

EFFECTS OF THE SINGLE EUROPEAN MARKET ON WELFARE OF THE PARTICIPATING COUNTRIES (theoretical approach)

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In this note we show how creation of a united market causes welfare surplus even if the market stays imperfect competitive. We discuss a simple partial equilibrium model with a monopoly initially supplying two national markets (subscripts 1, 2). We differentiate these markets by different characteristics of their demand functions, though we assume that the demand functions in both countries are linear:

$$P_1(q_1) = a_1 - b_1 \cdot q_1$$
$$P_2(q_2) = a_2 - b_2 \cdot q_2$$

with $a_1, a_2, b_1, b_2 > 0$.

We assume that $b_1 > b_2$ which means that the demand in market 1 is generally less elastic than the one in market 2 (the $P_1(q_1)$ line is steeper than the $P_2(q_2)$). We assume also that $a_1 > a_2$ which means that consumers in country 1 are generally ready to accept a higher price than consumers in country 2 (prohibition price for country 1 is higher than for country 2). Technology of production is described in the following way. There is a constant marginal cost ($MC = c > 0$) which is the only cost to cover. We ignore the constant cost of production (e.g. investment cost) because it doesn't influence the optimisation result (sunk cost). By assumption $a_2 > c$ (this is a technical condition necessary for positive quantity of goods supplied to both markets).

Hence, the profit function of a price differentiating monopoly has a form of sum of revenues on both markets subtracted costs of production):

(1)

$$\Pi = (a_1 - b_1 \cdot q_1) \cdot q_1 + (a_2 - b_2 \cdot q_2) \cdot q_2 - c(q_1 + q_2) = a_1 \cdot q_1 - b_1 \cdot q_1^2 + a_2 \cdot q_2 - b_2 q_2^2 - c \cdot q_1 - c \cdot q_2$$

First order condition allows us to calculate the profit maximising quantities supplied to both markets:

$$(2) \quad \frac{\partial \Pi}{\partial q_1} = a_1 - 2b_1 \cdot q_1 - c = 0 \Rightarrow q_1^* = \frac{a_1 - c}{2b_1} > 0$$

$$(3) \quad \frac{\partial \Pi}{\partial q_2} = a_2 - 2b_2 \cdot q_2 - c = 0 \Rightarrow q_2^* = \frac{a_2 - c}{2b_2} > 0$$

From the equations (2) and (3) and the assumptions concerning a_1 , a_2 , b_1 , b_2 we can't see whether the optimal quantity supplied to market 1 is bigger than the one sold in market 2. We have in the numerators of both fractions describing the optimal quantities: $a_1 - c > a_2 - c$, but also in the denominators: $b_1 > b_2$. As $a_2 > c$, both markets are supplied. The overall quantity produced is equal to Q^* :

$$(4) \quad Q^* = q_1^* + q_2^* = \frac{a_1 - c}{2b_1} + \frac{a_2 - c}{2b_2} = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c \cdot (b_1 + b_2)}{2b_1 \cdot b_2}$$

The prices in both markets are equal to (respectively):

$$(5) \quad P_1^*(q_1^*) = a_1 - b_1 \cdot \frac{a_1 - c}{2b_1} = \frac{a_1 + c}{2}$$

$$(6) \quad P_2^*(q_2^*) = a_2 - b_2 \cdot \frac{a_2 - c}{2b_2} = \frac{a_2 + c}{2}$$

As – by assumption – a_i ($i = 1, 2$) and c are positive, both prices are positive. Because we assume that $a_1 > a_2$, P_1^* is bigger than P_2^* .

