in the model with exogenous markup, an increase in population would not encourage the entry of new firms in our setup, but support integration. Only additional capital inflow can lead to an enhancement in specialization. However, as the demand for components gets more elastic, firms can expand both on the account of additional labor and additional capital.

In this model the final outcome depends on the number of firms in operation. As long as the number of firms in the new equilibrium stays small, capital augmentation allows firms to gain from integration and enhance any internalization gains acquired by an increase in labor availability. It is seen from (37) that the effect of an increase in labor endowment will be the same as in the model with exogenous markup: as population increases, *ceteris paribus*, integration would be encouraged by an increased demand and by raising the ratio of fixed to variable costs (i.e. $\frac{\partial \upsilon}{\partial \overline{L}} > 0$). Likewise, $\frac{\partial \upsilon}{\partial \overline{K}} > 0$, as long as $n < 1 + \frac{B}{(1-\beta)\left(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha)\right)}$, where $B = \frac{w\overline{L}}{w\overline{L}+r\overline{K}}$ is the labor share of income. In particular, if the number of firms in operation remains under this threshold, the firms benefit from internalization irrespective of additional labor availability, as the ratio of fixed to variable costs is higher than in the model with exogenous markup at the equal level of endowments. Only when the number of firms reaches

$$n = 1 + \frac{B}{(1-\beta)\left(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)\right)}$$, will the capital inflow have no influence on

integration (i.e. $\frac{\partial \upsilon}{\partial \overline{K}} = 0$). As the number of firms in the equilibrium grows large, specifically, if it

attains $n > 1 + \dfrac{B}{(1-\beta)\left(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)\right)}$, then $\frac{\partial \upsilon}{\partial \overline{K}} < 0$, and the augmentation of

capital leads to a decrease in the size of the firm similarly to the model with exogenous markup. Concluding, a small size of the capital stock in the economy will initially allow firms to expand, but as the size of the capital stock increases further, we will reach the exogenous markup solution where the firms will be scaled back as additional capital is utilized for specialization and fragmentation, *ceteris paribus*.

Hence capital plays an additionally important role in this endogenous markup framework: it can both enhance the degree of specialization and the degree of integration if the number of firms in the intermediate good sector remains small. If there are already many active component producers present and the industry is well fragmented, additional capital inflow will be used to increase specialization and the firm size decreases as the ratio of fixed to variable costs continues to decline. Then only an increase in population is able to enforce gains from internalization. While an increase in population would enhance integration, a possible simultaneous increase in capital stock could work against it, as under exogenous markup. The formation of equilibrium industrial structure is therefore directly dependent on the interaction between internal and external economies in this setup. Proposition 2 summarizes the above discussion.

Proposition 2 *If the component producing firms' markup is endogenous and two factors of production are employed, globalization would always increase both specialization and integration if the number of firms in the industry remains small. As the number of firms increases due to capital inflow, specialization continues to rise, but integration could either increase or decrease, depending on whether capital or labor stock is augmented more. With a large number of firms in operation, the internalization motive of the firms can only be supported if globalization enlarges the market through significant population increase.*

Proof   Available in the Appendix. ∎

4   Trade

Consider next free international trade, induced by globalization lead trade liberalization, taking place between two countries, home and foreign, that are identical in all respects other than possibly their factor endowments. In particular, we allow trade in both manufacturing final good Q_m and specialized components x. In a free trade equilibrium, the output of each distinct component would be concentrated in only one country, for the same reason that each component is produced by only one firm, and two countries would produce components belonging to a different stage of production and complete specialization cannot occur. Since the same number of components $n = n^h + n^f$ (where superscript h denotes home and f foreign) becomes available to both countries and free trade equalizes prices ($p^h = p^f$), the components will be used in identical relative amounts in each country.

