Vertical integration and trade with R&D and differentiation *

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Abstract

Vertical integration and outsourcing are compared in an international framework where firms distribute their production process over two countries separated by transport costs. In a first framework two international monopolies are compared: a vertically integrated one and a decentralised one. Both undertake process R&D with and without spillover. In a second framework a vertically integrated international duopoly selling differentiated goods is compared to one associated to international outsourcing. The incentives of firms to adopt the vertically integrated structure vis à vis the decentralised one are assessed in both frameworks.

JEL Classification: F12, L13, O31, R40.

Keywords: Vertical Integration Trade R&D Differentiation

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1 Introduction

Vertical integration is one among the objects of strategic decisions of firms whenever perfect competition is assumed away. Only in this case the choice between a vertically integrated production process or a decentralised arrangement, whereby intermediate goods are outsourced, becomes irrelevant. Buying an input from a perfectly competitive supplier by paying a price equal to the social opportunity cost is equivalent to internally transferring an input at its marginal cost. This irrelevance theorem vanishes as we abandon perfect competition (Perry, 1989; Williamson, 1971) or when we introduce international trade by considering markets segmented by transport costs. Moreover, as we move away from perfect competition product differentiation and process R&D are common features of the decisional set of each firm and they are often associated with a particular choice of the vertical organisation of the firm (Rossini, 2003; Rossini and Lambertini, 2003).

The issue of vertical integration and trade has been recently analysed by McLaren (1999, 2000), Grossman and Helpman (2002) and Markusen (2002). Vertical integration and differentiation has been dealt with in Lambertini and Rossini (2003), Pepall and Norman (2001), while that of process R&D within a vertical framework can be found in contributions of Banerjee and Lin (2001), Brocas (2003), and Rossini and Lambertini (2003) among others. A related question that can be traced in these contributions is that of the coexistence of vertically integrated and disintegrated firms in the same industry and its effects on welfare. Some contributions establish a sort of impossibility result for disintegrated and integrated firms to operate in the same industry (Grossman and Helpman, 2002) while others emphasize the circumstances in which the coexistence of the two arrangements is an equilibrium (Jansen, 2003; Lambertini and Rossini, 2003).

Opening of trade has a relevant influence on the vertical organisation of firms. As a matter of fact the increase in outsourcing that has accompanied the present process of trade liberalisation seems to point to a pro-decentralisation opening of trade. However, the apparent geographical segmentation of production of intermediate goods takes place within two distinct arrangements: one in which a multinational firm (MNF) spreads its production process crossborder and another in which firms decentralize (while often downsizing) the production of intermediate inputs to foreign producers on which they have no control since they entertain with them only market relationships. These market and/or internal arrangements that take place cross-
border occur in industries where differentiation of final products is quite common and/or firms venture in some kind of R&D. Empirical evidence on R&D and vertical integration is scarce and not recent; it can be found in Teece (1976) and in Armour and Teece (1980).

In an open trade scenario we cast our analysis of vertical integration where process R&D may give rise to a two ways, or a one way spillover along the vertical chain when the Downstream (D) firm outsources to an Upstream (U) producer the manufacture of an intermediate good. Surprisingly we associate the positive vertical spillover to the disintegrated structure rather than to the integrated one since we assume that independent firms have better incentives to exploit each other’s process R&D rather than a single MNF. The result will be to lessen the vertical Spenglerian externality (Spengler, 1951) that occurs in any imperfect market relationship between firms along the vertical chain. A negative externality arises each time the D firm increases its price, since this has a negative effect on the profits of the U firm. This externality - also called double marginalisation - is totally internalised, and, therefore, canceled, when firms integrate vertically and transfer the intermediate product internally at its marginal cost. The empirical evidence on that is somehow controversial (Slade, 1998,a; Slade, 1998,b) and does not provide a clearcut support for most of theoretical results. Here we wish to contrast this negative externality with the R&D spillover along the vertical chain when there is decentralisation taking place crossborder in the presence of international transport costs.

Beyond that, there are other reasons why many firms, rather than staying national, they undertake crossborder vertical integration (the entire chain of the productive process occurs within a unique firm whose branches are not located in only one country). For instance, they may wish to avoid antitrust discipline that is often against (domestic) vertical integration or they may simply aim at decreasing some costs along the vertical chain of production. In some cases process R&D may be cheaper abroad in external firms due to lower costs of skilled labour, as it is the case in China and India. However, in these countries the firm internal organisation is simpler if the firm is locally owned instead of being the vertical section of a MNF, making for outsourcing rather than crossborder vertical integration. Last but not least, in some countries there exist provisions for imported final goods requiring them to have a certain percentage of value represented by domestically produced input. Such an issue is dealt with in the fourth section of the paper within an international duopoly with vertically integrated and disintegrated firms having all their factories at home. If these firms wish
to export they have to buy the intermediate good to produce their exports from the foreign rival. Then each firm produces an intermediate good that is partly internally transferred and partly sold to the foreign rival. This scenario is contrasted with a disintegrated one with the same trade policy.

The common question we shall raise in all these arrangements is about the private and public incentives for firms either to stay integrated or to outsource. Some surprising results call for a revision of the assessment of the usual externality and explain the wave towards outsourcing on an international basis.

In next section we deal with the monopoly case with R&D and compare the performances of the two vertical arrangements in a crossborder framework. Subsequently we extend this framework to make room for vertical R&D spillovers. Finally, we shall go through the duopoly case with product differentiation. Conclusions are in the last section.

2 The monopoly case with R&D

2.1 A crossborder vertically disintegrated monopoly

We first consider an industry where production requires two stages, an upstream (U) stage where an intermediate product is manufactured and a downstream (D) stage where a final homogeneous good is assembled using the intermediate good as an input. In this subsection we concentrate on a vertically disintegrated market arrangement whereby two monopolists, based in two distinct countries, produce respectively in the U stage and in the D stage. In this framework we introduce international trade, since we assume that two countries are involved: Home (H) and Foreign (F). The final good is produced only in H, yet sold in H and exported in F, while bearing a transport cost of the traditional iceberg type (Samuelson, 1954; Lambertini and Rossini, 2001). The intermediate good is produced in trade of both the final and the intermediate good face transport costs, whereby only a fraction $t \in [0,1]$ of the produced item reaches the final buyer either in country F or H.

