

Trade costs in a model of pricing to market

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Abstract

This paper examines the role of trade costs as a form of imperfect international goods market integration. Following the approach of New Open Economy Macroeconomics, a two-country general equilibrium model is invoked to show that trade costs make individuals worse off since they reduce consumption, real wages and real profits, while labour input remains constant. The smaller a country is relative to the other, the more it suffers from trade costs. Moreover, trade costs exhibit a ‘containment effect’ such that spending tends to be kept in its country of origin. This finding is consistent with the empirical evidence on the ‘Home Bias in Trade Puzzle’. As a consequence, under full pricing to market (i.e. local-currency pricing) trade costs decrease the variability of the nominal exchange rate when an economy is hit by government spending shocks. However, with no pricing to market (i.e. producer-currency pricing) monetary shocks increase the exchange rate variability and make overshooting more likely. In addition, trade costs introduce asymmetries to real interest rate and output movements across countries in response to monetary shocks, whereas consumption and trade balance movements are not altered significantly. Finally, under full pricing to market trade costs diminish the negative international welfare spillover caused by a monetary shock. These qualitative results are robust towards the extension of rebating trade costs to consumers.

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1 Introduction

Consistent with recent empirical evidence on imperfect international goods market integration, this paper introduces trade costs into a two-country general equilibrium model of pricing to market. Some firms can set their prices in the local currency of sale, while others set prices in their domestic currency. Trade costs in this model are all-embracing in the sense that they do not only comprehend transportation costs but basically any kind of ‘border effect’ that can possibly exist between two countries, namely tariffs³, red tape, cultural and language barriers etc.

Indeed, trade costs turn out to have wide-ranging implications for the behaviour of key economic variables. In particular, in the flexible-price equilibrium trade costs reduce consumption, real wages and real profits but leave labour supply unaffected, hence making individuals worse off. The smaller a country is relative to the other, the more it suffers from trade costs. In the presence of monetary or government spending shocks, the variability of the exchange rate is altered.

As a general feature trade costs exhibit a ‘*containment effect*’, which means that they make imports relatively expensive and thus tend to contain spending in its country of origin. In other words, trade costs act like a buffer between countries and therefore obstruct the integration of international goods markets. Since imports are more expensive than domestically produced goods, consumption is biased towards domestically produced goods. Thus, trade costs give rise to a ‘Home Bias’ in consumption, which is consistent with the ‘*Home Bias in Trade Puzzle*’. Its empirical significance has been highlighted in several recent studies. One of the most eminent is probably McCallum (1995), who shows that trade volumes within Canada and within the United States are overwhelmingly bigger than those between the two countries.

Trade costs are also receiving increasing attention in economic theory. While the general importance of imperfect international goods market integration for economic outcomes has been known for a long time, e.g. as demonstrated in Krugman (1989), the potential of trade costs to elucidate some of the major puzzles in international macroeconomics is explicitly emphasised by Obstfeld and Rogoff (2000). However, they only use a very basic endowment economy model to illustrate their ideas. In contrast, this paper follows the approach of the New Open Economy Macroeconomics literature by analysing trade costs in the richer general equilibrium setting of Obstfeld and Rogoff (1995). Their two-country workhorse model incorporates a government, monopolistic competition and sticky prices. Furthermore, the pricing-to-market framework of Betts and Devereux (2000) is adopted.

The paper is structured as follows. Section 2 starts off with the model set-

³Trade costs in this model are fixed ‘iceberg’ trade costs in the sense that a certain fraction of goods vanishes during the trading process, i.e. there is a dead-weight loss. In particular, if tariffs are interpreted as trade costs, this may only pertain to the dead-weight loss incurred by tariffs since any potential revenue of tariffs could be rebated to consumers, cf. Section 8. Therefore, trade costs are purely exogenous.

up. Then Section 3 describes the model’s flexible-price equilibrium, whereas Sections 4 and 5 analyse the model’s behaviour under sticky prices. A welfare analysis is conducted in Section 6. Section 7 presents a numerical evaluation of the model. Section 8 checks the model’s robustness by rebating trade costs to consumers. The country size is varied in Section 9. Section 10 concludes.