It is then apparent, as in the lines of Krugman's (1980) modeling of the trade in consumption goods, that domestic residents will exhaust a fraction $\dfrac{n^f}{n^h + n^f}$ of their income on final manufacturing goods that are composed of foreign components as inputs, whereas foreigners will spend a fraction $\dfrac{n^h}{n^h + n^f}$ of their income on final manufacturing goods that have made use of home country components. The pattern of component production is determined utilizing the outcome presented by (14) and noting that the residents' total expenditure on domestic manufacturing industry goods is composed of a sum of domestic and foreign residents' expenditures or

$$n^h p^h x^h = (1-\gamma)\alpha \frac{n^h}{n^h + n^f} I^h + (1-\gamma)\alpha \frac{n^h}{n^h + n^f} I^f \quad , \tag{38}$$

and since the same reasoning applies towards foreign component production,

$$n^f p^f x^f = (1-\gamma)\alpha \frac{n^f}{n^h + n^f} I^h + (1-\gamma)\alpha \frac{n^f}{n^h + n^f} I^f \quad . \tag{39}$$

Dividing (38) by (39) allows us to reach, after canceling out n^h and n^f, $p^h x^h = p^f x^f$. As prices of the components equalize in trade, this leads to an equalization in the volume of specialized intermediate good production or $x^h = x^f$. Since component prices and outputs equalize across countries, from (11) we can also observe the equalization in factor prices (r and w), a result that is akin to Markusen (1989). The ability to trade intermediate goods I_m under these circumstances would become redundant.

Equation (38) implies that the number of component producing firms in operation at the world markets, $n^h + n^f$, can be expressed analogous to (15), such that in the model with exogenous markup,

$$n^h + n^f = \frac{(1-\gamma)\alpha(\overline{K}^h + \overline{K}^f + w^{\frac{1}{1-\beta}}(\overline{L}^h + \overline{L}^f))}{\sigma\theta} \quad . \tag{40}$$

After utilizing (16) in two separate equations for domestic and foreign markets and expressing the number of component producing firms at home and abroad (n^h and n^f), we substitute those into (40) to solve for the common wage rate

$$w = \left(\frac{(\overline{K}^h + \overline{K}^f)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))}{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)} \right)^{1-\beta} \quad . \tag{41}$$

Equation (41) shows that internationally equalized wage rate depends on the same parameters as earlier and on the factor endowments of *both* countries. As countries open up to component trade and allow international fragmentation, the wage rate in a labor abundant country would increase compared to autarky and the wage rate in a capital abundant country would

decrease (i.e. $w^{h(trade)} > w^{h(autarky)}$ if $\dfrac{\overline{L}^h}{\overline{K}^h} > \dfrac{\overline{L}^f}{\overline{K}^f}$). This contrasts to the results by Chakraborty (2003) attained in a closely related model, according to which free trade is able to rise wage rates in both countries, irrespective of labor being a scarce or abundant factor.

Common interest rate in this setup can again be found using (9) and it is equal to

$$ r = \left(\frac{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)}{(\overline{K}^h + \overline{K}^f)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))} \right)^{\beta} , \tag{42} $$

implying again that the factor endowments of *both* countries have an effect on the outcome. In particular, the interest rate in a labor abundant country would decrease compared to autarky and the interest rate in a capital abundant country would increase as trade opens up.

Equations (40) and (41) next allow us to derive the expression for the number of component producing firms or the equilibrium degree of fragmentation at home, which, after several substitutions, results in

$$ n^h = \frac{(1-\gamma)\alpha}{\theta} \frac{1}{(1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha} (\overline{K}^h + \overline{K}^f) + $$
$$ + \frac{(1-\alpha)(1-\beta)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))}{\theta((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)} \frac{\overline{L}^f(\overline{K}^h + \overline{K}^f)}{(\overline{L}^h + \overline{L}^f)} - $$
$$ - \frac{(\alpha + \beta - \alpha\beta)\overline{K}^f}{\theta}. \tag{43} $$

Equation (43) reveals that the number of components produced no longer solely depends on the parameter values and domestic capital endowment. In fact, even though $\dfrac{\partial n^h}{\partial \overline{K}^h} > 0$ as in

autarky, it follows from (43) that $\dfrac{\partial n^h}{\partial \overline{K}^f} < 0$, $\dfrac{\partial n^h}{\partial \overline{L}^h} < 0$ and $\dfrac{\partial n^h}{\partial \overline{L}^f} > 0$. Then an increase in the

capital endowment in another country hinders domestic vertical specialization, while an enlargement in labor endowment abroad encourages it. Population increase at home also discourages specialization, as it raises the ratio of fixed to variable costs. The solution for the equilibrium number of firms abroad, n^f, is a mirror image of (43), achieved by replacing the superscript h by f and vice versa.