Linear demand functions for the final good in the two countries are:

$$p_H = a - hh$$  \hspace{1cm} (1)

$$p_F = b - t h$$  \hspace{1cm} (2)
where \( hh \) and \( h \) are the quantities of the final good, produced in \( H \) and sold in \( H \) and \( F \) respectively, \( a \) and \( b \) stand for national market sizes and/or marginal willingness to pay of consumers in the two countries. Consumers in \( F \) get only \( t \) of the final good, due to transport costs.

Production of the final good requires the use of an intermediate input that is produced only in country \( F \), due to some exogenous comparative advantage. The \( fob \) (free on board) market price of the input is \( g \). The D firm based in \( H \) has to bear a transport cost \( t \) to ship the input to its factory in \( H \). Moreover, a constant marginal cost of production \( c \) has to be born making for the total cost function of the final good as:

\[
C_H = c(hh + h) + \frac{g}{t}(h + hh)
\]  

(3)

where \( g/t \) is the \( cif \) (cost insurance and freight) price paid by the D firm for the intermediate good.

We introduce the opportunity for the D firm to undertake process R&D, i.e. to invest so as to decrease the level of the marginal cost of production. Therefore, we assume that the R&D cost technology has the following convex shape (d’Aspremont - Jacquemin, 1988):

\[
k_H = \gamma \frac{x_H^2}{2}
\]  

(4)

where \( k_H \) represents total R&D commitment, \( x_H \) the amount of cost reduction and \( \gamma \) a cost parameter of the R&D activity. Therefore, the marginal cost is given by:

\[
c = c_H - x_H
\]  

(5)

where \( x_H \in [0, c_H] \) and \( c_H \) is a constant marginal cost. If we assume that the final good is produced by a monopoly in country \( H \), then the profit function of the D firm in country \( H \) is:

\[
\pi_H = hh \ p_H - \frac{g}{t} hh - c \ hh - \frac{\gamma}{2} x_H^2 + t \ h \ p_F - h \frac{g}{t} - c \ h.
\]  

(6)

As it appears the input is shipped from country \( F \) to country \( H \) and the transport cost is born by the D firm in \( H \). The final producer pays also for the transport cost of the final good from \( H \) to \( F \). The technical relationship linking the intermediate input to the production of the final good is one of fixed coefficients or perfect vertical complementarity (Tirole, 1988; Spencer and Jones, 1991).
As seen above, the input is bought from a firm in F which is the only producer of it. Again, we figure out this firm in F as having an analogous opportunity of undertaking process R&D with a convex technology. Then, the profit function of the monopoly based in F, producing the input needed for the final good based in F and selling uniquely to the foreign buyer in H, is:

$$\pi_F = (g - z)(hh + h) - \frac{\gamma x_F^2}{2}$$

(7)

where

$$z = c_F - x_F$$

(8)

with $x_F \in [0, c_F]$, and $c_F$ a costant marginal cost. Process R&D for the U monopoly firm based in F is:

$$k_F = \frac{\gamma x_F^2}{2}.$$  

(9)

Going through the optimal independent plans of profit maximization of the two firms we get the reduced form equilibrium values\(^1\) of endogeneous variables, respectively for firm H:

$$x_H^* = \frac{c_F(1 + t^2) + t(c_H(1 + t^2) - t(b + a t))}{1 + t(2 + t + t^2(2 - 4\gamma))}$$

(10)

$$hh^* = \frac{[(1 + 2t)(bt - a)(1 + t^2) + 2t^3((a - cd)t^3 - t(bt + c_H - 2a) - c_F(1 + t^2))\gamma]/\sqrt{[2(1 + t^2)(-1 + 2t)(1 + t^2) + 4t^3\gamma)]}}{1 + t(t^2 + 2t^3 - 2bt)\gamma}/[2(1 + t^2)(-1 + 2t)(1 + t^2) + 4t^3\gamma)]}$$

(11)

$$h^* = \frac{((1 + 2t)(a - bt)(1 + t^2) - 2(c_F + c_H t + (c_F - b)t^2 + (a + c_H)t^3 - 2bt)\gamma]}/[2(1 + t^2)(-1 + 2t)(1 + t^2) + 4t^3\gamma)]}$$

(12)

$$\pi_H^* = [4b(c_F + c_H t)\gamma((t + t^3)^2 - 2t^4(1 + t^2)^2)\gamma] +$$

(13)

\(^1\)Second order conditions (SOCs) are always met. Just in the case of the F firm it has to be:

$$\gamma \geq \gamma_1 = \frac{1 + t^2}{2t^2}.$$

6
and for firm $F$:

$$x^*_F = x^*_H$$

$$\pi^*_F = \frac{(c_F + c_H t + (c_F - b)t^2 + (c_H - a)t^3)2\gamma}{2(1 + t^2)(4t^3\gamma - (1 + 2t)(1 + t^2))}$$

$$g^* = \frac{1}{2} \left( b + c_F + (a - c_H) t - \frac{b + at}{1 + t^2} + \frac{c_F + c_H t + (c_F - b)t^2 + (c_H - a)t^3}{4t^3\gamma - 1 - t(2 + t + 2t^2)} \right)$$

Market prices of the final good in the two markets, $H$ and $F$ are, respectively:

$$p^*_F = \frac{2t(c_F + c_H t + (c_F + 3b)t^2 + (c_H + a)t^3 + 2bt^4)\gamma - (1 + 2t)(1 + t^2)(at + b(2 + t^2))}{(2(1 + t^2)(4t^3\gamma - (1 + 2t)(1 + t^2))}$$

$$p^*_H = \frac{2t^2(c_F + (2a + c_H) t + (c_F + b)t^2 + (c_H + 3a)t^3)\gamma - (1 + 2t)(1 + t^2)(a + bt + 2at^2)}{[2(1 + t^2)(4t^3\gamma - (1 + 2t)(1 + t^2)])}$$

Second best Social Welfare in the two countries, defined as the sum of consumers’ and producers’ surpluses, are reported in Appendix A.