The maximal profit is equal to:

$$(7) \quad \begin{aligned} \Pi &= (a_1 - b_1 \cdot q_1^*)q_1^* + (a_2 - b_2 \cdot q_2^*)q_2^* - c(q_1^* + q_2^*) = \left(a_1 - b_1 \cdot \frac{a_1 - c}{2b_1} \right) \cdot \frac{a_1 - c}{2b_1} + \\ &\left(a_2 - b_2 \cdot \frac{a_2 - c}{2b_2} \right) \cdot \frac{a_2 - c}{2b_2} - c \cdot \left(\frac{a_1 - c}{2b_1} + \frac{a_2 - c}{2b_2} \right) = \frac{(a_1 + c)(a_1 - c)}{4b_1} + \frac{(a_2 + c)(a_2 - c)}{4b_2} - \\ &\frac{2c(a_1 - c)}{2 \cdot 2b_1} - \frac{2c(a_2 - c)}{2 \cdot 2b_2} = \frac{(a_1 + c)(a_1 - c) - 2c(a_1 - c)}{4b_1} + \frac{(a_2 + c)(a_2 - c) - 2c(a_2 - c)}{4b_2} = \\ &= \frac{(a_1 - c)(a_1 + c - 2c)}{4b_1} + \frac{(a_2 - c)(a_2 + c - 2c)}{4b_2} = \frac{(a_1 - c)^2}{4b_1} + \frac{(a_2 - c)^2}{4b_2} > 0 \end{aligned}$$

The maximal profit calculated in the equation (7) is surely positive as it is sum of two positive fractions.

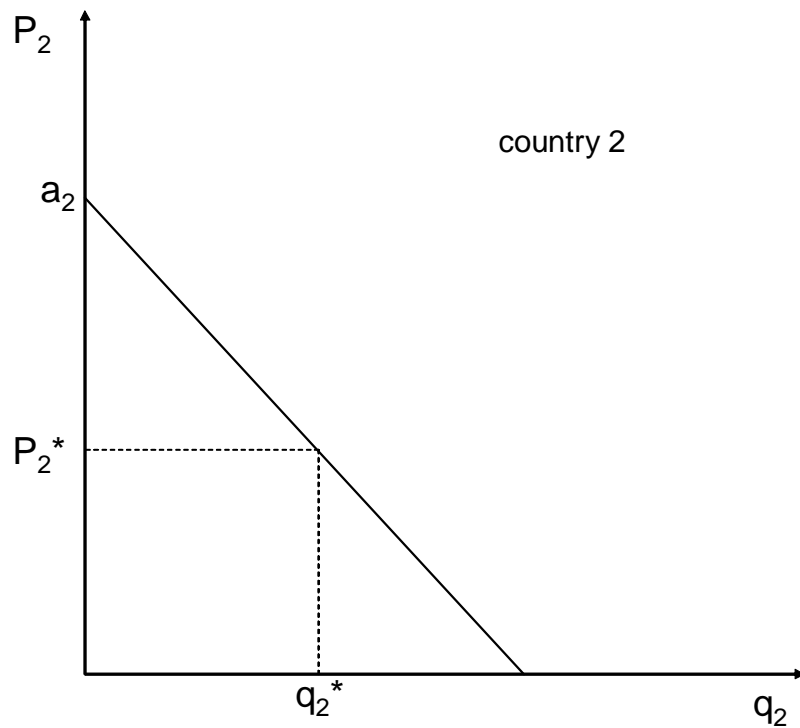
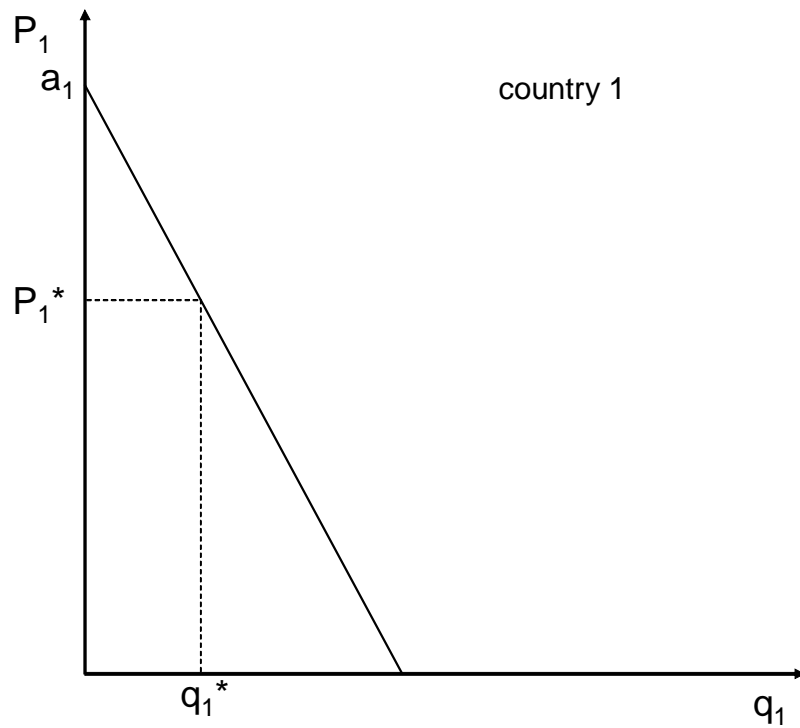
We define social welfare of a country as a sum of monopoly profit and consumers surplus. We assume that the changes in the analysed markets don't influence other markets in both countries (the partial equilibrium model, *ceteris paribus* condition). In both countries the consumer surpluses are equal to (respectively):

