

# A general equilibrium model of Vertical Intra-Industry Trade and Inter-Industry Trade with complete specialization of countries<sup>\*</sup>

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## Introduction

Since Abd-El-Rahman (1986) made the key distinction between vertical and horizontal intra-industry trade (VIIT and HIIT), an abundant empirical literature has proved that the former accounts for a larger share of total trade than the latter, even with regard to trade between similar countries. For instance, Fontagné and Freudenberg (2002) have observed that VIIT and HIIT represented respectively 45% and 19% of intra-EU trade in 1999, and that VIIT has sharply increased since 1980, while HIIT has stagnated. By the same token, Greenaway and Milner (2002) have shown that VIIT accounted for two thirds of the UK's bilateral intra-industry trade with EU countries.

The theoretical literature has identified different determinants for these two types of trade: mainly, scale economies for HIIT and relative factor endowments for VIIT. This explanation of VIIT, which was primarily developed by Falvey (1981), is based on the assumption that the production of higher-quality goods requires a higher quantity of capital per unit of labour.

In contrast to Falvey's model, the purpose of our paper is to explain trade in goods of different quality that have identical factor intensity at any relative factor price, i.e. trade in "perfectly intra-industry goods", according to Davis (1995). This approach is consistent with the evidence that vertical intra-industry trade is observed even at a very disaggregated level of statistical classifications, that is to say even within goods that have very similar factor intensity.

Therefore, in this paper we extend the definition of "perfectly intra-industry goods" to the case of vertically differentiated products, but differently from Davis (1995), we model explicitly the demand side of the economy, in order to define goods as intra-industry and vertically differentiated from both a production and a consumption point of view.

In the first section we define the supply side of the model, which consists in two industries and three goods, called  $x$ ,  $y$  and  $z$ . All the goods are produced using two factors (capital and labour) and constant-return-to-scale technologies. Goods  $x$  and  $y$  are vertically differentiated and belong to the same industry, called industry  $x$ - $y$ :  $y$  is characterized by a higher quality than  $x$ . Their production functions have the same factor intensity but the production of one unit of the higher-quality good requires more capital and more labour than one unit of the lower-quality good. Good  $z$  belongs to another industry (industry  $z$ ), since it has a lower capital intensity than the goods  $x$  and  $y$ . Absolute quantities of factors required for the production of the three goods (i.e. total factor productivity) can differ across countries, while factor intensity is identical in all countries.

In the second section we introduce the demand side of the model. From a consumption point of view, goods  $x$  and  $y$  are characterized as vertically differentiated by the assumption that all

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consumers demand only the higher-quality good ( $y$ ) if its price relative to the lower-quality ( $x$ ) is equal or lower than one. This assumption is consistent with the theoretical definition of pure quality differentiation. Hence, goods  $x$  and  $y$  are perfect substitutes and the marginal rate of substitution between them is constant and greater than one for each individual. The marginal rate of substitution can be different from one individual to another, representing different levels of “preference for quality” among people. Heterogeneous preferences with regard to quality allow demands of both  $x$  and  $y$  to be positive, at a given relative price.

In the third section we study the autarky equilibrium in a given country, hence determining the autarky relative prices of the goods  $y$  and  $z$  in terms of the good  $x$  (which is chosen as the numeraire) and the autarky relative price of factors.

In the last two sections, we deal with free trade equilibrium. Unlike Davis (1995), who has determined the patterns of trade between two countries in the integrated equilibrium framework (thus assuming that countries don’t specialize completely in the production of one good), we intend to explain the determinants of trade in the case of complete specialization of countries. In the fourth section we determine in which cases the complete specialization in the production of  $x$ ,  $y$  or  $z$  constitutes an equilibrium situation for a country in free trade. Finally, in the fifth section we construct a general equilibrium model of trade between three countries (called 1, 2 and 3). We demonstrate that the complete specialization of countries 1, 2 and 3 respectively in the production of goods  $x$ ,  $y$  and  $z$  constitutes an equilibrium situation for the three countries, provided that:

- country 1 and country 2 have similar relative endowments of factors;
- countries 1 and 2 are highly capital-abundant with regard to country 3;
- there are some (small) Hicks-neutral differences between the production functions of  $x$  or  $y$  in countries 1 and 2, which generate a relative technological advantage of country 1 in the production of  $x$  (and symmetrically a relative advantage of country 2 in the production of  $y$ ).

If these conditions are satisfied, the complete specialization of the three countries is an equilibrium, in which trade between countries 1 and 2 is totally intra-industry, whereas trade between country 3 on the one hand and countries 1 and 2 on the other hand is totally inter-industry.

## 1. The supply-side of the model

### Definitions and assumptions

Country  $i$  is endowed with capital  $K_i$  and labour  $L_i$ , which are mobile within industries in the same country and immobile between countries. The two factors of production are fully employed and they can be used to produce three final goods:  $x$ ,  $y$  and  $z$ . Perfect competition prevails in both goods and factor markets. Goods  $x$  and  $y$  belong to the same industry (which is called industry  $x$ - $y$ ) and they are vertically differentiated ( $y$  is of a higher quality than  $x$ ), whereas good  $z$  belongs to another industry (called industry  $z$ ). Following Davis (1995), we define an industry as a group of goods that are produced under identical factor intensity for any given factor price ratio.

The production functions of goods  $x$ ,  $y$  and  $z$  in country  $i$  are:

$$x_i = \left( \frac{1}{\gamma_i} \right) L_{ix}^\alpha K_{ix}^{1-\alpha} \quad (1.1)$$

$$y_i = \left( \frac{1}{\delta_i} \right) L_{iy}^\alpha K_{iy}^{1-\alpha} \quad (1.2)$$

$$z_i = \left( \frac{1}{\tau_i} \right) L_{iz}^\beta K_{iz}^{1-\beta} \quad (1.3)$$

where  $x_i, y_i$  and  $z_i$  are the quantities of goods  $x, y$  and  $z$  produced in country  $i$ ;  $L_{ix}, K_{ix}, L_{iy}, K_{iy}, L_{iz}, K_{iz}$  are the quantities of labour and capital devoted to the production of goods  $x, y$  and  $z$ , respectively; the parameters  $\gamma_i, \delta_i$  and  $\tau_i$  are the reversed coefficients of total factor productivity in sectors  $x, y$  and  $z$ , respectively. Since these parameters are country-specific, they capture Hicks-neutral productivity differences across countries in the production of the three goods. We assume that  $\gamma_i$  is smaller than  $\delta_i$  in every country  $i$ . This means that the production of one unit of the higher quality good ( $y$ ) needs more capital and more labour than the production of one unit of the low quality good ( $x$ ). We also make the following hypothesis:

$$0 < \alpha < \beta < 1$$

This means that: the production technology exhibits constant returns to scale in both industries; industry  $z$  is more labour intensive than industry  $x$ - $y$ ; factor intensity of industries do not vary across countries (because parameters  $\alpha$  and  $\beta$  are not country-specific).

### Conditional demand of factors and factor intensity

The representative firm producing good  $x$  in country  $i$  minimize its costs constrained by the production technology. The cost minimization problem for the firm producing  $x$  is:

$$\begin{aligned} \min \quad & w_i L_{ix} + r_i K_{ix} \\ \text{s.t.} \quad & x_i = \left( \frac{1}{\gamma_i} \right) L_{ix}^\alpha K_{ix}^{(1-\alpha)} \end{aligned} \quad (1.4)$$

where  $w_i$  and  $r_i$  are wage rate and capital rental, respectively.

Solving the cost minimization problem, we find the conditional demand functions for factors  $K$  and  $L$  of the representative firm producing  $x$ . Conditional demand functions depend on the level of output produced in country  $i$  ( $x_i$ ) as well as on the factor prices ratio:

$$K_{ix}^* = \gamma_i x_i \left( \frac{\alpha}{1-\alpha} \frac{r_i}{w_i} \right)^{(-\alpha)} \quad (1.5)$$

$$L_{ix}^* = \gamma_i x_i \left( \frac{\alpha}{1-\alpha} \frac{r_i}{w_i} \right)^{(1-\alpha)} \quad (1.6)$$

Symmetrically, the representative firms producing goods  $y$  and  $z$  minimize their costs constrained by the production technology in these sectors. Solving this cost minimization problem, we obtain the conditional demand functions for factors  $K$  and  $L$  of the representative firms producing  $y$  and  $z$ :

$$K_{iy}^* = \delta_i y_i \left( \frac{\alpha}{1-\alpha} \frac{r_i}{w_i} \right)^{(-\alpha)} \quad (1.7)$$

$$L_{iy}^* = \delta_i y_i \left( \frac{\alpha}{1-\alpha} \frac{r_i}{w_i} \right)^{(1-\alpha)} \quad (1.8)$$

$$K_{iz}^* = \tau_i z_i \left( \frac{\beta}{1-\beta} \frac{r_i}{w_i} \right)^{(-\beta)} \quad (1.9)$$

$$L_{iz}^* = \tau_i z_i \left( \frac{\beta}{1-\beta} \frac{r_i}{w_i} \right)^{(1-\beta)} \quad (1.10)$$

Conditional demand functions confirm that goods  $x$  and  $y$  belong to the same industry, because they are produced under the same factor intensity for any given relative price of factors<sup>1</sup>. In fact, from the expressions (1.5) and (1.6) and from the expressions (1.7) and (1.8) we obtain the relative demand of factors (i.e. factor intensity) of sectors  $x$  and  $y$  (respectively). These factor intensities are equal for a given factor price ratio.

$$\frac{K_{ix}^D}{L_{ix}^D} = \frac{K_{iy}^D}{L_{iy}^D} = \frac{(1-\alpha)}{\alpha} \left( \frac{w_i}{r_i} \right) \quad (1.11)$$

On the other hand, solving the expressions (1.9) and (1.10), we obtain the relative demand of factors of the representative firm producing good  $z$ , which is different, for any given factor price ratio, from the relative demand of factors of sectors  $x$  and  $y$ . Therefore, good  $z$  doesn't belong to the industry including goods  $x$  and  $y$ .

$$\frac{K_{iz}^D}{L_{iz}^D} = \frac{(1-\beta)}{\beta} \left( \frac{w_i}{r_i} \right) \quad (1.12)$$

Comparing the equations (1.11) and (1.12), we can observe that industry  $x$ - $y$  is more capital-intensive than industry  $z$ , for any given factor price ratio.

### Determination of cost functions of the representative firms

Using the conditional demand of capital and labour of the firm producing the good  $x$  -cf. equations (1.5) and (1.6)- we can write the cost function and the average cost function of the representative firm producing  $x$ :

$$C_{ix}(x_i) = r_i K_{ix}^* + w_i L_{ix}^* = G \gamma_i w_i^\alpha r_i^{(1-\alpha)} x_i \quad (1.13)$$

$$AC_{ix} = G \gamma_i r_i^{(1-\alpha)} w_i^\alpha \quad (1.14)$$

where  $G = \alpha^{(-\alpha)} (1-\alpha)^{(\alpha-1)}$ .