### 2.2 A crossborder vertically integrated monopoly

Here we go through the vertically integrated counterpart of the above scheme, i.e.: a monopoly that is a MNF producing an intermediate good in country $F$ and a final good in $H$. The integrated firm undertakes, as above, process R&D in both stages of production as in the previous scenario of the decentralised
firms. Demand functions of the final good are the same as above and R&D takes place along the same scheme with the only difference that the input is now transferred internally within the MNF at its marginal cost of production \( z \). Still a transport cost of the usual sort has to be born to both undertake this transfer and sell the final good in \( F \).

Then, the profit function of the vertically integrated MNF is:

\[
\pi_{\text{INT}} = hh \ p_H - \frac{\tilde{z}}{t} hh - c \ hh - \frac{\gamma x_{\text{HINT}}^2}{2} + t \ h \ p_F - \frac{\tilde{z}}{t} - c \ h - \frac{\gamma x_{\text{FINt}}^2}{2} \tag{18}
\]

where

\[
c = c_H - x_{\text{HINT}} \tag{19}
\]

and

\[
z = c_F - x_{\text{FINt}}. \tag{20}
\]

Optimization occurs with respect to the output sold in the two markets, respectively, and the levels of R&D commitment. As a result we get the equilibrium\(^2\) values of endogeneous variables:

\[
hh^{*}_{\text{INT}} = \frac{(bt - a)(1 + t^2) + 2t^3((a - c_H)t - c_F)\gamma}{4t^4\gamma - 2(1 + t^2)^2} \tag{21}
\]

\[
h^{*}_{\text{INT}} = \frac{(a - bt)(1 + t^2) + 2t((bt - c_H)t - c_F)\gamma}{4t^4\gamma - 2(1 + t^2)^2} \tag{22}
\]

\[
x^{*}_{\text{HINT}} = -t(c_F + c_H t + (c_F - b)t^2 + (c_H - a)t^3) \tag{23}
\]

\[
x^{*}_{\text{FINt}} = -\frac{c_F(1 + t^2) + t(-t(b + at) + c_H(1 + t^2))}{2t^4\gamma - (1 + t^2)^2}. \tag{24}
\]

From these equilibrium variables we obtain optimal profits, market prices and the social welfare in the two countries:

\[
\pi^{*}_{\text{INT}} = \frac{2t^2(c_F + c_H t)^2(1 + t^2)\gamma - 4bt^2(c_F + c_H t)\gamma + b^2t^2(2t^2\gamma - 1 - t^2) + a^2(2t^4\gamma - 1 - t^2) + 2at(b(1 + t^2) - 2t^2) - 2t^2(c_F + c_H t)\gamma]}{4(t^4(2\gamma - 1) - 1 - 2t^2)} \tag{25}
\]

\[
p^{*}_{\text{FINt}} = \frac{2t^2(c_F + t(c_H + bt))\gamma - (1 + t^2)(at + b(2 + t^2))}{4t^4\gamma - 2(1 + t^2)^2} \tag{26}
\]

\(^2\)SOCs are all met in the feasible set of parameters.
\[ p_{\text{HINT}}^* = \frac{(2t^3(c_F + t(c_H + a))\gamma - (1 + t^2)(a + bt + 2at^2))}{4t^4\gamma - 2(1 + t^2)^2} \]  

(27)

\[ SW_{\text{FINT}} = \frac{1}{2}(b - p_{\text{FINT}})(t h_{\text{INT}}). \]  

(28)

The level of social welfare in \( F \) does not include any profit since the head-
quarters of the MNF are in \( H \) and the input is assumed to be internally transfered at the marginal cost even though it has to bear a transport cost. In country \( H \) social welfare is:

\[ SW_{\text{HINT}} = \pi_{\text{HINT}}^* + \frac{1}{2}(a - p_H)(h h_{\text{INT}}). \]  

(29)

Detailed formulas for second best social welfare (SW) are in Appendix A.

2.3 Comparison between the integrated and the decentralised monopolies

We now compare the two different arrangements. In the first we have a decentralised production process in which no MNF exists and each country controls a section of the vertical production process. In the second, a vertically integrated MNF splits the vertical chain of production over two countries. We proceed through numerical simulations since the analytical comparisons are not liable of clear interpretation.

We calibrate the models taking into account of the constraints implied by SOC\$ and several nonnegativity requirements. We provide a control scenario and then see some abstracts of how the results are affected by exogeneous variables.

2.4 The control scenario and the results

The control scenario is one of symmetric countries, where \( a = b = 50 \) and \( c_H = c_F = 20 \), \( t = 0.9 \), \( \gamma = 9 \). This scenario is consistent with all constraints of both models of the previous sections. The results obtained are in Table 1 below.
Table 1

<table>
<thead>
<tr>
<th></th>
<th>No Vert. Integration (DIS)</th>
<th>Vert. Integr. MNF (VI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_H$</td>
<td>0.39</td>
<td>0.86</td>
</tr>
<tr>
<td>$x_F$</td>
<td>0.39</td>
<td>0.96</td>
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<tr>
<td>$h$</td>
<td>0.54</td>
<td>2.90</td>
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<td>$hh$</td>
<td>2.94</td>
<td>4.85</td>
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<tr>
<td>$p_H$</td>
<td>47.06</td>
<td>45.15</td>
</tr>
<tr>
<td>$p_F$</td>
<td>49.52</td>
<td>47.39</td>
</tr>
<tr>
<td>$g$</td>
<td>22.06</td>
<td></td>
</tr>
<tr>
<td>$\pi_H$</td>
<td>8.18</td>
<td></td>
</tr>
<tr>
<td>$\pi_F$</td>
<td>7.84</td>
<td></td>
</tr>
<tr>
<td>$\pi_{INT}$</td>
<td></td>
<td>22.90</td>
</tr>
<tr>
<td>$SW_H$</td>
<td>12.49</td>
<td>34.67</td>
</tr>
<tr>
<td>$SW_F$</td>
<td>7.95</td>
<td>3.41</td>
</tr>
</tbody>
</table>

From the above table we notice that: First, there is a superior performance of the vertically integrated arrangement confirming what is well known in the literature since Spengler (Spengler, 1951). Second, the welfare levels of the recipient country (of the input plant of the MNF) are quite lower than those of the country home of the MNF. Third, the disintegrated arrangement has the two firms investing the same amount in process R&D, while in the vertically integrated case, the MNF invests more in the R&D for the production of the input. Fourth, the exports of the final goods to the input producing country (F) are much higher than in the disintegrated case.