$$(8) \quad \wedge_1 = \frac{1}{2} q_1^* \cdot (a_1 - P_1^*) = \frac{1}{2} \left(\frac{a_1 - c}{2b_1} \right) \cdot \left(a_1 - \frac{a_1 + c}{2} \right) = \frac{(a_1 - c)^2}{8b_1} > 0$$

$$(9) \quad \wedge_2 = \frac{1}{2} q_2^* (a_2 - P_2^*) = \frac{1}{2} \left(\frac{a_2 - c}{2b_2} \right) \left(a_2 - \frac{(a_2 + c)}{2} \right) = \frac{(a_2 - c)^2}{8b_2} > 0$$

We illustrate both consumers' surpluses in graph 1.

Fig. 1: Consumer surplus in the segmented markets of countries 1 and 2



Source: own concept.

The aggregated consumers' surplus of both countries is a sum of the consumers' surpluses in both markets (expressions in equations (8) and (9)):

$$(10) \quad \Sigma(\wedge_1 + \wedge_2) = \frac{(a_1 - c)^2}{8b_1} + \frac{(a_2 - c)^2}{8b_2} = \frac{b_2(a_1 - c)^2 + b_1(a_2 - c)^2}{8b_1b_2}$$

Therefore, the overall welfare in both countries with national market segmentation is the sum of consumer surpluses (formula (10)) and monopoly profit (equation (7)).

$$(11) \quad W = \Pi + \Sigma(\wedge_1 + \wedge_2) = \frac{(a_1 - c)^2}{4b_1} + \frac{(a_2 - c)^2}{4b_2} + \frac{(a_1 - c)^2}{8b_1} + \frac{(a_2 - c)^2}{8b_2} = \frac{3(a_1 - c)^2}{8b_1} + \frac{3(a_2 - c)^2}{8b_2}$$

In this setting the most important result of the creation of a single market is liquidation of barriers to arbitrage between both national markets. We show it as a possibility to aggregate both national demand functions and get an unified demand function $Q(P)$:

$$b_1 \cdot q_1 = a_1 - P_1 \Rightarrow q_1 = \frac{a_1}{b_1} - \frac{P_1}{b_1}$$

$$b_2 \cdot q_2 = a_2 - P_2 \Rightarrow q_2 = \frac{a_2}{b_2} - \frac{P_2}{b_2}$$

Aggregating demand allows us to settle a unified price in both markets ($P_1 = P_2$):

$$Q(P) = q_1 + q_2 = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 \cdot b_2} - \left(\frac{P \cdot b_2 + P \cdot b_1}{b_1 \cdot b_2} \right)$$

$$(12) \quad Q(P) = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 \cdot b_2} - \frac{P \cdot (b_1 + b_2)}{b_1 \cdot b_2}$$