Next, the solution for the price and output of produced components can be derived analogously to the autarky by making use of (10) and (11), which leads to

$$ p = \frac{\sigma}{\sigma - 1} \lambda \left(\frac{(\overline{K}^h + \overline{K}^f)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))}{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)} \right)^{1-\beta} \tag{44} $$

and

$$x = \frac{\theta}{\lambda}(\sigma-1)\left(\frac{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}{(\overline{K}^h + \overline{K}^f)(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))}\right) \quad . \tag{45}$$

Equations (44) and (45) show the results very similar to the autarky model under exogenous markup, except that both price and output of the specialized components are dependent on worldwide labor and capital endowments. Compared to autarky, the price of the components would raise and the output of each component would fall in the labor abundant country. The opposite would occur in the capital abundant country. Also notice that the world aggregate output quantity of all stages of production $(n^h + n^f)x$ has a complicated solution and will depend on population and capital stocks in both countries.

The pattern of fragmentation in this model evolves from comparing the equilibrium number of firms operating in component production under trade with the same variable under autarky. Subtracting (19) from (43) and some manipulation results in $n^{h(trade)} > n^{h(autarky)}$ if $\frac{\overline{L}^f}{\overline{K}^f} > \frac{\overline{L}^h}{\overline{K}^h}$. The direction of trade becomes apparent from examining the relationship between the value of all domestically produced components and the domestic demand as presented by (14). In particular, it ensues after some manipulation that $n^h px > (1-\gamma)\alpha I^h$ if and only if $\frac{\overline{L}^f}{\overline{K}^f} > \frac{\overline{L}^h}{\overline{K}^h}$. Proposition 3 follows.

Proposition 3 *Free trade in components between two countries that are identical in all respects other than their factor endowments will result in a higher vertical specialization (compared to autarky) in a country that is relatively capital abundant and lower vertical specialization in a country that is relatively labor abundant. Relatively capital abundant country will be the net exporter of components and therefore the importer of final manufacturing goods.*

Notice that this intra-industry trade in components has a factor endowment basis and is not just occurring because of increasing returns. In case increasing returns would form the basis for trade, both countries would continue to produce exactly the same number of fragments as in autarky, simply producing a larger output of each component – and the direction of trade would be indeterminate (we do not know which country produces which components). The volume of trade in components occurring due to increasing returns could be however specified by $\frac{n^f}{n^h + n^f}I^h = \frac{I^f}{I^h + I^f}I^h$ and balanced trade in specialized components would hold, so that trade in Q_m would be redundant. Interestingly, the result above is reflected by a recent study by Kandogan (2003), who empirically analyses trade between transition economies and developed countries to find that vertical intra-industry trade (defined as the simultaneous export and import of goods in the same industry, but at different stages of production) is positively affected by the economies of scale and comparative advantage.

However, even though the pattern of trade can be determined here, it is not possible to disclose which country produces how much of the world intermediate good output I_m, how much labor in each country is employed in the intermediate good and manufacturing sectors (L_m and L_I) and what is the supply of final manufacturing good Q_m in each country. In particular, it is not possible to determine the volume of trade in components.