Numerical simulations allow some comparative statics to be obtained (see table A1 in Appendix B). These results may be summarized in the following:

**Remark 1** Considering the reaction of endogeneous variables to the 3 key parameters we may state that: I) when the cost of producing the input ($c_F$) goes down (first two columns of table A1 in Appendix B) the MNF exports are larger than the sales of the final good at home, while the disintegrated monopolies do just the opposite. Then, for the vertically integrated MNF the home market effect disappears. Moreover, when the disintegrated arrangement applies, the SW is higher in F than in H. While, when comparing the
two arrangements, \( F \) enjoys a higher welfare with the disintegrated firms than with vertically integrated MNF. Finally the R&D commitment of the MNF is higher in \( F \) than in \( H \), while for the disintegrated it is the same in both markets. II) when the cost of producing the final output \( (c_H) \) goes down vis à vis the cost of the input (columns 3 and 4 of table A1 in Appendix B), the vertically integrated (VI) MNF still sells more abroad. Yet, the gap is higher than under case I. Again the SW in \( F \) is higher than the one in \( H \) under the disintegrated arrangement. For the VI MNF the opposite obtains. III) as the market size increases in both countries (columns 5 and 6 of table A1 in Appendix B) everything replicates the results under I) except for the prices: the VI MNF has \( p_F < p_H \) while for the disintegrated arrangement it is the opposite. IV) as transport costs increase (columns 7-9 of table A1 in Appendix B) the VI firm loses viability while the disintegrated firm still operates crossborder. The VI MNF could survive and continue its crossborder production only thanks to a subsidy. The disintegrated market arrangement is able to survive and give rise to trade even when higher transport costs let the vertically integrated MNF cease being multinational. V) finally, as \( t \) decreases, the disintegrated firms increase their R&D commitment, exports become larger than domestic sales, reversing the home market effect, and the price of the input \( (g) \) decreases. This happens at the cost of lower profits for \( F \) and higher profits for \( H \), while SW follows the same pattern of profits in the two countries.

3 The case of a vertical spillover

So far we have assumed that the R&D activity does not give rise to any spillover along the vertical chain of production. In this section we abandon this hypothesis by assuming that the R&D undertaken by D and U firms causes a reciprocal beneficial spillover. Since the vertical market relationship seems to provide better incentives than vertical integration for an efficient R&D, we assume that the spillover occurs only with decentralised production. We realize that this is a totally non-Schumpeterian hypothesis that can be contrasted on many grounds, yet we make it the basis for this section of the analysis. Then, we figure out a spillover going two ways (from D to U and from U to D) in the vertical chain of the decentralised arrangement. Subsequently, we shall consider also a one way spillover, i.e: from U to D or
viceversa. One way spillover will also be considered for the VI MNF in the comparisons below, just for the sake of curiosity. The analytical presentation, that follows, is for the two way spillover. One way spillover can be obtained as a trivial subcase. By introducing the spillover in the R&D equations, we have that (5) becomes:

\[ c_S = c_H - x_H - \beta x_F \]  

while (8) becomes

\[ z_S = c_F - x_F - \beta x_H. \]  

and we have to be confined to \( x_H - \beta x_F \in [0, c_H] \) and \( x_F - \beta x_H \in [0, c_F] \).

With vertical decentralization profit functions replicate (7) and (6). Therefore, optimal plans by the two firms lead to the equilibrium variables:

\[
g_S^* = (t(c_F(1 + t^2))((1 + \beta + \beta^2 t)(1 + t^2) - 2t^2 \gamma)) + 
+ c_H(1 + t^2) - (1 + t^2)(1 + t + \beta(2 + t)) + 
+ 2t^3 \gamma + (b + at)((t + t^3)(1 + t + \beta(2 + t)) - 2t^4 \gamma)) / 
/((1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)) 
\]

\[
x_{FS}^* = \frac{(1 + \beta t)(c_F + c_H t + (c_F - b)t^2 + (c_H - a)t^3)}{(1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma} 
\]

\[
x_{HS}^* = \frac{c_F + c_H t + (c_F - b)t^2 + (c_H - a)t^3}{(1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma} 
\]

\[
p_{FS}^* = \frac{[((1 + t^2)at + b(2 + t^2))(1 + 2t + \beta(2 + t(2 + \beta))) - 
2t(c_F + c_H t + (c_F + 3b)t^2 + (c_H + a)t^3 + 2bt^4 \gamma)) / 
[2(1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)]}{2t(1 + t^2)((1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)} 
\]

\[
p_{HS}^* = \frac{[((1 + t^2)(a + bt + 2at^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 
2t^3(c_F + (c_H + a)t + (c_F + b)t^2 + (3a + c_H)t^3) \gamma)] / 
[2(1 + t^2)((1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)]}{2t^2(c_F + (c_H + a)t + (c_F + b)t^2 + (3a + c_H)t^3) \gamma)} 
\]

\[
hh_S^* = \frac{[(a - bt)(1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 2t^2((a - 
-c_H)t^3 - t(bt - 2a + c_H) - c_F(1 + t^2)) \gamma) / 
[2(1 + t^2)((1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)]}{2(1 + t^2)((1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)} 
\]

\[
h_S^* = \frac{[(a - bt)(1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) + 2(c_F + c_H t + (38) 
+ (c_F - b)t^2 + t^3(a + c_H) - 2bt^4 \gamma)] / 
[2(1 + t^2)((1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)]}{2(1 + t^2)((1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta))) - 4t^3 \gamma)} 
\]
Given these optimal values (SW formulas are in Appendix C) we have again to resort to numerical simulations calibrating the parameters according to SOCs and nonnegativity constraints on prices, quantities and profits. In Table 2 below we report the results of the control simulation, having calibrated the parameters as in the previous section, while assuming $\beta = 0.8$. The case of vertical integration reported below concerns one way spillover: from the H to F. The one without any spillover is reported in the previous section in Table 1 column 2.
Table 2