We reverse the demand function from equation (12) to calculate maximal profit of the monopoly:

$$(13) \quad P(Q) = \frac{-b_1 \cdot b_2 \cdot Q}{b_1 + b_2} + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2}$$

Now we define profit of the monopoly as:

$$(14) \quad \Pi(Q) = P(Q) \cdot Q - c \cdot Q$$

The equation (14) summarised the most important result of the introduction of a single market in our setting. A unique equilibrium price is set in both markets (in both analysed countries). For simplicity we assume zero transaction costs of supplying both markets (in case of segmented markets we made this unspoken assumption, too). Such a situation may occur in case of supplying markets of neighbouring countries. Before calculating the maximal profit we put the reversed demand function from the equation (13) into the profit function of the analysed monopoly (equation (14)):

$$(15) \quad \Pi(Q) = \left(\frac{-b_1 \cdot b_2 \cdot Q}{b_1 + b_2} + \frac{a_1 b_2 + a_2 \cdot b_1}{b_1 + b_2} \right) \cdot Q - cQ = \frac{a_1 \cdot b_2}{b_1 + b_2} \cdot Q + \frac{a_2 \cdot b_1}{b_1 + b_2} \cdot Q - \frac{b_1 \cdot b_2 \cdot Q^2}{b_1 + b_2} - cQ$$

Then we use first order condition for profit maximisation:

$$\frac{\partial \Pi}{\partial Q} = \frac{a_1 \cdot b_2}{b_1 + b_2} + \frac{a_2 \cdot b_1}{b_1 + b_2} - \frac{2b_1 \cdot b_2 \cdot Q}{b_1 + b_2} - c = 0$$

and calculate the profit maximising quantity supplied to the common market (Q_0) and the equilibrium price ($P(Q_0)$):

$$(16) \quad Q_0 = \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{2b_1 \cdot b_2},$$

$$(17) \quad P(Q_0) = \frac{-b_1 \cdot b_2 \cdot Q_0}{b_1 + b_2} + \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{b_1 + b_2} = \frac{-b_1 \cdot b_2}{b_1 + b_2} \cdot \left(\frac{a_1 \cdot b_2 + a_2 \cdot b_1 - cb_1 - cb_2}{2b_1 \cdot b_2} \right) + \frac{a_1 b_2 + a_2 \cdot b_1}{b_1 + b_2}$$

$$= \frac{a_1 \cdot b_2 + a_2 \cdot b_1 + c(b_1 + b_2)}{2(b_1 + b_2)} = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{2(b_1 + b_2)} + \frac{c}{2}$$

Importantly, the unified price set in both markets lies between differentiated prices under segmented markets:

$$P(Q_0) = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{2(b_1 + b_2)} + \frac{c}{2} < \frac{a_1 \cdot b_2 + a_1 \cdot b_1}{2(b_1 + b_2)} + \frac{c}{2} = \frac{a_1(b_1 + b_2)}{2(b_1 + b_2)} + \frac{c}{2} = \frac{a_1 + c}{2} = P_1^*(q_1^*)$$

$$P(Q_0) = \frac{a_1 \cdot b_2 + a_2 \cdot b_1}{2(b_1 + b_2)} + \frac{c}{2} > \frac{a_2 \cdot b_2 + a_2 \cdot b_1}{2(b_1 + b_2)} + \frac{c}{2} = \frac{a_2 + c}{2} = P_2^*(q_2^*).$$