Finally, since the prices and outputs of specialized components are equalized across countries, the expression for the total world output of intermediate goods evolves into

$$I_m = x \left(n^{h\left(\frac{1}{\rho}\frac{\sigma}{\sigma-1}\right)} + n^{f\left(\frac{1}{\rho}\frac{\sigma}{\sigma-1}\right)} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{and the respective composite price index becomes}$$

$$P_I = p \left(n^{h\left(\frac{\sigma-1}{\rho}-\sigma+1\right)} + n^{f\left(\frac{\sigma-1}{\rho}-\sigma+1\right)} \right)^{\frac{1}{1-\sigma}}.$$

Lemma 1 *Free trade in specialized components requires that* $\rho = \dfrac{\sigma-1}{\sigma}$.

Proof   Available in the Appendix. ■

If $\rho = \dfrac{\sigma-1}{\sigma}$, then the total world output in the intermediate good sector becomes

$I_m = (n^h + n^f)^{\frac{1}{\rho}} x$ and the composite price index solves for $P_I = \left(n^h + n^f\right)^{1-\frac{1}{\rho}} p$.

The implications of component trade in a model with endogenous markup can be examined analogously. We utilize equations (38) and (29) to reach

$$n^h + n^f = \frac{(1-\gamma)\alpha(\overline{K}^h + \overline{K}^f + w^{\frac{1}{1-\beta}}(\overline{L}^h + \overline{L}^f)) + \theta(\sigma-1)}{\sigma\theta} \tag{46}$$

and make use of (16) to derive, after substitution, the common wage and interest rates as earlier, which equal

$$w = \left(\frac{(\overline{K}^h + \overline{K}^f)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1)}{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)} \right)^{1-\beta} \tag{47}$$

and

$$r = \left(\frac{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)}{(\overline{K}^h + \overline{K}^f)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1)} \right)^{\beta}. \tag{48}$$

Equations (47) and (48) imply again that opening up to trade would increase the wage rate and decrease the interest rate in a labor abundant country, even though this change would be less than under exogenous markup due to the presence of a fixed input requirement. Then as countries open up to component trade, the wage rate in a labor abundant country would increase compared to autarky and the wage rate in a capital abundant country would decrease. The opposite would happened to the interest rate in this model.

The equilibrium degree of fragmentation in the model with endogenous markup solves from (46) and (47) to result in

$$n^h = \frac{(\sigma-1)(1-\beta)(1-\alpha)}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha} + \frac{(1-\gamma)\alpha}{\theta}\frac{1}{(1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha}(\overline{K}^h + \overline{K}^f) +$$

$$+ \frac{(1-\alpha)(1-\beta)(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))}{\theta((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}\frac{\overline{L}^f(\overline{K}^h + \overline{K}^f)}{(\overline{L}^h + \overline{L}^f)} - \tag{49}$$

$$- \frac{(\alpha+\beta-\alpha\beta)\overline{K}^f}{\theta}.$$

Equation (49) discloses that the effect from trade on the number of firms operating in component production under endogenous markup parallels that reached in (43). Finally, noticing that the price elasticity of demand is now equal to $\dfrac{\partial x_j}{\partial p_j}\dfrac{p_j}{x_j} = -\sigma + \dfrac{1}{n^h + n^f}(\sigma-1)$, which implies that the price of a component can be expressed by $p = \left(\dfrac{1}{\sigma-1}\left(\sigma + \dfrac{1}{n^h+n^f-1}\right)\right)w\lambda$, we can solve for the price and output of the component producing sector. In particular,

$$p = \lambda \frac{\left(\frac{\sigma}{\sigma-1}\right)(\overline{K}^h + \overline{K}^f) - \theta}{(\overline{K}^h + \overline{K}^f) - \theta\left(\frac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha}+1\right)} \cdot$$

$$\cdot \left(\frac{(\overline{K}^h + \overline{K}^f)(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))-\theta(\sigma-1)}{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}\right)^{1-\beta} \tag{50}$$

and

$$x = \frac{\theta}{\lambda}(\sigma-1)\frac{(\overline{K}^h + \overline{K}^f) - \theta\left(\frac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha}+1\right)}{(\overline{K}^h + \overline{K}^f) + 2\theta\left(\frac{(\sigma-1)(1-\beta)(1-\alpha)}{(1-\gamma)\alpha}\right)} \cdot$$

$$\cdot \left(\frac{(\overline{L}^h + \overline{L}^f)((1-\beta)\sigma(1-\alpha)+(1-\gamma)\alpha)}{(\overline{K}^h + \overline{K}^f)(\sigma-(1-\gamma)\alpha-(1-\beta)\sigma(1-\alpha))-\theta(\sigma-1)}\right). \tag{51}$$