<table>
<thead>
<tr>
<th></th>
<th>DIS spillover</th>
<th>VI one way spill</th>
<th>DIS one way spill</th>
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<tbody>
<tr>
<td>$x_H$</td>
<td>0.55</td>
<td>2.91</td>
<td>0.45</td>
</tr>
<tr>
<td>$x_F$</td>
<td>0.95</td>
<td>1.71</td>
<td>0.45</td>
</tr>
<tr>
<td>$h$</td>
<td>1.38</td>
<td>6.29</td>
<td>0.84</td>
</tr>
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<td>3.18</td>
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<td>$p_H$</td>
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<td>46.82</td>
</tr>
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<td>16.00</td>
<td>9.36</td>
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From the table above and the first column of Table 1 we are able to derive the following:

**Remark 2** Comparing a disintegrated crossborder arrangement that enjoys an externality in R&D activity and a vertically integrated crossborder arrangement that does not enjoy any spillover we find that I) VI is still preferable for country H but not for country F that has a larger SW with disintegration. II) aggregate profits of the two firms producing in the two stages of production separately are larger than those of the VI MNF. This provides an incentive for firms to disintegrate vertically, an event that may happen when they are not able to exploit internal spillovers due to the inability to build up a proper internal - crossborder - incentive system. III) as soon as we introduce some spillover within the VI MNF, it catches again its superiority.

In addition to that, numerical comparative statics, replicate mostly those of the previous section. However, the gap between VI and DIS is narrowed due to the obvious effect of the operating of the spillover in the DIS case. Moreover the profits of the firm in F become larger than those of H as the market size enlarges, in both the two way and the one way (from H to F) spillover cases. This allows for a larger SW in F than in H.
4 The duopoly case: differentiation and vertical restraints

Here we abandon the monopoly case and introduce an international differentiated duopoly that can be either vertically integrated or disintegrated. The question of integrated duopoly has been analysed in a different framework and with the aim of assessing the impact of trade policies, by Spencer and Jones (1991). Here, we follow a different route with product differentiation and the vertical relationships obeying to a restrained scheme and our aim remains that of evaluating the incentives of firms either to integrate or to outsource.

We assume that in each country there is a firm that sells both at home and abroad. Unlike the previous sections we assume that the two products manufactured in distinct countries are differentiated and the parameter measuring the marginal rate of substitution is represented by $s \in (0, 1]$.

4.1 Vertical integration

Consider first the symmetric international duopoly made up by two vertically integrated firms. Demand functions in the two countries are respectively:

\[
p_H = a - hh - stf \tag{41}
\]
\[
p_F = b - f s h. \tag{42}
\]

Each firm uses an intermediate input that is internally produced and transferred from the U section of production to the D section at the marginal cost. However, we assume that each firm is obliged to use the input produced by the foreign rival to manufacture its exports. In other words, each firm produces internally an intermediate good partly transferred to its own final production and partly sold to the rival that has to use it to produce its exports. This may be the result of either an antitrust policy aimed at making firms compete not just in the production of the final good but also in intermediate inputs. Alternatively, it may be the outcome of a trade policy meant to protect somehow the domestic production of the intermediate input or even to preserve some domestic features of the good that is sold in the domestic country, in other words, a sort of “antiglobal” measure.

As a consequence of these hypotheses, profit functions are, respectively,

\[
\pi_H = (p_H - c - z)hh + p_F th - ch - (p_{m_F}/t)h + (p_{m_H} - z)f \tag{43}
\]
\[ \pi_F = (p_F - c - z)f f + p_H t f - c f - (p_{m_H}/t)f + (p_{m_F} - z)h \]  

(44)

where \(c\) is the constant marginal cost of production of the final good, \(p_{m_F}\) is the price of the input bought by the firm based in \(H\) from the foreign rival \(F\) to produce its exports to \(F\), i.e. \(h\), while \(p_{m_H}\) is the price of the input bought by the firm based in \(F\) from the rival \(H\) so as to produce \(f\). Of course the input bought from the foreign rival has to be imported in the other country bearing a transport cost.

Firms set prices in the market for the intermediate products. In this market each firm has a quasi-monopoly position since each firm is compelled to buy from the rival an amount that is determined by its export of the final good. Then they compete in the market for the final good setting their optimal quantity.

From the two stage optimal plan, backward solved to get subgame perfection, we get the equilibrium\(^3\) controls, profits and second best social welfare\(^4\):

\[ p^*_H = \frac{2z + t(c(2 - 2s + 5t) - 2s z + 5t(a + z))}{2t(6t - s)} \]  

(45)

\[ p^*_F = \frac{2z + t(c(2 - 2s + 5t) - 2s z + 5t(b + z))}{2t(6t - s)} \]  

(46)

\[ h^* = \frac{2z - t(b(t - s) + c(s + t - 2) + z(s + t))}{s(s - 6t)t^2} \]  

(47)

\[ hh^* = \frac{t(7tz - 5at + c(7t - 2)) - 2z}{2(s - 6t)t} \]  

(48)

\[ f^* = \frac{2z - t(a(t - s) + c(s + t - 2) + z(s + t))}{s(s - 6t)t^2} \]  

(49)

\[ ff^* = \frac{t(7tz - 5bt + c(7t - 2)) - 2z}{2(s - 6t)t} \]  

(50)

\(^3\)SOCs are met provided that:

\[ t \geq \frac{1}{6}s. \]