Note also that it holds:

$$P(Q_0) < \frac{a_1 + c}{2} < a_1$$

and

$$P(Q_0) > \frac{a_2 + c}{2} > c.$$

If we additionally assume that

$$(18) \quad a_2 > \frac{a_1 + c}{2},$$

then we get:

$$(19) \quad P(Q_0) < \frac{a_1 + c}{2} < a_2.$$

Equation (19) is a necessary condition for a monopoly to supply both national markets after introducing a single market regulation (and – as a consequence – a unified price in both markets). The maximal monopoly profit in the unified market is equal to:

$$(20) \quad \begin{aligned} \Pi^* &= P(Q) \cdot Q - c \cdot Q = (P(Q) - c) \cdot Q = \\ &\left(\frac{a_1 b_2 + a_2 b_1 + c}{2(b_1 + b_2)} - c \right) \cdot \frac{a_1 b_2 + a_2 b_1 - c(b_1 + b_2)}{2b_1 b_2} = \frac{a_1 b_2 + a_2 b_1 - c(b_1 + b_2)}{2(b_1 + b_2)} \cdot \frac{a_1 b_2 + a_2 b_1 - c(b_1 + b_2)}{2b_1 b_2} \\ &= \frac{[a_1 b_2 + a_2 b_1 - c(b_1 + b_2)]^2}{4b_1 b_2 (b_1 + b_2)} > 0 \end{aligned}$$

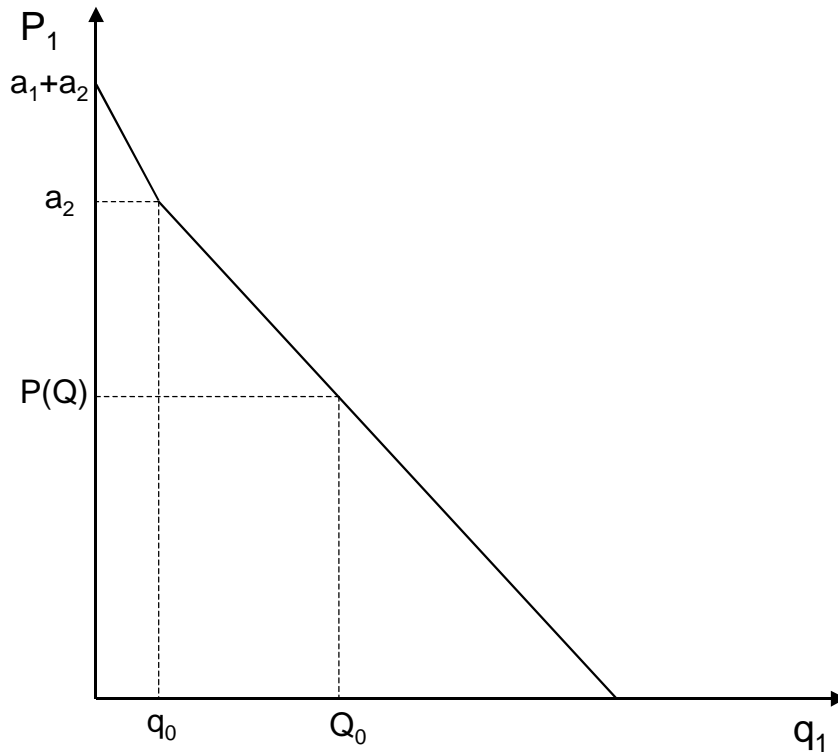
Π^* in formula (20) is obviously positive, because $a_1 > a_2 > c$, which means that $a_1 > c$ and $a_2 > c$. In a numerator of the fraction both parts of the sum exceed the subtracted part. Now we can calculate a new consumer surplus in both countries under unified equilibrium price. We present this result also graphically in graph 2. We first calculate q_0 in graph 2. We use demand function in country 1 with the price $P_1(q_0)$ equal to a_2 :

$$q_0 = \frac{a_1 - a_2}{b_1}.$$

The equilibrium quantity Q_0 is bigger than q_0 :

$$\begin{aligned}
Q_0 &= \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{2b_1 \cdot b_2} > \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - (2a_2 - a_1)(b_1 + b_2)}{2b_1 \cdot b_2} = \\
(21) \quad &\frac{a_1 \cdot b_2 + a_2 \cdot b_1 - 2a_2 \cdot b_1 + a_1 \cdot b_1 - 2a_2 \cdot b_2 + a_1 \cdot b_2}{2b_1 \cdot b_2} = \frac{(a_1 - a_2)b_1 + 2b_2(a_1 - a_2)}{2b_1 \cdot b_2} = \\
&\frac{(a_1 - a_2)(b_1 + 2b_2)}{2b_1 \cdot b_2} > \frac{(a_1 - a_2)2b_2}{2b_1 \cdot b_2} = \frac{a_1 - a_2}{b_1} = q_0.
\end{aligned}$$

Fig. 2: Consumers' surpluses in the single market



Source: own concept.