Symmetrically, we determine the cost and the average cost functions of the representative firm producing  $y$ :

$$C_{iy}(y_i) = G \delta_i w_i^\alpha r_i^{(1-\alpha)} y_i \quad (1.15)$$

$$AC_{iy} = G \delta_i r_i^{(1-\alpha)} w_i^\alpha \quad (1.16)$$

and those of the representative firm producing  $z$ :

$$C_{iz}(z_i) = F \tau_i w_i^\beta r_i^{(1-\beta)} z_i \quad (1.17)$$

$$AC_{iz} = F \tau_i r_i^{(1-\beta)} w_i^\beta \quad (1.18)$$

where  $F = \beta^{(-\beta)} (1-\beta)^{(\beta-1)}$ .

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<sup>1</sup> According to a definition introduced by Davis (1995), goods  $x$  and  $y$  are “perfectly intra-industry goods”.

In a context of perfect competition, firms don't make profits in the long-run equilibrium. Therefore, if a good is produced in country  $i$ , its price must be equal to its average cost. For example, if good  $x$  is produced in country  $i$ , the following condition must be verified:

$$p_{ix} = G\gamma_i r_i^{(1-\alpha)} w_i^\alpha \quad (1.19)$$

where  $p_{ix}$  is the price of good  $x$  in country  $i$ .

## 2. The demand-side of the model

### Preliminary assumptions and determination of the individual budget constraint

We assume that the endowment of labour (i.e. the number of workers) of country  $i$  ( $L_i$ ) coincides with the number of consumers in the country. Every worker-consumer (indexed by  $j$ ) owns an amount of capital  $K_j$ , which is a share of the country's endowment  $K_i$ .

The individual supply of factors is assumed to be exogenous and amounts to one unit of labour and  $K_j$  units of capital. Therefore, the budget constraint of the individual consumer  $j$  is:

$$w_i + r_i K_j = p_{ix} x_j + p_{iy} y_j + p_{iz} z_j \quad (2.1)$$

where  $x_j$ ,  $y_j$  and  $z_j$  are the quantities of goods  $x$ ,  $y$  and  $z$  bought by the consumer  $j$ .

In this paper, we suppose that the domestic endowment of capital is equally distributed among people in the country, so that  $K_j = \frac{K_i}{L_i}$  for each individual consumer  $j$  in country  $i$ .

### Utility function

We define the utility function of the individual consumer  $j$  as:

$$U_j = (x_j + A_j y_j)^\lambda z_j^{(1-\lambda)} \quad (2.2)$$

where  $A_j > 1$  for each consumer  $j$  and  $0 < \lambda < 1$ .

Since goods  $x$  and  $y$  are vertically differentiated, i.e. they differ only in their quality, it seems reasonable to suppose that every consumer considers them as perfect substitutes. Therefore we assume that, for all consumers, the marginal rate of substitution between  $y$  and  $x$  is constant and bigger than one<sup>2</sup>. Thus, for any given relative price ( $p_y/p_x$ ), each consumer demands only one type of the differentiated good and if both goods have the same price, all the consumers will demand only good  $y$ , because everyone acknowledges that  $y$  is of a higher quality than  $x$ . Nevertheless, the marginal rate of substitution can vary from one to infinity across individuals, meaning that the "preference for quality" can differ across people, i.e. some consumers appreciate quality more than others.

### Individual demand functions

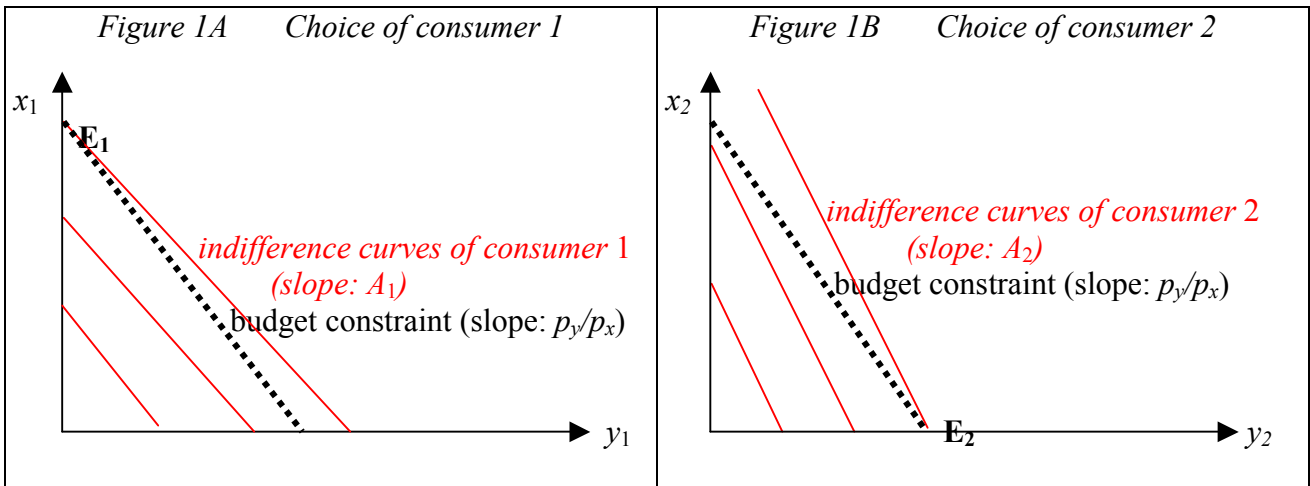
The decision problem of consumer  $j$  in country  $i$  is:

$$\begin{aligned} \max U_j &= (x_j + A_j y_j)^\lambda z_j^{(1-\lambda)} \\ \text{s.t. } p_{ix} x_j + p_{iy} y_j + p_{iz} z_j &= w_i + r_i K_j \end{aligned} \quad (2.3)$$

<sup>2</sup> In the equation (2.2),  $A_j$  represent the marginal rate of substitution between goods  $y$  and  $x$  for the consumer  $j$ , i.e. the rate at which the consumer  $j$  is willing to give up good  $x$  in exchange for one more unit of good  $y$ .

Every consumer  $j$  solves this problem in two stages: firstly, he allocates his resources to the consumption of  $z$  on the one hand and  $x$  or  $y$  on the other hand. Secondly, he makes a choice between  $x$  and  $y$ , comparing their relative price to his marginal rate of substitution  $A_j$ : if the relative price of  $y$  in terms of  $x$  is lower than  $A_j$ , the consumer will demand only good  $y$ ; if it is higher than  $A_j$ , he will demand only good  $x$ ; if it is equal to  $A_j$ , the consumer's choice between  $x$  and  $y$  of the consumer  $j$  will be indeterminate.

As regards the choice between  $x$  and  $y$ , consider the example represented in the figures 1A and 1B. Assume that two individuals, called consumer 1 and consumer 2, have identical resources and that consumer 2 has a stronger preference for quality than consumer 1, i.e.  $A_2 > A_1$ . Suppose that the relative price of  $y$  in terms of  $x$  is included between the marginal rates of substitution of the two consumers, i.e.  $A_1 < \frac{p_y}{p_x} < A_2$ . In this situation, in order to maximize their utility under their budget constraint, consumer 1 will demand only good  $x$  and consumer 2 will demand only good  $y$ . Equilibrium consumptions of goods  $x$  and  $y$  for the consumers 1 and 2 are represented in the figures 1A and 1B by the points  $E_1$  and  $E_2$ , respectively.



Considering the prices of goods and factors as given, every consumer  $j$  solves the problem (2.3) and determines his individual demands of goods  $x$ ,  $y$  and  $z$ . These demands are called respectively:  $x_j^D$ ,  $y_j^D$  and  $z_j^D$ . For a given relative price  $(p_{iy} / p_{ix})$ , the individual demands of each consumer  $j$  characterized by  $A_j > (p_{iy} / p_{ix})$  are:

$$\begin{aligned} x_j^D &= 0 \\ y_j^D &= \lambda \left( \frac{w_i + r_i K_j}{p_{iy}} \right) \\ z_j^D &= (1 - \lambda) \left( \frac{w_i + r_i K_j}{p_{iz}} \right) \end{aligned} \quad (2.4)$$

whereas the individual demands of each consumer  $j$  characterized by  $A_j < (p_{iy} / p_{ix})$  are:

$$\begin{aligned} x_j^D &= \lambda \left( \frac{w_i + r_i K_j}{p_{ix}} \right) \\ y_j^D &= 0 \\ z_j^D &= (1 - \lambda) \left( \frac{w_i + r_i K_j}{p_{iz}} \right) \end{aligned} \quad (2.5)$$

Evidently,  $x_j^D$  and  $y_j^D$  are indeterminate for all consumers  $j$  for whom  $A_j = (p_{iy} / p_{ix})$ .

### Global demand functions in country $i$

We define  $P_{iy}$  as the relative price of good  $y$  in terms of the good  $x$ :

$$P_{iy} = \frac{p_{iy}}{p_{ix}}$$

For any given relative price  $P_{iy}$ , the population of country  $i$  is divided into consumers of  $y$  (the individuals  $j$  who have  $A_j > P_{iy}$ ) and consumers of  $x$  (the individuals  $j$  who have  $A_j < P_{iy}$ ). The former have individual demands of the type described in equations (2.4); the latter have individual demands of the type described in equations (2.5). We suppose that no consumer has  $A_j = P_{iy}$ , i.e. all the consumers in country  $i$  have determinate individual demands of  $x$  and  $y$ .

For any given relative price  $P_{iy}$ , we define  $\varphi_i$  as the share of individuals characterized by  $A_j > P_{iy}$  in the total population of country  $i$ .  $\varphi_i$  takes values within the interval  $[0,1]$  and it decreases monotonically in  $P_{iy}$ . In fact, the higher  $P_{iy}$ , the smaller the number of persons characterized by  $A_j > P_{iy}$ , i.e. the higher is  $P_{iy}$ , the fewer are the consumers of  $y$  and the more are the consumers of  $x$ .