\(^4\)Reduced form equations are not reported. They may be provided upon request by the author. These SW are used in numerical simulations used in the discussion.
\[ p_{mH}^* = \frac{t(c(s(2 - 4t) + 3(t - 2)t) + 6z + t(2as + 3at - 4sz + 3tz))}{2(6t - s)} \]  
\[ p_{mF}^* = \frac{t(c(s(2 - 4t) + 3(t - 2)t) + 6z + t(2bs + 3bt - 4sz + 3tz))}{2(6t - s)} \]  
\[ \pi_H^* = \frac{[(t^2(4b^2(s - t)^2 - a^2(s - 6t)t(4s + t) + 2c(-4b(s - t)(s + t - 2) + a(s - 6t)(2t - 2s + 6st - t^2)) + c^2(-8s^2(t - 1) + 2(t - 2)^2(2 + 3t) + s(-20 + t(47t - 12)))] + 2t(a(s - 6t)(2t - 2s + 6st^2 - t^3) + 4b(2s - (2 + s^2)t + t^3) + c(-2s^2(t - 1)(1 + 4t) + 2(t - 2)(2 + 3t)(t^2 - 2) + s(-12 + t(-18 + t(47t - 2)))) + (4s^2t(1 + t - 2t^2) + 2(2 + 3t)(t^2 - 2)^2 + s(-4 + t(-16 + t(-20 + t(8 + 47t)))))z + 2t(6s - 6t)(2t - 2s + 6st - t^3) + 4a(2s - (2 + s^2)t + t^3) + c(-2s^2(t - 1)(1 + 4t) + 2(t - 2)(2 + 3t)(t^2 - 2) + s(-12 + t(-18 + t(47t - 2)))) + (4s^2t(1 + t - 2t^2) + 2(2 + 3t)(t^2 - 2)^2 + s(-4 + t(-16 + t(-20 + t(8 + 47t)))))z^2)]}{[4s(s - 6t)^2t^2]} \]  
\[ \pi_F^* = \frac{[(t^2(4a^2(s - t)^2 - b^2(s - 6t)t(4s + t) + 2c(-4b(s - t)(s + t - 2) + b(s - 6t)(2t - 2s + 6st - t^2)) + c^2(-8s^2(t - 1) + 2(t - 2)^2(2 + 3t) + s(-20 + t(47t - 12)))] + 2t(b(s - 6t)(2t - 2s + 6st^2 - t^3) + 4a(2s - (2 + s^2)t + t^3) + c(-2s^2(t - 1)(1 + 4t) + 2(t - 2)(2 + 3t)(t^2 - 2) + s(-12 + t(-18 + t(47t - 2)))) + (4s^2t(1 + t - 2t^2) + 2(2 + 3t)(t^2 - 2)^2 + s(-4 + t(-16 + t(-20 + t(8 + 47t)))))z + 2t(6s - 6t)(2t - 2s + 6st - t^3) + 4a(2s - (2 + s^2)t + t^3) + c(-2s^2(t - 1)(1 + 4t) + 2(t - 2)(2 + 3t)(t^2 - 2) + s(-12 + t(-18 + t(47t - 2)))) + (4s^2t(1 + t - 2t^2) + 2(2 + 3t)(t^2 - 2)^2 + s(-4 + t(-16 + t(-20 + t(8 + 47t)))))z^2)]}{[4s(s - 6t)^2t^2]} \]  
\[ SW_H = \pi_H^* + \frac{1}{2}(a - p_H^*)(hh^* + t f^*) \]  
\[ SW_F = \pi_F^* + \frac{1}{2}(b - p_F^*)(ff^* + t h^*). \]
4.2 Vertically disintegrated international duopoly

Here we go through the parallel environment of the previous subsection, with vertically disintegrated firms. As above, each firm has to buy the amount of input needed for the production of exports from the firm based in the country to which exports are shipped. While demand functions replicate (41) and (42) the profit functions are no longer two but four since we have two D firms and two U firms distributed in the two countries, F and H. Again we proceed in two stages, assuming the same sequence as in the previous subsection. Then going backwards we first solve the Cournot game in the D stage and then go through the U stage where firms compete in prices.

Then, the profit functions of the D firms are:

\[ \pi_{HD} = (p_H - c - g_H)hh + p_Fh - ch - g_F h/t \]  
\[ \pi_{FD} = (p_F - c - g_F)ff + p_H f - cf - g_H f/t \]

while the profit functions of the U firms are:

\[ \pi_{HU} = (g_H - z)(hh + f) \]  
\[ \pi_{FU} = (g_F - z)(ff + h). \]

As above, the inputs are sold fob to the D firms which bear all transport costs since they are based abroad where they manufacture the final good.

The optimal controls\(^5\) and profits are\(^6\):

\[ hh^* = \frac{[at^2(5 - st - 4t^2 + 2st^3) + ct(2 - (7 + s)t + 2(1 + s)t^3 - 4st^4) - (2t^2 - 1)(2 - st - t^2 + 2st^3)z]}{[6t^3(2 - st - t^2 + 2st^3)]} \]  
\[ h^* = \frac{[bt^2(2 - 4st - t^2 + 5st^3) + ct(-4 + 2(1 + s)t + 2(1 + s)t^2 - (1 + 7s)t^3 + 2st^4) + (t^2 - 2)(2 - st - t^2 + 2st^3)z]}{[6st^3(2 - st - t^2 + 2st^3)]} \]

\(^5\)Socs are always met.
\(^6\)Social welfare equations are provided upon request and are used in numerical simulations. Moreover the optimal variables for country F are not reported for the sake of space: they can be found easily by changing \(a\) with \(b\) and viceversa, since everything is symmetric.
\[
g^*_H = \frac{2z - 2ct + t(a + c + cs + (a - 2c)s) + (s(2t^2 - 1) - t)z}{2(2 - t(s + t - 2st^2))}
\]  
\[
p^*_H = \frac{[at^2(5 - t^2 + st(5t^2 - 1)) + ct(2 + t((1 - t)(5 + t) + s(-1 + t(-4 + t(5 + 2t))))) + (1 + t^2)(2 - t^2 + st(2t^2 - 1))z]/[6t^2(2 - st - t^2 + 2st^3)]}{2 - t(s + t - 2st^2)}
\]  
\[
\pi^*_{HD} = \frac{1}{36t^4} \left( -\frac{1}{s(2 - st - t^2 + 2st^3)} (6ct(bt^2(2 - 4st - t^2 + 5st^3) + ct(-4 + 2(1 + s)t^2 - (1 + 7s)st^3 + 2st^4) + (t^2 - 2)(2 - t^2 + st(2t^2 - 1))z) + (3(-bt^2(1 + st) + ct(2 - (1 + s)t^2 + st(2t^2 - 1)) + (st - 2 + t^2 - 2st^3)z)bt^2(2 - 4st - t^2 + 5st^3) + ct(-4 + 2(1 + s)t^2 + (1 + 7st)st^3 + 2st^4) + (t^2 - 2)(2 - t^2 + st(2t^2 - 1))z)z) + st(2t^2 - 1)^2 + (bt^2(2 - 4st - t^2 + 5st^3) + ct(2t(1 + s) - 4 + 2(1 + s)t^2 - (1 + 7st)st^3 + 2st^4) + (t^2 - 2)(2 - t^2 + st(2t^2 - 1))z(bt^2(5 - st - t^2 + st^3) + ct(2 - (s - 5)t - 4(1 + s)t^2 + (5s - 1)t^3 + 2st^4) + (1 + t^2)(2 - t^2 + st(2t^2 - 1))z))/(s(2 - t^2 + st(2t^2 - 1))^2) + [1/(2 - t^2 + st(2t^2 - 1))^2]((at^2(-5 + st + 4t^2 - 2st^3) + ct(-2 + (7 + s)t - 2(1 + s)t^2 - 2(1 + s)t^3 + 4st^4) + (2t - 2)(1 + st) + 2t^2 - 2st^3))z^2) + 2t^2 - 2st^3)z^2) + 12st^3(2 - st - t^2 + 2st^3)
\]  
We are again confined to numerical simulations. Samples of them are reported in tables D1 and D2 in Appendix D. From their scrutiny we state the following.