Before we calculate the social welfare of both countries under the single market equilibrium (without price differentiation) we have to calculate the consumer surplus as a sum of the areas A, B and C in graph 2:

$$(22) \quad A = \frac{1}{2} q_0 \cdot a_1 = \frac{a_1 \cdot (a_1 - a_2)}{2b_1}$$

As – by assumption – $a_1 > a_2$, the area A in formula (22) is positive.

$$\begin{aligned}
(23) \quad B + C &= \frac{1}{2}(q_0 + Q_0) \cdot (a_2 - P(Q_0)) = \frac{1}{2} \left(\frac{(a_1 - a_2)}{b_1} + \right. \\
&\left. \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c(b_1 + b_2)}{2b_1 \cdot b_2} \right) \cdot \left(a_2 - \frac{a_1 \cdot b_2 + a_2 \cdot b_1 - c}{2(b_1 + b_2)} - \frac{c}{2} \right) \\
&= \frac{1}{2} \cdot \frac{(2a_1 \cdot b_2 - 2a_2 \cdot b_2 + a_1 \cdot b_2 + a_2 \cdot b_1 - cb_1 - cb_2)}{2b_1 \cdot b_2} \cdot \frac{(2a_2 \cdot b_1 + 2b_2 \cdot a_2 - a_1 \cdot b_2 - a_2 \cdot b_1 - c \cdot (b_1 + b_2))}{2(b_1 + b_2)} \\
&= \frac{[(a_2 - c) \cdot b_1 + (3a_1 - 2a_2 - c) \cdot b_2] \cdot [(a_2 - c) \cdot b_1 + (2a_2 - a_1 - c) \cdot b_2]}{8b_1 \cdot b_2(b_1 + b_2)} \\
&= \frac{(a_2 - c)^2 b_1^2 + (a_2 - c)b_1 b_2(3a_1 - 2a_2 - c + 2a_2 - a_1 - c) + (3a_1 - 2a_2 - c)(2a_2 - a_1 - c)b_2^2}{8b_1 b_2(b_1 + b_2)} \\
&= \frac{(a_2 - c)^2 b_1^2 + (a_2 - c)b_1 b_2(2a_1 - 2c) + (3a_1 - 2a_2 - c)(2a_2 - a_1 - c)b_2^2}{8b_1 b_2(b_1 + b_2)} \\
&= \frac{(a_2 - c)^2 b_1^2 + 2(a_1 - c)(a_2 - c)b_1 b_2 + (8a_1 a_2 - 4a_2^2 - 3a_1^2 - 2a_1 c + c^2)b_2^2}{8b_1 b_2(b_1 + b_2)} \\
&= \frac{(a_2 - c)^2 b_1^2 + 2(a_1 - c)(a_2 - c)b_1 b_2 + (8a_1 a_2 - 4a_2^2 - 4a_1^2 + a_1^2 - 2a_1 c + c^2)b_2^2}{8b_1 b_2(b_1 + b_2)} \\
&= \frac{(a_2 - c)^2 b_1^2 + 2(a_1 - c)(a_2 - c)b_1 b_2 + (a_1 - c)^2 b_2^2 - 4(a_1 - a_2)^2 b_2^2}{8b_1 b_2(b_1 + b_2)} \\
&= \frac{[(a_2 - c)b_1 + (a_1 - c)b_2]^2 - 4(a_1 - a_2)^2 b_2^2}{8b_1 b_2(b_1 + b_2)}
\end{aligned}$$