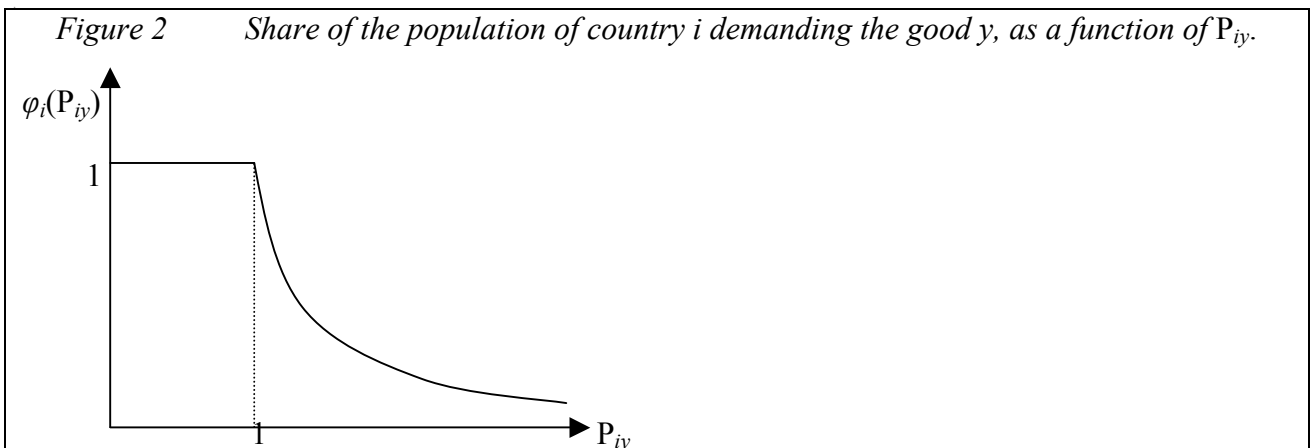
We can conclude that  $\varphi_i$  is a monotonically decreasing function of  $P_{iy}$ :  $\varphi_i = \varphi_i(P_{iy})$ .

For any given price  $P_{iy}$ ,  $\varphi_i(P_{iy}) L_i$  consumers in country  $i$  are characterized by  $A_j > P_{iy}$  and consequently have individual demands of the type (2.4);  $[1-\varphi_i(P_{iy})] L_i$  consumers in country  $i$  are characterized by  $A_j < P_{iy}$  and consequently have individual demands of the type (2.5).

We can give a simple explicit form to the function  $\varphi_i(P_{iy})$ , assuming that the distribution of parameters  $A_j$  among the population of country  $i$  is consistent with this form of the function:

$$\begin{aligned} \varphi_i(P_{iy}) &= \frac{1}{P_{iy}} & \text{if } P_{iy} \geq 1 \\ \varphi_i(P_{iy}) &= 1 & \text{if } P_{iy} < 1 \end{aligned} \quad (2.6)$$

This particular explicit form of the function  $\varphi_i(P_{iy})$  is consistent with the assumption that goods  $x$  and  $y$  are vertically differentiated: if  $P_{iy}$  is equal to or smaller than one (i.e.  $x$  and  $y$  have the same price or  $y$  is cheaper than  $x$ ), nobody will demand good  $x$  (because of its lower quality) and all the consumers in country  $i$  will demand good  $y$ . The function (2.6) is represented in figure 2.



We can now calculate the total demands of  $x$ ,  $y$  and  $z$  in country  $i$ , as a sum of the individual

$$\text{demands of consumers in country } i: \quad x_i^D = \sum_{j=1}^{L_i} x_j^D \quad ; \quad y_i^D = \sum_{j=1}^{L_i} y_j^D \quad ; \quad z_i^D = \sum_{j=1}^{L_i} z_j^D .$$

We have assumed that the stock of capital of country  $i$  ( $K_i$ ) is equally distributed among the consumers-workers of country  $i$ <sup>3</sup> (cf. page 4). In such a case, the total demand functions in country  $i$  are:

$$x_i^D = \lambda(1 - \varphi_i) \left( \frac{w_i L_i + r_i K_i}{p_{ix}} \right) \quad (2.7)$$

$$y_i^D = \lambda \varphi_i \left( \frac{w_i L_i + r_i K_i}{p_{iy}} \right) \quad (2.8)$$

$$z_i^D = (1 - \lambda) \left( \frac{w_i L_i + r_i K_i}{p_{iz}} \right) \quad (2.9)$$

where  $\varphi_i$  is a decreasing function of the relative price  $P_{iy}$ :  $\varphi_i = \varphi_i(P_{iy})$

### 3. Autarky equilibrium in country $i$

#### Equilibrium prices of goods

In autarky, positive quantities of the three goods  $x$ ,  $y$  and  $z$  are produced in country  $i$ . Therefore, equilibrium prices of  $x$ ,  $y$  and  $z$  must equal the average costs of production of the three goods in country  $i$ <sup>4</sup>. The three following conditions must be verified in equilibrium:

$$p_{ix}^A = G \gamma_i r_i^{(1-\alpha)} w_i^\alpha \quad (3.1)$$

$$p_{iy}^A = G \delta_i r_i^{(1-\alpha)} w_i^\alpha \quad (3.2)$$

$$p_{iz}^A = F \tau_i r_i^{(1-\beta)} w_i^\beta \quad (3.3)$$

where  $p_{ix}^A$ ,  $p_{iy}^A$ ,  $p_{iz}^A$  are respectively the autarky equilibrium prices of  $x$ ,  $y$  and  $z$ , in country  $i$ , as a function of the prices of factors.

We choose good  $x$  as the numeraire and call  $P_{iy}$  and  $P_{iz}$ , respectively, the relative prices of  $y$  and  $z$  in terms of  $x$ . With these notations, autarky equilibrium prices of  $y$  and  $z$  in terms of  $x$  are:

$$P_{iy}^A = \left( \frac{\delta_i}{\gamma_i} \right) \quad (3.4)$$

$$P_{iz}^A = \left( \frac{\tau_i F}{\gamma_i G} \right) \left( \frac{w_i}{r_i} \right)^{(\beta-\alpha)} \quad (3.5)$$

The autarky equilibrium price of  $y$  in terms of  $x$  depends only on the technological parameters  $\delta_i$  and  $\gamma_i$  and it is completely determined by the supply-side of the model, whereas the autarky equilibrium price of  $z$  in terms of  $x$  depends crucially on the relative price of factors, which will be determined by posing the equilibrium condition of factor markets.

<sup>3</sup> This assumption means that  $K_j = \frac{K_i}{L_i}$  for each individual consumer  $j$  in country  $i$ .

<sup>4</sup> The average cost functions of the representative firms producing  $x$ ,  $y$  and  $z$  in country  $i$  have been calculated in the section 1, cf. equations (1.14), (1.16) and (1.18).

## Total demands of goods

Now we can substitute in the general demand functions, which appear in expressions (2.7), (2.8) and (2.9), the autarky equilibrium value of the prices of  $x$ ,  $y$  and  $z$ , as determined in equations (3.1), (3.2) and (3.3). In this way, we obtain the total demands of goods  $x$ ,  $y$  and  $z$  in an autarky situation<sup>5</sup>, as a function of the relative price of factors in country  $i$ :

$$x_i^{DA} = \frac{\lambda [1 - \varphi_i(P_{iy}^A)]}{\gamma_i G} \left[ \left( \frac{w_i}{r_i} \right)^{(1-\alpha)} L_i + \left( \frac{r_i}{w_i} \right)^\alpha K_i \right] \quad (3.6)$$

$$y_i^{DA} = \frac{\lambda \varphi_i(P_{iy}^A)}{\delta_i G} \left[ \left( \frac{w_i}{r_i} \right)^{(1-\alpha)} L_i + \left( \frac{r_i}{w_i} \right)^\alpha K_i \right] \quad (3.7)$$

$$z_i^{DA} = \left( \frac{1-\lambda}{\tau_i F} \right) \left[ \left( \frac{w_i}{r_i} \right)^{(1-\beta)} L_i + \left( \frac{r_i}{w_i} \right)^\beta K_i \right] \quad (3.8)$$

It should be noted here that in autarky  $\varphi_i$  is completely determined at this stage. In fact,  $\varphi_i$  is a function of  $P_{iy}$  (the relative price of  $y$  in terms of  $x$ ), which depends only on technological parameters [cf. equation (3.4)].

## Sectorial and total demands of factors

At the autarky equilibrium in the goods markets, total productions of  $x$ ,  $y$  and  $z$  of the country  $i$  are equal to total demands (3.6), (3.7) and (3.8). Therefore, we can obtain the demands of capital and labour of each sector<sup>6</sup> by replacing the autarky production of  $x$ ,  $y$  and  $z$  in the conditional demands of factors determined in the section 1, in the equations (1.5) to (1.10). The demands of factors of each sector in the country  $i$  are written in the following table:

	<i>Autarky demand of capital</i>	<i>Autarky demand of labour</i>
<i>Sector x</i>	$K_{ix}^{DA} = \lambda(1-\alpha) [1 - \varphi_i(P_{iy}^A)] \left[ \left( \frac{w_i}{r_i} \right) L_i + K_i \right]$	$L_{ix}^{DA} = \lambda\alpha [1 - \varphi_i(P_{iy}^A)] \left[ L_i + \left( \frac{r_i}{w_i} \right) K_i \right]$
<i>Sector y</i>	$K_{iy}^{DA} = \lambda(1-\alpha) \varphi_i(P_{iy}^A) \left[ \left( \frac{w_i}{r_i} \right) L_i + K_i \right]$	$L_{iy}^{DA} = \lambda\alpha \varphi_i(P_{iy}^A) \left[ L_i + \left( \frac{r_i}{w_i} \right) K_i \right]$
<i>Sector z</i>	$K_{iz}^{DA} = (1-\lambda)(1-\beta) \left[ \left( \frac{w_i}{r_i} \right) L_i + K_i \right]$	$L_{iz}^{DA} = (1-\lambda)\beta \left[ L_i + \left( \frac{r_i}{w_i} \right) K_i \right]$

Summing up the demands of capital of the three sectors, we determine the total demand of capital of country  $i$  in autarky, as a function of the relative price of factors ( $w_i / r_i$ ):

<sup>5</sup> The notation  $x_i^{DA}$  means: demand of good  $x$ , in autarky, in country  $i$ .