**Remark 3** With product differentiation and firms obliged to buy part of their inputs from the foreign producer (either vertically integrated or independently operating) we can notice that: \( I \) the disintegrated duopolists export a larger
share of production making for a deeper trade integration than a vertically integrated international duopoly. \(\text{II})\) aggregate profits are larger for disintegrated firms than integrated ones. This points again to a clear incentive to disintegrate and to outsource as a way to enhance the value of firms. \(\text{III})\) the level of final prices is always lower with an integrated production structure. This is also the effect of the fact that integration is partial since each firm transfers internally only a chunk of its production of the intermediate good since it has to sell part of it to the rival firm for the production of exports. \(\text{IV})\) social welfare is always higher with the integrated arrangement. \(\text{V})\) in spite of the quasi-monopoly market structure the price of the input sold by the integrated firm to its rival is lower than the one of the disintegrated arrangement. \(\text{VI})\) as transport costs increase, in the disintegrated arrangement, profits of D firms increase while those of U firms decrease; for the integrated firm profits decrease altogether; while the market price of the intermediate good \((g)\) decreases faster than that of the integrated arrangement \((p_{m,F,H})\).

5 Concluding discussion

We have gone through three distinct vertical arrangements in the presence of trade and transport costs. Our aim has been mostly to evaluate the private and social incentives to adopt either a vertically integrated production structure or a disintegrated one. Our curiosity comes from the casual observation that a great deal of international outsourcing is taking place both between advanced and less advanced countries and even among countries which are homogeneous in their degree of affluence (as for instance Japan, the US and the EU).

When process R&D along the vertical chain is considered the traditional advantage of the vertically integrated arrangement fades away from a private point of view, i.e. for the firms. This is the case when the market is able to "channel" the vertical spillover while the VI firm is not. In a more competitive framework with product differentiation, duopoly and a trade policy constraint on the buying policies of inputs of firms we find again that in many circumstances firms have an incentive to outsource, i.e. to disintegrate since they are able to make higher aggregate profits. Moreover, we have seen that the vertically integrated firms operate internationally on a more limited range of the parameters space of its disintegrated counterpart.
(a similar result can be found in Rossini and Lambertini, 2003) that appears to be more trade prone.

As a partial conclusion we find that vertical integration still keeps some superior properties in terms of social welfare, yet neither for all countries involved nor for all firms.

Competiton policies towards vertically integrated firms operating cross-border should be carefully assessed to avoid the reduction of social welfare in many circumstances. In some case they may - ironically- favour firms, since they are able to pocket higher aggregated profits when they disintegrate.
6 APPENDICES

6.1 Appendix A: SW when R&D has no spillover

In the case of a disintegrated production process, the level of social welfare in country F is given by:

\[
SW_F = \pi_F^* + \frac{1}{2}(b - p_F)th = \\
= [4b(c_F + c_Ht)\gamma((1 + 2t)(2 + t + 2t^2)(t + t^3)^2 - 2t^4(1 + t^2)(1 + \\
+2t(2 + t + 2t^2))\gamma) + 4(c_F + c_Ht)^2\gamma(-(1 + 2t)(1 + t^2)^4 +
+(t + t^3)^2(1 + 4(t + t^3))\gamma) + b^2t^4((1 + 2t)^2(1 + t^2)^2 - 4(1 + 2t)(1 + \\
+2t^2)(1 + t + t^3)\gamma + 4t^2(1 + 4t(1 + t)(1 + t^3))\gamma^2 + a^2((t +
+2t^2 + t^3 + 2t^4)^2 - 4t^6(1 + 2t)(1 + t^2)(1 + t + t^3)\gamma + 4t^8(1 +
+4(t + t^3))\gamma^2) + 2at^2(bt(-1 + 2t)^2(1 + t^2)^2 - 2t(-1 + t^3(-4 + t((5 +
+4(t - 1)t))\gamma + 4t^4(-1 + 2t(2 + t(2t - 1)))\gamma^2) + 2(c_F + c_Ht)(1 +
+t^2)\gamma(-1 + t^2(3 + 2t(1 + t(4 + t + t^2(2 - 4\gamma) - 4\gamma) + \gamma)))))/
/[8(1 + t^2)^2(1 + t(2 + t + t^2(2 - 4\gamma)))^2]
\]

while in country H it is:

\[
SW_H = \pi_H^* + \frac{1}{2}(a - p_H)(th) = \\
= [4(c_F + c_Ht)^2\gamma(-(-1 + t^2)^4 + (2 + 3t^2)(t + t^3)^2\gamma + 4b\gamma(c_F +
+c_Ht)(-(1 + 2t)(t + t^3)^2 - 2t^4(2 + 3t^2 + t^4)\gamma) + a^2((1 +
+2t^2)(3 + 2t^2) - 4t^3(1 + t^2)(6 + t(12 + t(5 +
+11t + t^3))))\gamma + 4t^6(12 + 14t^2 + 3t^4)\gamma^2 + 2at(2(c_F +
+c_Ht)\gamma(1 + t^2)^2(1 + 4t + t^3) -
-2t^4(4 + 7t^2 + 3t^4)\gamma) + b((-1 + 2t)^2(1 +
+t^2)^2(3 + 2t^2) + 2t^3(1 + t^2)(11 +
+t(20 + t(9 + 16t))))\gamma - 4t^6(8 + 7t^2)\gamma^2)]/
/[8(1 + t^2)^2(1 + t(2 + t + t^2(2 - 4\gamma)))^2]
\]