We can prove that sum of areas B and C calculated in the formula (24) is positive. We transform the equation as follows:

$$\frac{[(a_2 - c)b_1 + (a_1 - c)b_2 + 2(a_1 - a_2)b_2] \cdot [(a_2 - c)b_1 + (a_1 - c)b_2 - 2(a_1 - a_2)b_2]}{8b_1 b_2(b_1 + b_2)}$$

which is equal to:

$$(24) \quad \frac{[(a_2 - c)b_1 + (a_1 - c)b_2 + 2(a_1 - a_2)b_2] \cdot [(a_2 - c)b_1 + b_2(a_1 - c - 2a_1 + 2a_2)]}{8b_1 b_2(b_1 + b_2)}$$

In equation (24) the denominator is positive, as b_1 and b_2 are the positive parameters of the demand functions. All elements in the first set of brackets and in the first multiplication in the

second set of brackets are positive (assumed relation between parameters a_1 , a_2 and c). The last multiplication in the second set of brackets is positive because:

$$a_1 - c - 2a_1 + 2a_2 = 2a_2 - a_1 - c,$$

where $(2a_2 - a_1)$ is bigger than c due to assumption (18). It means that also the last part of the multiplication in the numerator is positive and the total fraction in the formula (24) is positive. Adding up the areas calculated in formulas (22) and (23) we get the consumers' surplus in both countries:

$$\begin{aligned}
 (25) \quad A + B + C &= \frac{a_1(a_1 - a_2)}{2b_1} + \frac{[(a_2 - c)b_1 + (a_1 - c)b_2]^2 - 4(a_1 - a_2)^2 b_2^2}{8b_1 b_2 (b_1 + b_2)} \\
 &= \frac{4a_1(a_1 - a_2)b_2(b_1 + b_2) + [(a_2 - c)b_1 + (a_1 - c)b_2]^2 - 4(a_1 - a_2)^2 b_2^2}{8b_1 b_2 (b_1 + b_2)} \\
 &= \frac{[(a_2 - c)b_1 + (a_1 - c)b_2]^2 + 4(a_1 - a_2)b_2[a_1(b_1 + b_2) - (a_1 - a_2)b_2]}{8b_1 b_2 (b_1 + b_2)} \\
 &= \frac{[a_1 b_2 + a_2 b_1 - c(b_1 + b_2)]^2 + 4(a_1 - a_2)b_2(a_1 b_1 + a_2 b_2)}{8b_1 b_2 (b_1 + b_2)}
 \end{aligned}$$

The social welfare of both countries under single market equilibrium (without price differentiation) can be calculated as sum of the monopoly profit Π^* (formula (21)) and the consumers' surplus $A + B + C$ (graph 2; formula (25)):

$$\begin{aligned}
 (26) \quad W^* = \Pi^* + A + B + C &= \frac{[a_1 b_2 + a_2 b_1 - c(b_1 + b_2)]^2}{4b_1 b_2 (b_1 + b_2)} + \frac{[a_1 b_2 + a_2 b_1 - c(b_1 + b_2)]^2 + 4(a_1 - a_2)b_2(a_1 b_1 + a_2 b_2)}{8b_1 b_2 (b_1 + b_2)} \\
 &= \frac{3[a_1 b_2 + a_2 b_1 - c(b_1 + b_2)]^2 + 4b_2(a_1 - a_2)(a_1 b_1 + a_2 b_2)}{8b_1 b_2 (b_1 + b_2)}
 \end{aligned}$$