<sup>6</sup> In this model there are three sectors (which coincide with the three goods  $x$ ,  $y$  and  $z$ ) and two industries ( $x$ - $y$  and  $z$ ).

$$K_i^{DA} = K_{ix}^{DA} + K_{iy}^{DA} + K_{iz}^{DA} = [1 - \lambda\alpha - (1 - \lambda)\beta] \left[ \left( \frac{w_i}{r_i} \right) L_i + K_i \right] \quad (3.9)$$

Symmetrically, summing up the demands of labour of sectors  $x$ ,  $y$  and  $z$ , we determine the total demand of labour of country  $i$  in autarky, as a function of the relative price of factors:

$$L_i^{DA} = L_{ix}^{DA} + L_{iy}^{DA} + L_{iz}^{DA} = [\lambda\alpha + (1 - \lambda)\beta] \left[ L_i + \left( \frac{r_i}{w_i} \right) K_i \right] \quad (3.10)$$

### The autarky equilibrium relative price of factors

In order to determine the autarky equilibrium price of factors, we impose the condition of equilibrium of the factor markets in country  $i$ . Total supply of factors of country  $i$  is exogenous and coincides with the endowments of capital and labour of country  $i$ . Therefore, we can establish the equilibrium conditions for the factor markets as follows<sup>7</sup>:

$$K_i^{DA} = K_i^S \Leftrightarrow [1 - \lambda\alpha - (1 - \lambda)\beta] \left[ L_i \left( \frac{w_i}{r_i} \right) + K_i \right] = K_i \quad (3.11)$$

$$L_i^{DA} = L_i^S \Leftrightarrow [\lambda\alpha + (1 - \lambda)\beta] \left[ L_i + \left( \frac{r_i}{w_i} \right) K_i \right] = L_i \quad (3.12)$$

The relative price of factors which equalize the supply and demand of factors can be determined either from equation (3.11) or from equation (3.12):

$$\left( \frac{w_i}{r_i} \right)^A = \left[ \frac{\lambda\alpha + (1 - \lambda)\beta}{1 - \lambda\alpha - (1 - \lambda)\beta} \right] \frac{K_i}{L_i} \quad (3.13)$$

The autarky equilibrium relative price of factors ( $w_i / r_i$ ) is an increasing function of the ratio of the endowments in country  $i$  ( $K_i / L_i$ ).

We can now substitute the autarky equilibrium relative price of factors in the expression of the relative price of  $z$  in terms of  $x$  [equation (3.5)], in order to define the autarky equilibrium value of this price (as a function of the exogenous variables). Remember that the relative price of  $y$  in terms of  $x$  in autarky doesn't depend on the relative price of factors, but it depends only on technological parameters [cf. equation (3.4)]. The autarky equilibrium value of price of  $z$  in terms of  $x$  is:

$$P_{iz}^A = \frac{F\tau_i}{G\gamma_i} \left[ \frac{\alpha\lambda + \beta(1 - \lambda)}{(1 - \alpha)\lambda + (1 - \beta)(1 - \lambda)} \right]^{(\beta - \alpha)} \left( \frac{K_i}{L_i} \right)^{(\beta - \alpha)} \quad (3.14)$$

Substituting the autarky equilibrium relative price of factors in equations (3.6), (3.7) and (3.8), we can determine the quantities of  $x$ ,  $y$ ,  $z$  produced and consumed at the autarky equilibrium in country  $i$ .

<sup>7</sup> Equations (3.11) and (3.12) represent respectively the equilibrium conditions for the markets of capital and labour in country  $i$ : the autarky demands of capital and labour are made equal to the exogenous supplies of the two factors.

#### 4. Free trade equilibrium in country $i$ with complete specialization

We suppose now that country  $i$  is a small economy which opens to international trade. In such a situation, we can assume that the international prices of goods  $x$ ,  $y$  and  $z$  are exogenously fixed by the world market. If a good is produced in country  $i$  at the free trade equilibrium, this means that its average cost of production in country  $i$  is equal to its price<sup>8</sup> (which is now exogenous). If a good is not produced in country  $i$  at the free trade equilibrium, this means that its average cost of production in country  $i$  is higher than its international price, so that the firms producing this good in country  $i$  would make losses.

We are interested here in three particular free trade equilibrium situations which correspond to complete specialization of country  $i$  in the production of, respectively,  $x$ ,  $y$  or  $z$ . The complete specialization of country  $i$  has implications on the nature of its trade with the rest of the world, as is shown in the following table:

Specialization of country $i$	Country $i$ 's exports	Country $i$ 's imports	Nature of multilateral trade
in good $x$	$x$	$y$ and $z$	<i>partly intra-industry</i>
in good $y$	$y$	$x$ and $z$	<i>partly intra-industry</i>
in good $z$	$z$	$x$ and $y$	<i>only inter-industry</i>

In this section, we study the conditions for the complete specialization of country  $i$  in a free trade context, i.e. we determine for which values of the exogenous variables complete specialization in the production of  $x$ ,  $y$  or  $z$  constitutes an equilibrium situation for country  $i$ .

##### The free-trade equilibrium relative price of factors in the case of complete specialization

If country  $i$  specializes in good  $x$ , its endowments of capital and labour (respectively:  $K_i$  and  $L_i$ ) are entirely used for the production of good  $x$ . The equilibrium conditions for the two factor markets are:

$$K_i = \gamma_i x_i \left( \frac{\alpha}{1-\alpha} \frac{r_i}{w_i} \right)^{(-\alpha)} \quad ; \quad L_i = \gamma_i x_i \left( \frac{\alpha}{1-\alpha} \frac{r_i}{w_i} \right)^{(1-\alpha)}$$

The terms on the left in these equilibrium conditions are the exogenous endowments of capital and labour of country  $i$ , while the terms on the right are the conditional demands of factors of the representative firm producing good  $x$ <sup>9</sup> ( $x_i$  is the national production of good  $x$ ).

Solving these conditions we obtain the equilibrium relative price of factors:

$$\left( \frac{w_i}{r_i} \right)^T = \left( \frac{\alpha}{1-\alpha} \right) \frac{K_i}{L_i} \quad (4.1)$$

This is the equilibrium relative price of factors if country  $i$ , in free trade, specializes completely in the production of  $x$ .

<sup>8</sup> Cf. the condition (1.19). The price of a good which is produced in country  $i$  at equilibrium cannot be lower than the average cost of production (because in such a case firms producing this good would make losses) and it cannot be higher than the average cost, because this would not be consistent with the condition of long-run equilibrium in a context of perfect competition (i.e. the null profit condition).

<sup>9</sup> Cf. equations (1.5) and (1.6).

In an analogous manner, we can determine the equilibrium relative price of factors if country  $i$  specializes in goods  $y$  or  $z$ : if country  $i$  specializes in good  $y$ , the equilibrium relative price of factor is identical to that in the equation (4.1), whereas if it specializes in good  $z$ , it is:

$$\left(\frac{w_i}{r_i}\right)^T = \left(\frac{\beta}{1-\beta}\right) \frac{K_i}{L_i} \quad (4.2)$$

If we compare equations (4.1) and (4.2) with equation (3.13), which determines the equilibrium relative price of factors in autarky, we will observe that the relative price of labour in terms of capital in free trade is lower than in autarky if country  $i$  specializes in the industry capital intensive (i.e. in good  $x$  or  $y$ ), whereas it is higher than in autarky if the country  $i$  specializes in the industry labour-intensive (i.e. in good  $z$ ).

### 1<sup>st</sup> case: complete specialization in good $x$

The complete specialization of country  $i$  in the production of good  $x$  in free trade, constitutes an equilibrium situation for country  $i$  if the exogenous variables satisfy the following conditions:

$$p_x^T = G\gamma_i r_i^{(1-\alpha)} w_i^\alpha \quad (4.3)$$

$$p_y^T < G\delta_i r_i^{(1-\alpha)} w_i^\alpha \quad (4.4)$$

$$p_z^T < F\tau_i r_i^{(1-\beta)} w_i^\beta \quad (4.5)$$

Condition (4.3) states that the price of good  $x$  in free trade<sup>10</sup> is equal to the average cost of production of  $x$  in country  $i$ , so that  $x$  can be produced in country  $i$  at the free trade equilibrium. Conditions (4.4) and (4.5) affirm that the prices of goods  $y$  and  $z$  in free trade are lower than the average cost of production of these two goods in country  $i$ , so that  $y$  and  $z$  can't be produced in country  $i$  at the free trade equilibrium.

Dividing by  $p_x^T$  the two terms of inequality (4.4) and substituting condition (4.3) in the term on the right, we obtain the following condition:

$$P_y^T < \frac{\delta_i}{\gamma_i} \quad (4.6)$$

where  $P_y^T$  is the relative price of  $y$  in terms of  $x$  in free trade, i.e:  $P_y^T = \frac{p_y^T}{p_x^T}$

If condition (4.6) is verified, i.e. if the free trade price of  $y$  in terms of  $x$  is lower than the autarky price of  $y$  in terms of  $x$  in country  $i$ <sup>11</sup>, country  $i$  doesn't produce good  $y$  and produces good  $x$  at equilibrium in free trade.

Dividing by  $p_x^T$  the two terms of inequality (4.5) and substituting condition (4.3) in the term on the right, we obtain the following condition:

<sup>10</sup> We have called  $p_x^T$  the price of  $x$  in free trade. Unlike for the autarky price  $p_{ix}^A$ , the country index  $i$  has been suppressed here, because in free trade, good  $x$  has an identical price in all countries ("One price" law).

<sup>11</sup> Cf. the equation (3.4):  $\frac{\delta_i}{\gamma_i}$  is the autarky price of equilibrium of  $y$  in terms of  $x$  in country  $i$ .

$$P_z^T < \frac{F\tau_i}{G\gamma_i} \left( \frac{w_i}{r_i} \right)^{(\beta-\alpha)} \quad (4.7)$$

where  $P_z^T$  is the relative price of  $z$  in terms of  $x$  in free trade. The term on the right in this inequality looks like the autarky price of  $z$  in terms of  $x$  [cf. equation (3.5)], but here the relative price of factors ( $w_i/r_i$ ) is smaller than in autarky because country  $i$  is completely specialized in good  $x$ . Substituting in condition (4.7) the equilibrium relative price of factors when country  $i$  is specialized in good  $x$  [cf. equation (4.1)], we obtain:

$$P_z^T < \frac{\tau_i}{\gamma_i} \left( \frac{\alpha}{\beta} \right)^\beta \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\beta)} \left( \frac{K_i}{L_i} \right)^{(\beta-\alpha)} \quad (4.8)$$

If condition (4.8) is verified, country  $i$  doesn't produce good  $z$  and produces good  $x$  at equilibrium in free trade. The less abundant in capital (and the more abundant in labour) is country  $i$ , the lower must be the free trade price of  $z$  in terms of  $x$  to force country  $i$  to give up completely the production of  $z$ . Note that the term on the right in this condition is smaller than the autarky price of  $z$  in terms of  $x$ <sup>12</sup>.

In conclusion, the complete specialization in good  $x$  constitutes an equilibrium solution for country  $i$  in free trade if the exogenous variables (i.e. the free trade equilibrium prices of goods, the technological parameters and the endowments of factors) verify conditions (4.6) and (4.8).