When we consider a vertically integrated MNF we have, in the two countries, respectively:

\[
SW_{FINT} = \frac{t^2((a - bt)(1 + t^2) + 2t(-c_F + t(-c_H + bt))\gamma)^2}{8(-2t^4\gamma - 2(1 + t^2)^2}
\]
and

\[ SW_{HINT} = \frac{1}{8(-2t^4\gamma - 2(1 + t^2)^2)} \left[ b^2t^2(3 + t^4(7 - 12\gamma) + 2t^6(1 - 2\gamma)^2 - 
- 4t^2(\gamma - 2)) + 4bt^2(c_F + c_H t)\gamma(2 + 3t^2 + t^4 - 4t^4\gamma) + 
+ 4(c_F + c_H t)^2\gamma(-1 + t^2) + t^4(2 + 3t^2)\gamma) + 
+ a^2(-1 - t^2 + 2t^4\gamma)(-3 - 5t^2 + t^4(-2 + 6\gamma)) + 
+ 2at(b(1 + t^2) - 2t^2(c_F + 
+ c_H t)\gamma)(-3 - 5t^2 + t^4(-2 + 6\gamma)) \right]. \]

### 6.2 Appendix B Numerical Comparative Statics

Comparative statics of integrated versus disintegrated crossborder production under monopoly. (VI stands for vertically integrated, while DIS stands for vertically disintegrated).
Table A1

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6.3 Appendix C: SW with monopoly and R&D spillover

Here we provide the social welfare of country H

\[ SW_{H,SNOINT} = \]
\[ (4(c_F + c_H t)^2 \gamma(-1 + t^2)^4 + (2 + 3t^2)(t + t^3)^2 \gamma) + \]
\[ + 4b(c_F + c_H t)\gamma(-(t + t^3)^2(-2 + t(1 + \beta(2 + t(2 + \beta t)))) - \]
\[ - 2t^4(2 + 3t^2 + t^4)\gamma + a^2((1 + t^2)^2(3 + 2t^2)(1 + \]
\[ + 2t(2 + \beta t)))^2 - 4t^3(1 + t^2)(6 + \beta^2t^2(6 + 5t^2) + \]
\[ + 2\beta(1 + t)(6 + 5t^2) + t(12 + t(5 + 11t + t^3)))\gamma + 4t^6(12 + 14t^2 + 3t^4)\gamma^2) + \]
\[ + b^2((3 + 2t^2)(t + t^3)^2(1 + 2t + \beta(2 + t(2 + \beta t)))^2 - \]
\[ - 4t^4(1 + t^2)(1 + t(5 + 10\beta + (11 + 10\beta)t + (4 + \beta(8 + 5\beta)t)^2 + \]
\[ + 8(1 + \beta)t^3 + 4\beta^2t^4)\gamma + 4t^6(2 + 11t^2 + 8t^4)\gamma^2) + \]
\[ + 2at(2t(c_F + c_H t)\gamma((1 + t^2)^2(1 + 2\beta + 2(2 + \beta)t + \beta^2t^2 + 2t^3) - \]
\[ - 2t^3(4 + 7t^2 + 3t^4)\gamma + b(-1 + t^2)^2(3 + 2t^2)(1 + 2t + \beta(2 + t(2 + \beta t)))^2 + \]
\[ + 2t^3(1 + t^2)(11 + 22\beta + 20t + 22\beta t + (9 + \beta(18 + 11\beta))t^2 + \]
\[ + 2(8 + 9\beta)t^3 + 9\beta^2t^4)\gamma - 4t^6(8 + 7t^2)\gamma^2)) / \]
\[ /[8(1 + t^2)^2((1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta t))) - 4t^3\gamma)^2] \]

and of country F

\[ SW_{F,SNOINT} = \]
\[ [(a - bt)^2(t + t^3)^2(1 + 2t + \beta(2 + t(3 + \beta t)))^2 - 4(1 + t^2) \]
\[ (1 + 2t + \beta(2 + t(2 + \beta t)))(c_F^2(1 + t^2)^3 + t^4(c_H^2(1 + t^2)^3 - \]
\[ - c_H^2t(1 + t^2)(-a + 2b + 2at + bt + 2bt^2 + 2at^3) + \]
\[ + t^2(b(b - a) + (a + b)^2t + (a^2 - 3ab + b^2)t^2 + 2b(a + b)t^3 + a^2t^4)) + \]
\[ + c_F^2t(1 + t^2)(t(a - 2at - b(2 + t)) + 2(-t^3(b + at) + c_H(1 + t^2)^2))\gamma + \]
\[ + 4t^2(c_F^2(1 + t^2)^2(1 + 4t + t^3) + 2c_F^2t(1 + t^2)(-t(b + at + 4bt + 2(2a + \]
\[ + b)t^2 + 4bt^3 + 4at^4) + c_H(1 + t^2)(1 + 4(t + t^3)) + t^2(-2c_H^2t(1 + t^2)(b - \]
\[ - at + 4bt + 2(2a + b)t^2 + 4bt^3 + 4at^4) + t^2(b^2 + 2b(2b - a)t + \]
\[ +(a^2 + 8ab + 4b^2)t^2 + 4(a^2 - ab + b^2)t^3 + 4at^4) + t^2(b^2 + 2b(2b - a)t + \]
\[ +(a^2 + 8ab + 4b^2)t^2 + 4(a^2 - ab + b^2)t^3 + 4b(2a + b)t^4 + 4a^2t^5) + \]
\[ + c_H^2(1 + t^2)^2(1 + 4(t + t^3))\gamma)^2) / \]
\[ /[8(1 + t^2)^2((1 + t^2)(1 + 2t + \beta(2 + t(2 + \beta t))) - 4t^3\gamma)^2]. \]
6.4 Appendix D: Tables on duopoly with differentiation

Table D1: The disintegrated duopoly (DIS)

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<td>21.11</td>
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<td>19.23</td>
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<td>200.47</td>
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<td>66.05</td>
</tr>
<tr>
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<td>32.03</td>
<td>16.30</td>
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Table D2: The vertically integrated duopoly (VI)

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References