Equations (50) and (51) reveal a similar outcome to the autarky model, except that now the factor endowments of both countries play a role. The results for the pattern of fragmentation and the direction of trade in the model with endogenous markup parallel those reached under exogenous markup.

5   Conclusions

A three-sector model of final, intermediate and non-tradable goods was developed, incorporating increasing returns, to analyze the interaction between internal and external

economies and their subsequent impact on the formation of equilibrium industrial structure. Compared to the often utilized one factor of production Dixit-Stiglitz-Ethier framework, significant differences emerge.

With two factors of production employed in the monopolistically competitive intermediate good sector, where component producers face exogenous markup, only increase in the capital stock can enhance vertical specialization, whereas only an increase in population is able to enforce integration. As the size of the market is determined by total income, globalization driven market enlargement can occur because of capital or labor inflow. Then globalization would always increase specialization and it could either increase, decrease or not affect integration, depending on how much capital augmentation is able to offset that of labor as the market enlarges. Only in a special case in which two endowments equal can the firm size remain constant.

We next endogenized the markup in the model as a function of industry's degree of fragmentation in order to study the effect of expansion effects on the industry's vertical structure. This new setup allowed us to show that globalization would always increase both specialization and integration if the number of firms in the intermediate good industry remains small. If the number of firms would increase due to capital inflow, specialization would rise, but integration could either increase or decrease, depending on whether capital or labor stock is augmented more similar to the case with exogenous markup. With a large number of firms in operation, the internalization motive of the firms could only be supported if globalization enlarges the market through significant population increase.

Finally, if the size of the market is augmented through trade, it was shown that free trade in intermediate specialized inputs and a final consumption good would equalize factor prices and that the endowment basis determined the direction of trade. Accordingly, relatively capital abundant country would be the net exporter of components and therefore the importer of final manufacturing goods under both exogenous and endogenous markup. Compared to autarky, trade would enhance fragmentation in a relatively capital abundant country and diminish it in a relatively labor abundant country.

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Appendix

Proof of $\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha) > 0$

This proof relates to the variable solutions in the model with exogenous markup, in particular, it shows that (17), (18), (20), (21) and (24) are positive.

Since $\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha) = \sigma(1-(1-\beta)(1-\alpha)) - (1-\gamma)\alpha$ is increasing in σ and $\sigma > 1$, this expression achieves it's minimum at $\sigma \to 1$. Rewrite the above with $\sigma = 1$ to reach $1 - (1-\beta)(1-\alpha) - (1-\gamma)\alpha = \beta(1-\alpha) + \alpha\gamma > 0$. ∎

Proof of $\left(\dfrac{\sigma}{\sigma-1}\right)\overline{K} - \theta > 0$

This and all the remaining proofs of positive expressions relate to the model with endogenous markup, in particular, they show that (30), (31), (33), (34) and (37) are positive.

From capital full employment condition in (7) we have $\overline{K} > n\theta$, then substitute for n utilizing (32) to reach, after some manipulation, $\sigma\overline{K} > \theta(\sigma-1)$. ∎