If country  $i$  specializes completely in the production of good  $x$ , the free trade equilibrium demands of goods  $x$ ,  $y$  and can be calculated as follows<sup>13</sup>:

$$x_i^{DT} = \lambda \left[ 1 - \varphi_i(P_y^T) \right] \left( \frac{w_i L_i + r_i K_i}{p_x^T} \right) \quad (4.9)$$

$$y_i^{DT} = \left[ \frac{\lambda \varphi_i(P_y^T)}{P_y^T} \right] \left( \frac{w_i L_i + r_i K_i}{p_x^T} \right) \quad (4.10)$$

$$z_i^{DT} = \left( \frac{1-\lambda}{P_z^T} \right) \left( \frac{w_i L_i + r_i K_i}{p_x^T} \right) \quad (4.11)$$

In these equations, we substitute the condition (4.3), which is verified if country  $i$  is completely specialized in the production of  $x$ , and we obtain:

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<sup>12</sup> Autarky price of  $z$  in terms of  $x$  appears in equation (3.14) as:  $P_{iz}^A = \frac{F\tau_i}{G\gamma_i} \left[ \frac{\alpha\lambda + \beta(1-\lambda)}{(1-\alpha)\lambda + (1-\beta)(1-\lambda)} \left( \frac{K_i}{L_i} \right) \right]^{(\beta-\alpha)}$ .

Condition (4.8) can be rewritten as:  $P_z^T < \frac{F\tau_i}{G\gamma_i} \left[ \frac{\alpha}{(1-\alpha)} \frac{K_i}{L_i} \right]^{(\beta-\alpha)}$ . Since we have assumed  $\alpha < \beta$ , the term on the right in this inequality is always smaller than the autarky price of  $z$  in terms of  $x$ .

<sup>13</sup> We use the general expressions of demand of goods, determined in section two [cf. equations (2.7), (2.8) and (2.9)] and we multiply and divide the equations (2.8) and (2.9) by  $p_x^T$ , obtaining the equations (4.10) and (4.11).

$$x_i^{DT} = \frac{\lambda [1 - \varphi_i(P_{iy}^T)]}{\gamma_i G} \left[ \left( \frac{w_i}{r_i} \right)^{(1-\alpha)} L_i + \left( \frac{r_i}{w_i} \right)^\alpha K_i \right] \quad (4.12)$$

$$y_i^{DT} = \frac{\lambda \varphi_i(P_{iy}^T)}{\gamma_i G P_{iy}^T} \left[ \left( \frac{w_i}{r_i} \right)^{(1-\alpha)} L_i + \left( \frac{r_i}{w_i} \right)^\alpha K_i \right] \quad (4.13)$$

$$z_i^{DT} = \left( \frac{1-\lambda}{P_z^T \gamma_i G} \right) \left[ \left( \frac{w_i}{r_i} \right)^{(1-\alpha)} L_i + \left( \frac{r_i}{w_i} \right)^\alpha K_i \right] \quad (4.14)$$

Finally, substituting in these equations the equilibrium relative price of factors when country  $i$  is specialized in good  $x$  [cf. equation (4.1)], and assuming that function  $\varphi_i(P_y)$  has the explicit form defined in expression (2.6), we determine the equilibrium demand of goods as a function of the exogenous variables. The free equilibrium demands of goods  $x$ ,  $y$  and  $z$  in country  $i$ , if country  $i$  is completely specialized in the production of good  $x$ , are:

$$x_i^{DT} = \lambda \left( 1 - \frac{1}{P_y^T} \right) \left( \frac{1}{\gamma_i} L_i^\alpha K_i^{1-\alpha} \right) \quad (4.15)$$

$$y_i^{DT} = \lambda \left( \frac{1}{P_y^T} \right)^2 \left( \frac{1}{\gamma_i} L_i^\alpha K_i^{1-\alpha} \right) \quad (4.16)$$

$$z_i^{DT} = (1-\lambda) \left( \frac{1}{P_z^T} \right) \left( \frac{1}{\gamma_i} L_i^\alpha K_i^{1-\alpha} \right) \quad (4.17)$$

## 2<sup>nd</sup> case: complete specialization in good $y$

The complete specialization of country  $i$  in the production of  $y$  constitutes an equilibrium situation for country  $i$  if the exogenous variables satisfy the following conditions:

$$p_x^T < G \gamma_i r_i^{(1-\alpha)} w_i^\alpha \quad (4.18)$$

$$p_y^T = G \delta_i r_i^{(1-\alpha)} w_i^\alpha \quad (4.19)$$

$$p_z^T < F \tau_i r_i^{(1-\beta)} w_i^\beta \quad (4.20)$$

Condition (4.19) states that the price of good  $y$  in free trade is equal to the average cost of production of  $y$  in country  $i$ , so that  $y$  can be produced in the country  $i$  at the free trade equilibrium. Conditions (4.18) and (4.20) affirm that the prices of goods  $x$  and  $z$  in free trade are lower than the average cost of production of these two goods in country  $i$ , so that  $x$  and  $z$  can't be produced in country  $i$  at the free trade equilibrium.

Dividing by  $p_y^T$  the two terms of inequality (4.18) and substituting condition (4.19) in the term on the right, we obtain the following condition:

$$P_y^T > \frac{\delta_i}{\gamma_i} \quad (4.21)$$

where  $P_y^T$  is the relative price of  $y$  in terms of  $x$  in free trade.

If condition (4.21) is verified, i.e. if the free trade price of  $y$  in terms of  $x$  is higher than the autarky price of  $y$  in terms of  $x$  in country  $i$ , country  $i$  doesn't produce good  $x$  and produces good  $y$  at the equilibrium in free trade.

Dividing by  $p_y^T$  the two terms of inequality (4.20) and substituting condition (4.19) in the term on the right, we obtain the following condition:

$$\frac{p_z^T}{p_y^T} < \frac{F\tau_i}{G\delta_i} \left( \frac{w_i}{r_i} \right)^{(\beta-\alpha)} \quad (4.22)$$

Substituting in condition (4.22) the equilibrium relative price of factors when country  $i$  is specialized in good  $y$  [cf. equation (4.1)], we obtain:

$$\frac{p_z^T}{p_y^T} < \frac{\tau_i}{\delta_i} \left( \frac{\alpha}{\beta} \right)^\beta \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\beta)} \left( \frac{K_i}{L_i} \right)^{(\beta-\alpha)} \quad (4.23)$$

If condition (4.23) is verified, country  $i$  doesn't produce good  $z$  and produces good  $y$  at the equilibrium in free trade. The less abundant in capital (and the more abundant in labour) is the country  $i$ , the lower must be the free trade price of  $z$  in terms of  $y$  to force country  $i$  to give up completely the production of  $z$ . Once again, the term on the right in this condition is smaller than the autarky price of  $z$  in terms of  $y$ <sup>14</sup>.

In conclusion, the complete specialization in good  $y$  constitutes an equilibrium solution for country  $i$  in free trade if the exogenous variables (i.e. the free trade equilibrium prices, the technological parameters and the endowments of factors) satisfy the conditions (4.21) and (4.23).

Proceeding as in the previous subsection, concerning a country specialized in good  $x$  [cf. (4.9) to (4.17)], we determine the equilibrium demand of goods as a function of the exogenous variables when country  $i$  is specialized in  $y$ . The free equilibrium demands of goods  $x$ ,  $y$  and  $z$  in country  $i$ , if country  $i$  is specialized in the production of good  $y$ , are:

$$x_i^{DT} = \lambda (P_y^T - 1) \left( \frac{1}{\delta_i} L_i^\alpha K_i^{1-\alpha} \right) \quad (4.24)$$

$$y_i^{DT} = \lambda \left( \frac{1}{P_y^T} \right) \left( \frac{1}{\delta_i} L_i^\alpha K_i^{1-\alpha} \right) \quad (4.25)$$

$$z_i^{DT} = (1-\lambda) \left( \frac{P_y^T}{P_z^T} \right) \left( \frac{1}{\delta_i} L_i^\alpha K_i^{1-\alpha} \right) \quad (4.26)$$

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<sup>14</sup> Autarky price of  $z$  in terms of  $y$  is:  $\frac{p_{iz}^A}{p_{iy}^A} = \frac{F\tau_i}{G\delta_i} \left[ \frac{\alpha\lambda + \beta(1-\lambda)}{(1-\alpha)\lambda + (1-\beta)(1-\lambda)} \left( \frac{K_i}{L_i} \right) \right]^{(\beta-\alpha)}$ . Condition (4.23) can be rewritten

as:  $\frac{p_z^T}{p_y^T} < \frac{F\tau_i}{G\delta_i} \left[ \frac{\alpha}{(1-\alpha)} \frac{K_i}{L_i} \right]^{(\beta-\alpha)}$ . Since we have assumed  $\alpha < \beta$ , the term on the right in this inequality is always smaller than the autarky price of  $z$  in terms of  $y$ .

### 3<sup>rd</sup> case: complete specialization in good $z$

The complete specialization of country  $i$  in the production of  $z$  constitutes an equilibrium situation for country  $i$  if the exogenous variables satisfy the following conditions:

$$p_x^T < G\gamma_i r_i^{(1-\alpha)} w_i^\alpha \quad (4.27)$$

$$p_y^T < G\delta_i r_i^{(1-\alpha)} w_i^\alpha \quad (4.28)$$

$$p_z^T = F\tau_i r_i^{(1-\beta)} w_i^\beta \quad (4.29)$$

Condition (4.29) states that the price of good  $z$  in free trade is equal to the average cost of production of  $z$  in country  $i$ , so that  $z$  can be produced in the country  $i$  at the free trade equilibrium. Conditions (4.27) and (4.28) affirm that the prices of goods  $x$  and  $y$  in free trade are lower than the average cost of production of these two goods in country  $i$ , so that  $x$  and  $y$  can't be produced in country  $i$  at the free trade equilibrium.

Dividing by  $p_z^T$  the two terms of inequality (4.27) and substituting condition (4.29) in the term on the right, we obtain the following condition:

$$P_z^T > \frac{F\tau_i}{G\gamma_i} \left( \frac{w_i}{r_i} \right)^{(\beta-\alpha)} \quad (4.30)$$

where  $P_z^T$  is the relative price of  $z$  in terms of  $x$  in free trade, i.e:  $P_z^T = \frac{p_z^T}{p_x^T}$

Symmetrically, dividing by  $p_z^T$  the two terms of the inequality (4.28) and substituting condition (4.29) in the term on the right, we obtain:

$$\frac{p_z^T}{p_y^T} > \frac{F\tau_i}{G\delta_i} \left( \frac{w_i}{r_i} \right)^{(\beta-\alpha)} \quad (4.31)$$

Substituting in conditions (4.30) and (4.31) the equilibrium relative price of factors when country  $i$  is specialized in good  $z$  [cf. equation (4.2)], we obtain, respectively:

$$P_z^T > \frac{\tau_i}{\gamma_i} \left( \frac{\alpha}{\beta} \right)^\alpha \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\alpha)} \left( \frac{K_i}{L_i} \right)^{(\beta-\alpha)} \quad (4.32)$$

$$\frac{p_z^T}{p_y^T} > \frac{\tau_i}{\delta_i} \left( \frac{\alpha}{\beta} \right)^\alpha \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\alpha)} \left( \frac{K_i}{L_i} \right)^{(\beta-\alpha)} \quad (4.33)$$

The complete specialization in the production of good  $z$  constitutes an equilibrium solution for country  $i$  in free trade if the exogenous variables verify these last two conditions.