The last step of our analysis is a computation of the levels of welfare under the initial segmentation of both national markets (W calculated in the formula (12)) and under the rules of the single market (D' in the formula (26)). Let us start with transforming the formula (11) as follows:

$$(11a) \quad W = \frac{3(a_1 - c)^2}{8b_1} + \frac{3(a_2 - c)^2}{8b_2}$$

$$\begin{aligned}
&= \frac{3(a_1 - c)^2 b_2 (b_1 + b_2) + 3(a_2 - c)^2 b_1 (b_1 + b_2)}{8b_1 b_2 (b_1 + b_2)} \\
&= \frac{3(a_1 - c)^2 b_2^2 + 3(a_2 - c)^2 b_1^2 + 3(a_1 - c)^2 b_1 b_2 + 3(a_2 - c)^2 b_1 b_2}{8b_1 b_2 (b_1 + b_2)} \\
&= \frac{3[(a_1 - c)b_2 + (a_2 - c)b_1]^2 - 6(a_1 - c)(a_2 - c)b_1 b_2 + 3(a_1 - c)^2 b_1 b_2 + 3(a_2 - c)^2 b_1 b_2}{8b_1 b_2 (b_1 + b_2)} \\
&= \frac{3[(a_1 - c)b_2 + (a_2 - c)b_1]^2 + 3b_1 b_2 [(a_1 - c) - (a_2 - c)]^2}{8b_1 b_2 (b_1 + b_2)}
\end{aligned}$$

We have finally:

$$(11b) \quad W = \frac{3[(a_1 - c)b_2 + (a_2 - c)b_1]^2}{8b_1 b_2 (b_1 + b_2)} + \frac{3(a_1 - a_2)^2}{8(b_1 + b_2)}$$

The social welfare under the single market rules (W' from the formula (26)) is equal to:

$$\begin{aligned}
W' &= \frac{3[a_1 b_2 + a_2 b_1 - c(b_1 + b_2)]^2 + 4b_2(a_1 - a_2)(a_1 b_1 + a_2 b_2)}{8b_1 b_2 (b_1 + b_2)} = \\
&= \frac{3[(a_1 - c)b_2 + (a_2 - c)b_1]^2}{8b_1 b_2 (b_1 + b_2)} + \frac{(a_1 - a_2)(a_1 b_1 + a_2 b_2)}{2b_1 (b_1 + b_2)}
\end{aligned}$$

We can transform it as an expression equal to the initial social welfare under price differentiation (11a) and some additional positive element. We proceed as follows:

$$W' - \frac{(a_1 - a_2)(a_1 b_1 + a_2 b_2)}{2b_1 (b_1 + b_2)} = W - \frac{3(a_1 - a_2)^2}{8(b_1 + b_2)}$$

and hence,

$$\begin{aligned}
(27) \quad W' &= W + \frac{(a_1 - a_2)(a_1 b_1 + a_2 b_2)}{2b_1 (b_1 + b_2)} - \frac{3(a_1 - a_2)^2}{8(b_1 + b_2)} \\
&= W + \frac{(a_1 - a_2)(a_1 b_1 + 3a_2 b_1 + 4a_2 b_2)}{8b_1 (b_1 + b_2)} > W
\end{aligned}$$

The expression added to the initial social welfare (W) is certainly positive, because $(a_1 - a_2)$ is positive by assumption about characteristics of both markets and that the sum in the second set of brackets contains only positive expressions. Also $(b_1 + b_2)$ in denominator are the positive parameters of the demand functions.

In equation (27) we have proved that creation of a united market results in welfare surplus even if both markets are still supplied by the monopoly. We are aware that our simple model doesn't take into account many important aspects of introducing the Single European Market. We put our attention only to the influence of price unification on the international level (here represented by two economies constituting a united market).

We ignored e.g. possible results of uniting markets for the competition (it is possible that more than one producer will be supplying both markets). We also assumed zero transaction costs (and other transaction costs). One reason is that some of them are existent (or negligible) in both situations: with and without a single market. Another reason is that in consequence of uniting the markets some transaction costs are eliminated (e.g. administrative costs of supplying a foreign market). The easiest way to build it into our model is to assume decreasing marginal cost after the market unification. It would make increase in welfare even bigger. We also didn't calculate the economic consequences of uniting markets from the point of view of every country separately (especially interesting can be results for the producer country).

We hope that our setting shows an interesting aspect of the complex phenomena of unification of the European market. It can be also helpful in analysing a more general case of national economies aiming at unification of their markets.