Proof of $\overline{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1) > 0$

Rewrite $(1-\gamma)\alpha$ using (14) and (28) to reach $(1-\gamma)\alpha = \dfrac{n\theta r\sigma - r\theta(\sigma-1)}{I}$, and $(1-\alpha)(1-\beta)$ after substituting from (8) and the solution to the utility maximization $Q_s = (1-\alpha)I$ to reach $(1-\alpha)(1-\beta) = \dfrac{K_s r}{I}$. Also notice that from (7), $K_s = \overline{K} - n\theta$ and that total income in the

economy $I = w\bar{L} + r\bar{K}$. Then $\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha) = \sigma\left(1 - \dfrac{r\bar{K}}{w\bar{L}+r\bar{K}}\right) + \dfrac{r\theta(\sigma-1)}{w\bar{L}+r\bar{K}}$ and

$$\bar{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1) = \bar{K}\sigma\left(1 - \dfrac{r\bar{K}}{w\bar{L}+r\bar{K}}\right) + \dfrac{\bar{K}r\theta(\sigma-1)}{w\bar{L}+r\bar{K}} - \theta(\sigma-1) =$$

$$= \bar{K}\sigma\left(1 - \dfrac{r\bar{K}}{w\bar{L}+r\bar{K}}\right) - \theta(\sigma-1)\left(1 - \dfrac{r\bar{K}}{w\bar{L}+r\bar{K}}\right) = \left(\dfrac{w\bar{L}}{w\bar{L}+r\bar{K}}\right)\left(\sigma\bar{K} - \theta(\sigma-1)\right) > 0,$$ since the last

term is positive from the proof above and $0 < B < 1$, where $B = \dfrac{w\bar{L}}{w\bar{L}+r\bar{K}}$ (the labor share of

income). ∎

Proof of $\bar{K} - \theta\left(\dfrac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} + 1\right) > 0$

We again rewrite $\dfrac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} = \dfrac{\bar{K} - n\theta}{n\theta\sigma - \theta(\sigma-1)}$ and after substituting this to the

expression we wish to prove, it becomes $\bar{K} - \theta\left(\dfrac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} + 1\right) = \bar{K} - \theta\left(\dfrac{\bar{K} - n\theta}{n\theta\sigma - \theta(\sigma-1)} + 1\right)$,

yielding, after some manipulation, $\bar{K} - \theta\left(\dfrac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} + 1\right) = \dfrac{\sigma\bar{K} - \theta(\sigma-1)}{\sigma + \dfrac{1}{n-1}} > 0$. ∎

Proof of Proposition 2

To prove Proposition 2, we need to show that the sign of the derivative of internal economies with respect to $\bar{K}$ is dependent on the number of firms operating in the intermediate good industry.

The derivative of internal economies expressed by (37) with respect to $\bar{K}$ is equal to

$$\dfrac{\partial \upsilon}{\partial \bar{K}} = \dfrac{1}{\lambda}\left(\dfrac{1}{\left(\dfrac{\sigma}{\sigma-1}\right)\bar{K} - \theta} - \dfrac{\bar{K} - \theta\left(\dfrac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} + 1\right)}{\left(\left(\dfrac{\sigma}{\sigma-1}\right)\bar{K} - \theta\right)^2}\left(\dfrac{\sigma}{\sigma-1}\right)\right) \cdot$$

$$\cdot\left(\dfrac{\bar{L}((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)}{\bar{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1)}\right)^{1-\beta} + \dfrac{1}{\lambda}\dfrac{\bar{K} - \theta\left(\dfrac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} + 1\right)}{\left(\dfrac{\sigma}{\sigma-1}\right)\bar{K} - \theta} \cdot$$

$$\cdot\left(\bar{L}((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)\right)^{1-\beta}(-1)(1-\beta)\dfrac{\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)}{\left(\bar{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma-1)\right)^{2-\beta}} \cdot$$