Proceeding as in the case of the country specialized in good  $x$  [cf. (4.9) to (4.17)], we determine the equilibrium demand of goods as a function of the exogenous variables. The free equilibrium demands of goods  $x$ ,  $y$  and  $z$  in country  $i$ , if country  $i$  is completely specialized in the production of good  $z$ , are:

$$x_i^{DT} = \lambda P_z^T \left( 1 - \frac{1}{P_y^T} \right) \left( \frac{1}{\tau_i} L_i^\beta K_i^{1-\beta} \right) \quad (4.34)$$

$$y_i^{DT} = \lambda P_z^T \left( \frac{1}{P_y^T} \right)^2 \left( \frac{1}{\tau_i} L_i^\beta K_i^{1-\beta} \right) \quad (4.35)$$

$$z_i^{DT} = (1-\lambda) \left( \frac{1}{\tau_i} L_i^\beta K_i^{1-\beta} \right) \quad (4.36)$$

## 5. General equilibrium in free trade: three countries completely specialized

### Conditions for the complete specialization of the three countries in free trade

We develop now a general equilibrium model, in which three countries, called 1, 2, and 3, are opened to trade and specialized respectively in the production of goods  $x$ ,  $y$  and  $z$ .

According to the results of the previous section, the complete specialization of the three countries constitutes an equilibrium situation if the free trade equilibrium prices respect the following conditions:

$$\frac{\delta_2}{\gamma_2} < P_y^T < \frac{\delta_1}{\gamma_1} \quad (5.1)$$

$$\frac{\tau_3}{\gamma_3} \left( \frac{\alpha}{\beta} \right)^\alpha \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\alpha)} \left( \frac{K_3}{L_3} \right)^{(\beta-\alpha)} < P_z^T < \frac{\tau_1}{\gamma_1} \left( \frac{\alpha}{\beta} \right)^\beta \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\beta)} \left( \frac{K_1}{L_1} \right)^{(\beta-\alpha)} \quad (5.2)$$

$$\frac{\tau_3}{\delta_3} \left( \frac{\alpha}{\beta} \right)^\alpha \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\alpha)} \left( \frac{K_3}{L_3} \right)^{(\beta-\alpha)} < \frac{p_z^T}{p_y^T} < \frac{\tau_2}{\delta_2} \left( \frac{\alpha}{\beta} \right)^\beta \left( \frac{1-\alpha}{1-\beta} \right)^{(1-\beta)} \left( \frac{K_2}{L_2} \right)^{(\beta-\alpha)} \quad (5.3)$$

These conditions correspond to those we have defined in section 4. Condition (5.1) assembles inequalities (4.6) and (4.21): if it is verified, at the free trade equilibrium country 1 can produce good  $x$  but it can't produce good  $y$ , whereas country 2 can produce  $y$  but can't produce  $x$ . The condition (5.2) assembles inequalities (4.8) and (4.32): if it is verified, country 1 can produce  $x$  but it can't produce  $z$ , while country 3 can produce  $z$  but it can't produce  $x$ . Finally, condition (5.3) assembles inequalities (4.23) and (4.33): if it is verified, country 2 can produce  $y$  but it can't produce  $z$ , whereas country 3 can produce  $z$  but it can't produce  $y$ . In conclusion if the relative prices of goods at the free trade equilibrium are consistent with the three conditions stated above, then the complete specialization of countries 1, 2 and 3 in the production of  $x$ ,  $y$  and  $z$  respectively, constitutes an equilibrium situation for the three countries at the same time.

The complete specialization of country 1 and country 2 in the production of, respectively,  $x$  and  $y$  constitutes an equilibrium in free trade only if condition (5.1) is verified. This condition states that the free trade equilibrium price of  $y$  in terms of  $x$  must be strictly included between the autarky equilibrium price in country 2 (which then specializes in good  $y$ ) and the autarky equilibrium price in country 1 (which specializes in good  $x$ ). The autarky equilibrium price of  $y$  in terms of  $x$  in country  $i$  has been determined in section 3, equation (3.4), as the ratio of the technological parameters  $\delta_i$  and  $\gamma_i$ . Therefore, a necessary condition for the complete specialization of country 1

and 2 (respectively in goods  $x$  and  $y$ ) is that country 1 has a *relative*<sup>15</sup> technological advantage in the production of  $x$  and country 2 has a *relative* technological advantage in the production of good  $y$ , i.e:

$$\frac{\delta_2}{\gamma_2} < \frac{\delta_1}{\gamma_1} \quad (5.4)$$

The complete specialization of country 1 and country 3 in the production of, respectively,  $x$  and  $z$  constitutes an equilibrium in free trade only if condition (5.2) is verified. This is possible only if:

$$\left(\frac{\tau_3}{\gamma_3}\right)\left(\frac{K_3}{L_3}\right)^{(\beta-\alpha)} < \left[\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right]^{(\beta-\alpha)} \left(\frac{\tau_1}{\gamma_1}\right)\left(\frac{K_1}{L_1}\right)^{(\beta-\alpha)} \quad (5.5)$$

We define  $\left[\frac{\alpha(1-\beta)}{\beta(1-\alpha)}\right] = H$ . Since we have assumed that  $\beta$  is greater than  $\alpha$ ,  $H$  is always smaller than one, and condition (5.5) can be restated as follows:

$$\left(\frac{\tau_3}{\gamma_3}\right)^{\left(\frac{1}{\beta-\alpha}\right)}\left(\frac{K_3}{L_3}\right) < H \left(\frac{\tau_1}{\gamma_1}\right)^{\left(\frac{1}{\beta-\alpha}\right)}\left(\frac{K_1}{L_1}\right) \quad (5.5)$$

Condition (5.5) concerns both technology parameters and factor endowments of countries 1 and 3.

If we assume for convenience that the technological parameters are identical in countries 1 and 3 (i.e.  $\gamma_1 = \gamma_3$  and  $\tau_1 = \tau_3$ ), condition (5.5) is verified only if country 1 has a capital to labour ratio substantially bigger than country 3. Since  $H$  decreases as the difference between  $\beta$  and  $\alpha$  increases, the more important is this difference, the bigger must be the difference between the factor endowments ratio in the two countries to permit their complete specialization.

We release now the assumption of identical technology and we come back to the general case allowing for Hicks-neutral differences in the production functions across countries; we can conclude that technological differences contribute to the consistency of the assumption of complete specialization if  $\left(\frac{\tau_3}{\gamma_3}\right) < \left(\frac{\tau_1}{\gamma_1}\right)$ , i.e. if country 1 has a relative technological advantage in the production of  $x$  and symmetrically country 3 has a relative technological advantage in the production of  $z$ .

Finally, condition (5.3), which has to be verified to allow the complete specialization of countries 2 and 3 respectively in  $y$  and  $z$ , can be rewritten as follows:

$$\left(\frac{\tau_3}{\delta_3}\right)^{\left(\frac{1}{\beta-\alpha}\right)}\left(\frac{K_3}{L_3}\right) < H \left(\frac{\tau_2}{\delta_2}\right)^{\left(\frac{1}{\beta-\alpha}\right)}\left(\frac{K_2}{L_2}\right) \quad (5.6)$$

As in the previous case, this condition is verified either if the relative endowment of capital of country 2 is substantially bigger than the relative endowment of capital of country 3, or if country 2 has a strong technological (relative) advantage in the production of good  $y$ .

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<sup>15</sup> A relative (and not absolute) technological advantage determines the pattern of the specialization in free trade. Assume for example that technological parameters in the two countries verifies the following conditions:  $\delta_2 < \delta_1$ ;  $\gamma_2 < \gamma_1$ ;  $\frac{\delta_2}{\gamma_2} < \frac{\delta_1}{\gamma_1}$ . In this case, even if country 2 has an absolute technological advantage in the production of both  $y$  and  $z$ ,

complete specialization can occur, because country 1 has a relative advantage in the production of  $x$  and country 2 has a relative advantage in the production of  $y$ .

In conclusion, the complete specialization of countries 1, 2 and 3, in the production of, respectively,  $x$ ,  $y$  and  $z$  constitutes an equilibrium situation in free trade, if:

- countries 1 and 2 have similar ratios of factor endowments and some Hicks-neutral difference in the production functions of  $x$  and  $y$ ;
- country 3 has a capital to labour ratio substantially smaller than countries 1 and 2, or a strong technological disadvantage in the production of  $x$  and  $y$ .

### Determination of the free trade equilibrium prices of goods

If we assume that countries 1, 2 and 3 are respectively specialized in the production of goods  $x$ ,  $y$  and  $z$ , the equilibrium conditions of the goods markets in free trade are:

$$x_1^D + x_2^D + x_3^D = x_1^S \quad (5.7)$$

$$y_1^D + y_2^D + y_3^D = y_2^S \quad (5.8)$$

$$z_1^D + z_2^D + z_3^D = z_3^S \quad (5.9)$$

where  $x_i^D$  is the demand of  $x$  in country  $i$  and  $x_i^S$  is the production of  $x$  of country  $i$ .

The demand functions of goods  $x$ ,  $y$  and  $z$  have been determined in section two<sup>16</sup>. If we assume that the function  $\varphi_i(P_y)$  is identical in the three countries, equations (5.7), (5.8) and (5.9) can be rewritten as:

$$\frac{\lambda[1 - \varphi(P_y^T)]}{p_x^T} (w_1 L_1 + r_1 K_1 + w_2 L_2 + r_2 K_2 + w_3 L_3 + r_3 K_3) = \frac{1}{\gamma_1} L_1^\alpha K_1^{(1-\alpha)} \quad (5.7)$$

$$\frac{\lambda \varphi(P_y^T)}{p_y^T} (w_1 L_1 + r_1 K_1 + w_2 L_2 + r_2 K_2 + w_3 L_3 + r_3 K_3) = \frac{1}{\delta_2} L_2^\alpha K_2^{(1-\alpha)} \quad (5.8)$$

$$\frac{(1-\lambda)}{p_z^{LE}} (w_1 L_1 + r_1 K_1 + w_2 L_2 + r_2 K_2 + w_3 L_3 + r_3 K_3) = \frac{1}{\tau_3} L_3^\beta K_3^{(1-\beta)} \quad (5.9)$$

where in the terms on the right we have substituted, respectively, the production function of  $x$  in country 1, the production function of  $y$  in country 2 and the production function of  $z$  in country 3.

From equations (5.7) and (5.8), we obtain:

$$\frac{1 - \varphi(P_y^{LE})}{\varphi(P_y^{LE})} P_y^{LE} = \frac{\delta_2}{\gamma_1} \left( \frac{L_1}{L_2} \right)^\alpha \left( \frac{K_1}{K_2} \right)^{(1-\alpha)} \quad (5.10)$$

In order to determine the free trade equilibrium price of  $y$  in terms of  $x$ , we must give an explicit form to the function  $\varphi(P_y)$ . Therefore, we assume that the function  $\varphi(P_y)$  has the following form, which has been introduced in section two:

$$\varphi(P_y) = \frac{1}{P_y} \quad \text{if } P_y \geq 1$$

$$\varphi(P_y) = 1 \quad \text{if } P_y < 1$$

Using this form of the function  $\varphi(P_y)$ , we obtain from equation (5.10) the free trade equilibrium price of  $y$  in terms of  $x$ :

<sup>16</sup> Cf. equations (2.7), (2.8) and (2.9).

$$P_y^{LE} = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\delta_2 L_1^\alpha K_1^{(1-\alpha)}}{\gamma_1 L_2^\alpha K_2^{(1-\alpha)}}} \quad (5.11)$$

Using equations (5.7) and (5.9) we obtain the free trade equilibrium price of  $z$  in terms of  $x$ :

$$P_z^T = \left( \frac{P_y^T}{P_y^T - 1} \right) \left( \frac{1-\lambda}{\lambda} \right) \left[ \frac{\tau_3 L_1^\alpha K_1^{(1-\alpha)}}{\gamma_1 L_3^\beta K_3^{(1-\beta)}} \right] \quad (5.12)$$

which can be determined also from equations (5.8) and (5.9) as :

$$P_z^T = (P_y^T)^2 \left( \frac{1-\lambda}{\lambda} \right) \left[ \frac{\tau_3 L_2^\alpha K_2^{(1-\alpha)}}{\delta_2 L_3^\beta K_3^{(1-\beta)}} \right] \quad (5.12)$$

These are two different and equivalent expressions for the equilibrium price of  $z$  in terms of  $y$ .

The relative prices (5.11) and (5.12) have been determined by assuming that the three countries are completely specialized. Therefore, they are to be considered as the free trade equilibrium prices only if they satisfy the conditions (5.1), (5.2) and (5.3), because otherwise the complete specialization doesn't constitute an equilibrium situation for the three countries.

### A numerical example of free trade equilibrium of complete specialization

We develop now a numerical example in which the complete specialization of countries 1, 2 and 3 in the production respectively of  $x$ ,  $y$  and  $z$  constitutes an equilibrium situation for the three countries in free trade (i.e. the conditions (5.1), (5.2) and (5.3) are verified at the free trade equilibrium). Then we demonstrate that the free trade equilibrium is revealed preferred to autarky equilibrium for the three countries.

In this numerical example, we assume:  $\alpha = 0.3$ ;  $\beta = 0.7$ ;  $\lambda = 0.5$ . These parameters have the same values in the three countries, consistently with the assumptions made in sections one and two. Industry  $x$ - $y$  is capital-intensive, while industry  $z$  is labour-intensive. The following table defines the values of country-specific technological parameters.

	<i>Country 1</i>	<i>Country 2</i>	<i>Country 3</i>
$\gamma_i$	2	2	2
$\delta_i$	4	3	4
$\tau_i$	1	1	1

In order to limit the influence of technological determinants of trade at a minimum necessary, we introduce one only difference in the production functions of the three countries: parameter  $\delta_2$  is assumed to be smaller than  $\delta_1$  and  $\delta_3$ , meaning that country 2 has an absolute technological advantage in the production of the good  $y$ , with respect to countries 1 and 3.

Factor endowments in the three countries are:

	<i>Country 1</i>	<i>Country 2</i>	<i>Country 3</i>
$K_i$	150	200	50
$L_i$	10	10	200

In countries 1 and 2, the ratio of the endowments is not identical but quite similar, characterizing these countries as capital abundant relative to country 3.

The equilibrium relative prices of  $y$  and  $z$  in terms of  $x$  and the relative price of factors in autarky and in free trade are shown in the following tables:

<i>Autarky equilibrium prices</i>			
	<i>Country 1</i>	<i>Country 2</i>	<i>Country 3</i>
$P_y^A$	2	1.5	2
$P_z^A$	1.5749	1.7670	0.3062
$(w_i/k_i)^A$	17.6087	23.4780	0.2935

<i>Free trade equilibrium prices</i>			
	<i>Country 1</i>	<i>Country 2</i>	<i>Country 3</i>
$P_y^T$	1.7151	1.7151	1.7151
$P_z^T$	0.9075	0.9075	0.9075
$(w_i/k_i)^T$	6.4286	8.5714	0.5833

In autarky (cf. section three), good  $y$  is cheaper (in terms of  $x$ ) in country 2 than in the two other countries, whereas good  $z$  is cheaper in country 3 than in countries 1 and 2. The price of labour relative to capital is substantially higher in countries 2 and 1 than in country 3.

In free trade, with complete specialization of the three countries (cf. sections four and five), the relative prices of  $y$  and  $z$  in terms of  $x$  are identical in the three countries (for the “One price” law). The equilibrium prices determined with these values of the exogenous variables respect the conditions (5.1), (5.2) and (5.3). Therefore, in this numerical example, the complete specialization of countries 1, 2 and 3, respectively in the production of  $x$ ,  $y$  and  $z$ , constitutes an equilibrium for the three countries, in free trade. At equilibrium, international trade is partly inter-industry and partly intra-industry in vertically differentiated goods (VIIT), since the bilateral trade between country 3 on the one hand and countries 1 and 2 on the other hand is totally inter-industry, whereas the bilateral trade between country 1 and country 2 is totally VIIT.

We can observe that, with complete specialization, the relative prices of factors in the three countries are not identical at the free trade equilibrium, so that there is no factor price equalization. Nevertheless, they converge with respect to the autarky equilibrium: in free trade, the price of labour relative to capital is lower than in autarky in the capital-abundant countries, while it is higher than in autarky in the labour-abundant country.

Using the Lerner-Pearce diagram, we represent now the autarky and the free trade equilibrium situations for the three countries. The essential feature of the Lerner-Pearce diagram is that it uses unit-value isoquants. In equilibrium, the unit-value isoquants of the goods which are produced in country  $i$  are tangent to the unit isocost line in country  $i$ , whereas the unit-value isoquants of the goods that are not produced in country  $i$  are located above the unit isocost line, meaning that the these goods *cannot* be produced (i.e. the production of these goods in country  $i$  would generate losses).

We takes good  $x$  as the numeraire and we define the equations of the unit-value<sup>17</sup> isoquants for the country  $i$ , and the equation of the unit isocost line.

The equation of the unit-value isoquant for good  $x$  is:

$$1 = \frac{1}{\gamma_i} L_i^\alpha K_i^{1-\alpha} \quad (\text{i})$$

The equation of the unit-value isoquant for good  $y$  is:

$$1 = \frac{P_y}{\delta_i} L_i^\alpha K_i^{1-\alpha} \quad (\text{ii})$$

<sup>17</sup> Since we have assumed that the good  $x$  is the numeraire, the isoquants are unit-value in terms of  $x$ .

Equation (ii) represents all the bundles of factors that produce a quantity of  $y$  which is worth one unit of  $x$ . Note that in autarky, the unit-value isoquants of  $x$  and  $y$  coincide, because :  $P_y^A = \frac{\delta_i}{\gamma_i}$ .

Finally, the equation of the unit-value isoquant for good  $z$  is: 
$$1 = \frac{P_z}{\tau_i} L_i^\beta K_i^{1-\beta} \quad (\text{iii})$$

Equation (iii) represents all the bundles of factors that produce a quantity of  $z$  which is worth one unit of  $x$ .

The equation of the unit isocost line is: 
$$1 = \left( \frac{w_i}{p_{ix}} \right) L_i + \left( \frac{r_i}{p_{ix}} \right) K_i \quad (\text{iv})$$

To represent graphically the isocost line, we need to substitute the equilibrium value of  $p_{ix}$  in equation (iv). If good  $x$  is produced in country  $i$ ,  $p_{ix}$  is equal to the average cost of production of  $x$  in country  $i$ , i.e:  $p_{ix} = G\gamma_i r_i^{(1-\alpha)} w_i^\alpha$  [cf. equation (1.19)]. Substituting this expression in equation (iv), we obtain:

$$1 = \frac{1}{G\gamma_i} \left[ \left( \frac{w_i}{r_i} \right)^{(1-\alpha)} L_i + \left( \frac{r_i}{w_i} \right)^\alpha K_i \right]$$

which is the equation of the unit isocost line in country  $i$ , if good  $x$  is produced in country  $i$ <sup>18</sup>.

If good  $x$  is not produced at the equilibrium in country  $i$ , equation (iv) has to be restated, either as:

$$1 = \frac{P_y}{G\delta_i} \left[ \left( \frac{w_i}{r_i} \right)^{(1-\alpha)} L_i + \left( \frac{r_i}{w_i} \right)^\alpha K_i \right]$$
, if country  $i$  produces only good  $y$  at equilibrium<sup>19</sup> (that is the

case of country 2 in free trade), or as: 
$$1 = \frac{P_z}{F\tau_i} \left[ \left( \frac{w_i}{r_i} \right)^{(1-\beta)} L_i + \left( \frac{r_i}{w_i} \right)^\beta K_i \right]$$
, if country  $i$  produces only

the good  $z$  at equilibrium<sup>20</sup> (that is the case of country 3 in free trade).

Applying these equations to our numerical example, we obtain the following representations for the autarky and the free trade equilibrium in countries 1, 2 and 3. The unit isocost line is coloured in black, while the unit-value isoquants are coloured respectively in blue (good  $x$ ), in brown (good  $y$ ) and in red (good  $z$ ).

<sup>18</sup> Good  $x$  is always produced in country  $i$  in autarky, whereas in free trade it is produced in country  $i$  only if country  $i$  specializes in  $x$ . Thus, equation (iv') represents the unit isocost line in countries 1, 2 and 3 in autarky and the unit isocost line in country 1 in free trade.