To determine the sign of this derivative we collect common positive terms and substitute as in the previous proof to reach

$$\frac{\partial \upsilon}{\partial \overline{K}} = \left(\frac{\sigma + \frac{1}{n-1}}{\sigma \overline{K} - \theta(\sigma - 1)} - \frac{\sigma}{\sigma \overline{K} - \theta(\sigma - 1)} - (1-\beta)\frac{\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)}{B(\sigma \overline{K} - \theta(\sigma - 1))} \right) \cdot$$

$$\cdot \frac{1}{\lambda} \frac{\overline{K} - \theta\left(\frac{(1-\beta)(1-\alpha)}{(1-\gamma)\alpha} + 1 \right)}{\left(\frac{\sigma}{\sigma - 1} \right)\overline{K} - \theta} \left(\frac{\overline{L}((1-\beta)\sigma(1-\alpha) + (1-\gamma)\alpha)}{\overline{K}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) - \theta(\sigma - 1)} \right)^{1-\beta} =$$

$$= \left(\frac{1}{n-1} - (1-\beta)\frac{1}{B}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) \right) \frac{\upsilon}{\left(\sigma \overline{K} - \theta(\sigma - 1) \right)}, \text{ where } B = \frac{w\overline{L}}{w\overline{L} + r\overline{K}}. \text{ From}$$

this expression it clearly follows that the last multiplicative term of the derivative is positive and will therefore not influence the sign, hence we can leave it out from our consideration. Hence we need to show what sign does $\frac{1}{n-1} - (1-\beta)\frac{1}{B}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))$ have. As $\lim_{n \to \infty}$,

$\frac{\partial \upsilon}{\partial \overline{K}} < 0$, since $-(1-\beta)\frac{1}{B}(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha)) < 0$ and we approach our result for the

model with exogenous markup. However, as $\lim_{n \to 1}$, $\frac{\partial \upsilon}{\partial \overline{K}} > 0$, since $\frac{1}{n-1} \to \infty$, and there is no

solution for $n = 1$. Finally, $\frac{\partial \upsilon}{\partial \overline{K}} = 0$ for $n = 1 + \frac{B}{(1-\beta)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))}$.

Consequently, $\frac{\partial \upsilon}{\partial \overline{K}} > 0$ for $n < 1 + \frac{B}{(1-\beta)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))}$ and $\frac{\partial \upsilon}{\partial \overline{K}} < 0$ for

$n > 1 + \frac{B}{(1-\beta)(\sigma - (1-\gamma)\alpha - (1-\beta)\sigma(1-\alpha))}$. ∎

Proof of Lemma 1

To prove Lemma 1, we make use of the expressions for the intermediate good output

$I_m = x \left(n^{h\left(\frac{1}{\rho}\frac{\sigma}{\sigma - 1} \right)} + n^{f\left(\frac{1}{\rho}\frac{\sigma}{\sigma - 1} \right)} \right)^{\frac{\sigma}{\sigma - 1}}$ and the composite price index $P_I = p \left(n^{h\left(\frac{\sigma - 1}{\rho} - \sigma + 1 \right)} + n^{f\left(\frac{\sigma - 1}{\rho} - \sigma + 1 \right)} \right)^{\frac{1}{1-\sigma}}$.

Notice that analogous to (12), at the world level $P_I I_m = (n^h + n^f)px$, since components from both countries are available as inputs. But then

$P_I I_m = px \left(n^{h\left(\frac{1}{\rho}\frac{\sigma}{\sigma - 1} \right)} + n^{f\left(\frac{1}{\rho}\frac{\sigma}{\sigma - 1} \right)} \right)^{\frac{\sigma}{\sigma - 1}} \cdot \left(n^{h\left(\frac{\sigma - 1}{\rho} - \sigma + 1 \right)} + n^{f\left(\frac{\sigma - 1}{\rho} - \sigma + 1 \right)} \right)^{\frac{1}{1-\sigma}} = (n^h + n^f)px$. Rewrite this

expression to reach $n^{h\left(\frac{\sigma - 1}{\rho} - \sigma + 1 \right)} + n^{f\left(\frac{\sigma - 1}{\rho} - \sigma + 1 \right)} = (n^h + n^f)^{1-\sigma} \left(n^{h\left(\frac{1}{\rho}\frac{\sigma}{\sigma - 1} \right)} + n^{f\left(\frac{1}{\rho}\frac{\sigma}{\sigma - 1} \right)} \right)^{\sigma}$. These two

sides are equal if and only if $\rho = \frac{\sigma - 1}{\sigma}$. ∎