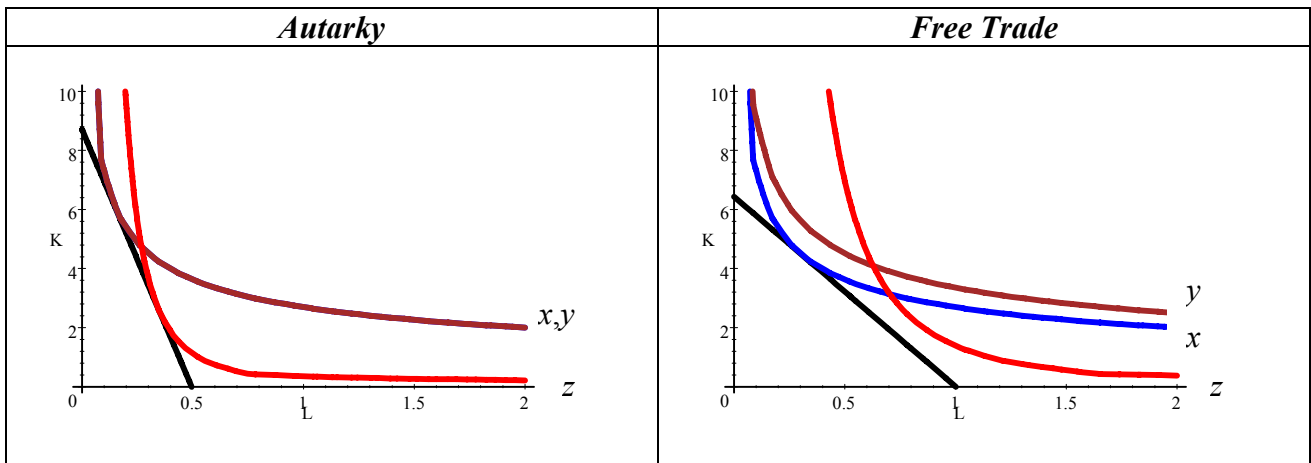
<sup>19</sup> To determine this expression of the unit isocost line, we have multiplied and divided by  $p_{iy}$  the term on the right of the equation (iv), obtaining: 
$$1 = P_y \left[ \left( \frac{w_i}{p_{iy}} \right) L_i + \left( \frac{r_i}{p_{iy}} \right) K_i \right]$$
, where  $P_y = \frac{p_{iy}}{p_{ix}}$ . Then we have substituted the expression

$p_{iy} = G\delta_i r_i^{(1-\alpha)} w_i^\alpha$ , meaning that the price of  $y$  is equal to its average cost in country  $i$ , which is true in this case because the country  $i$  is assumed to produce good  $y$ .

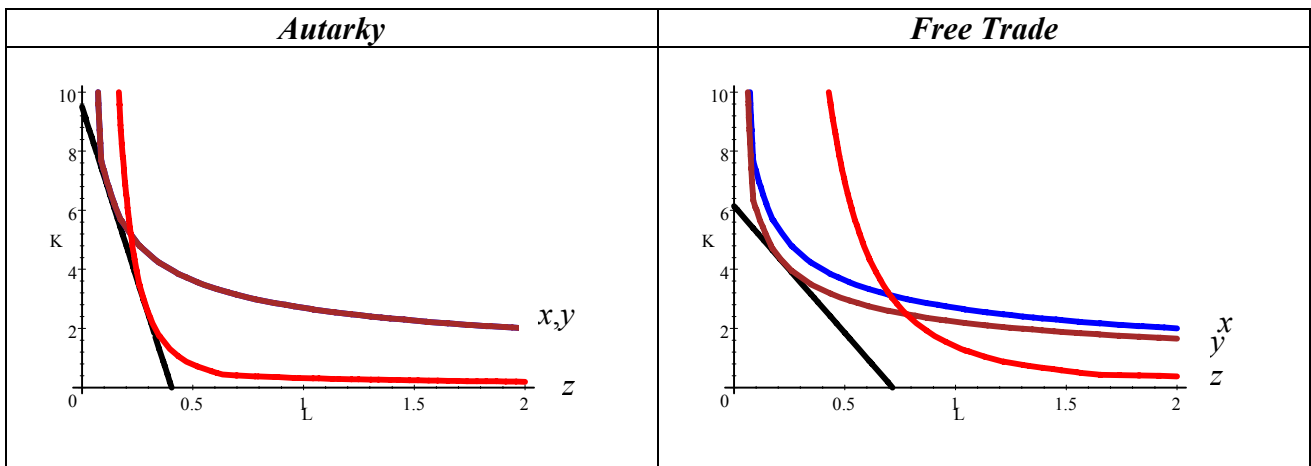
<sup>20</sup> To determine this expression of the unit isocost line, we have multiplied and divided by  $p_{iz}$  the term on the right of the equation (iv), obtaining: 
$$1 = P_z \left[ \left( \frac{w_i}{p_{iz}} \right) L_i + \left( \frac{r_i}{p_{iz}} \right) K_i \right]$$
, where  $P_z = \frac{p_{iz}}{p_{ix}}$ . Then we have substituted the expression

$p_{iz} = F\tau_i r_i^{(1-\beta)} w_i^\beta$ , meaning that the price of  $z$  is equal to its average cost in country  $i$ , which is true in this case because the country  $i$  is assumed to produce good  $z$ .

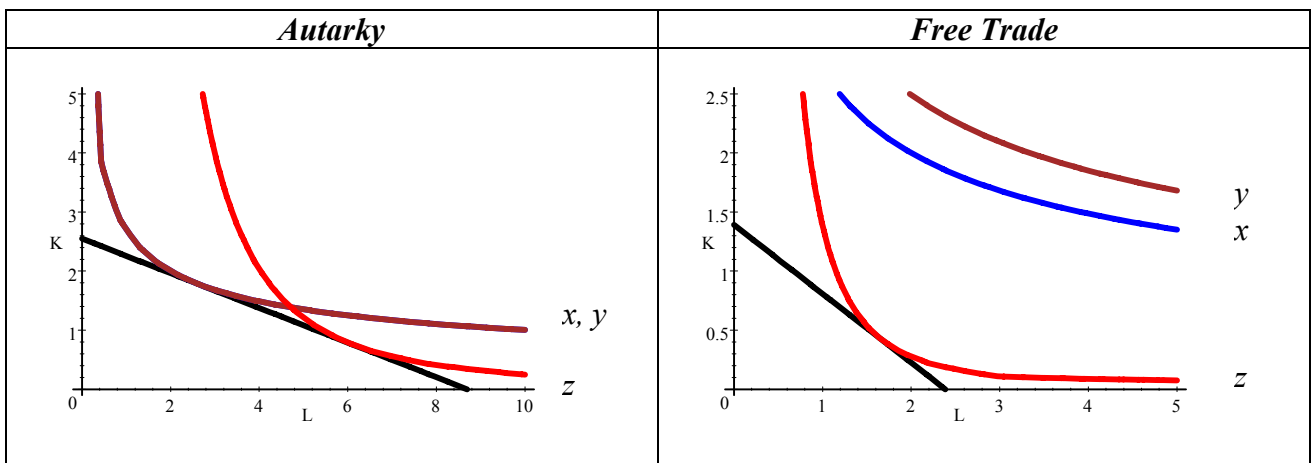
**Equilibrium in Country 1:**



**Equilibrium in Country 2:**



**Equilibrium in Country 3:**



We observe in these figures that in the autarky equilibrium the three goods are produced in each country: in fact, in autarky, the three unit-value isoquants are tangent to the unit isocost line in the three countries<sup>21</sup>. In free trade, complete specialization constitutes an equilibrium for the three

<sup>21</sup> The blue isoquant (which refers to the good  $x$ ) doesn't appear in these figures because it coincides with the brown one (which refers to the good  $y$ ).

countries, since only one unit-value isoquant is tangent to the isocost line: the blue one for country 1 (which specializes in the production of  $x$ ), the brown one for country 2 (which specializes in  $y$ ) and the red one for country 3 (which specializes in  $z$ ).

It remains to be seen if the free trade equilibrium (with complete specialization) constitutes a better situation than the autarky equilibrium for the three countries. To answer this question, we use revealed preferences: the free trade equilibrium is preferred to the autarky equilibrium if the consumption in free trade, evaluated at autarky prices, is greater than the consumption in autarky.

Using good  $x$  as the numeraire, we define  $C_i^A$  and  $C_i^T$ , respectively, as total consumption in autarky and in free trade, evaluated at the autarky equilibrium relative prices:

$$C_i^A = x_i^A + P_{iy}^A y_i^A + P_{iz}^A z_i^A$$

$$C_i^T = x_i^T + P_y^A y_i^T + P_z^A z_i^T$$

In the following tables, we have calculated, with the particular values of parameters and factor endowments of this numerical example, consumption of goods  $x$ ,  $y$ , and  $z$  in the three countries, in autarky and in free trade<sup>22</sup>, and total consumption in terms of the numeraire. We observe from these data that, in the three countries, total consumption in free trade, evaluated at autarky prices, is greater than total consumption in autarky. Thus, we can conclude that free trade equilibrium (with complete specialization) is revealed preferred to autarky equilibrium for the three countries.

<i>Autarky</i>	<i>Country 1</i>	<i>Country 2</i>	<i>Country 3</i>
$x_i^A$	7.4872	6.1050	8.5240
$y_i^A$	3.7436	8.1400	4.2620
$z_i^A$	14.2620	15.5476	83.5152
$C_i^A$	<b>37.4361</b>	<b>45.7876</b>	<b>42.6201</b>

<i>Free trade</i>	<i>Country 1</i>	<i>Country 2</i>	<i>Country 3</i>
$x_i^T$	5.5509	7.7627	19.9703
$y_i^T$	4.5261	6.3296	16.2836
$z_i^T$	22.0060	30.7744	79.1705
$C_i^T$	<b>49.2610</b>	<b>71.6354</b>	<b>76.7793</b>

## Conclusion

We have demonstrated that the complete specialization of country  $i$  in the production of goods  $x$ ,  $y$  (which are capital intensive and belong to the same industry) or  $z$  (which is labour intensive) constitutes a free trade equilibrium, provided that technological parameters and factor endowments of country  $i$  satisfy the conditions stated in section four.

<sup>22</sup> We use equations (3.6), (3.7) and (3.8), in which we substitute the autarky equilibrium relative price of factors, to calculate the autarky equilibrium consumption of the three goods. As regards the free trade equilibrium consumption of goods, they have been determined in section four, according to the specialization of the country in the production of goods  $x$ ,  $y$  or  $z$ . Hence, to calculate the free trade equilibrium consumption in country 1 (specialized in good  $x$ ), we use equations (4.15), (4.16) and (4.17), whereas for country 2 (specialized in  $y$ ) we use equations (4.24), (4.25) and (4.26).

In a three countries world, the complete specialization of countries 1, 2 and 3 in the production of goods  $x$ ,  $y$  and  $z$  respectively, is an equilibrium situation for the three countries, provided that:

- country 1 and country 2 have similar relative endowments of factors;
- countries 1 and 2 are highly capital-abundant with regard to country 3;
- there are some (small) Hicks-neutral differences between the production functions of  $x$  or  $y$  in countries 1 and 2, which generate a relative technological advantage of country 1 in the production of  $x$  (and symmetrically a relative advantage of country 2 in the production of  $y$ ).

If these conditions are satisfied, the complete specialization of the three countries constitutes an equilibrium situation, in which trade between countries 1 and 2 is totally intra-industry, whereas trade between country 3 on the one hand and countries 1 and 2 on the other hand is totally inter-industry.

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