2 The model

The model is based on the two-country setting in Betts and Devereux (2000). Households choose among a range $[0, 1]$ of differentiated, nondurable and tradable goods which are produced by monopolistic firms. Both the Home and the Foreign countries are of equal size⁴ so that they have the same number of inhabitants and produce half of the goods range each. Furthermore, it is assumed that s with $0 \leq s \leq 1$ is the fraction of firms in each country that is able to price to market, i.e. that can price-discriminate across the two countries. Therefore, the implicit assumption is made that households cannot arbitrage away potential cross-country price differences for goods. As a new ingredient the existence of fixed ‘iceberg’ trade costs, τ , is assumed where τ represents the fraction of goods that gets lost in the trading process so that $0 \leq \tau \leq 1$. Appendix A outlines the derivations of this section and gives the corresponding expressions without trade costs for reference.

2.1 Households

Households derive utility from consumption and also from real money balances due to a transactionary motive and disutility from work. All households and firms are assumed to be identical within one country and preferences are the same across countries. In Home country notation utility is given by

$$U_t = \sum_{v=t}^{\infty} \beta^{v-t} \left(\ln C_v + \frac{\gamma}{1-\epsilon} \left(\frac{M_v}{P_v} \right)^{1-\epsilon} + \eta \ln(1-h_v) \right) \quad (1)$$

with the CES composite consumption index defined as

$$C_t \equiv \left(\int_0^1 c_{it}^{\left(\frac{\rho-1}{\rho}\right)} di \right)^{\frac{\rho}{\rho-1}} \quad (2)$$

where $\rho > 1$.⁵ c_{it} is consumption of good i at time t , β is the subjective discount factor with $\frac{1}{2} < \beta < 1$, M_t is the money supply and h_t represents labour. β , γ , ϵ , η , ρ are positive parameters and the same in both countries.

⁴For differing country sizes cf. Section 9.

⁵ ρ is the price elasticity of demand for each good and also the elasticity of substitution between goods, which is defined as $\frac{d \ln(c_{it}/c_{jt})}{d \ln(p_{jt}/p_{it})}$. The assumption $\rho > 1$ is necessary to guarantee existence of an equilibrium. In the case of $\rho < 1$ marginal revenue would be negative and an economically meaningful equilibrium would not exist. For $\rho = 1$ the CES specification reduces to Cobb-Douglas.

The Home consumption based price index is defined as the minimum expenditure subject to $C_t = 1$

$$P_t = \left[\int_0^{\frac{1}{2}} p_{it}^{1-\rho} di + \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{1}{2}s} \left(\frac{1}{1-\tau} p_{it}^* \right)^{1-\rho} di + \int_{\frac{1}{2}+\frac{1}{2}s}^1 \left(\frac{1}{1-\tau} e_t q_{it}^* \right)^{1-\rho} di \right]^{\frac{1}{1-\rho}} \quad (3)$$

Accordingly, the Foreign price index is given by

$$P_t^* = \left[\int_0^{\frac{1}{2}s} \left(\frac{1}{1-\tau} q_{it} \right)^{1-\rho} di + \int_{\frac{1}{2}s}^{\frac{1}{2}} \left(\frac{1}{1-\tau} \frac{1}{e_t} p_{it} \right)^{1-\rho} di + \int_{\frac{1}{2}}^1 q_{it}^{*1-\rho} di \right]^{\frac{1}{1-\rho}} \quad (4)$$

where prices denoted p represent Home currency prices, while prices denoted q represent Foreign currency prices. In general asterisks indicate Foreign country variables. In this context an asterisk means that a price is set by a Foreign firm, i.e. all p_{it}^* are set by Foreign firms in Home currency and all q_{it}^* are set by Foreign firms in Foreign currency. The goods on the range $[0, \frac{1}{2}]$ are produced by Home firms and the goods on the range $[\frac{1}{2}, 1]$ are produced by Foreign firms. So the Home country price index, P_t , consists of the prices p_{it} of goods on the range $[0, \frac{1}{2}]$ that are produced by the Home country and of goods on the range $[\frac{1}{2}, 1]$ that are produced by the Foreign country. For the goods on the range $[\frac{1}{2}, \frac{1}{2} + \frac{1}{2}s]$ the Foreign firms are able to price to market, i.e. they set the corresponding prices p_{it}^* in Home currency. The range $[\frac{1}{2} + \frac{1}{2}s, 1]$ represents the goods produced by Foreign firms that are not able to price to market and therefore set their prices q_{it}^* in Foreign currency. These are converted into Home currency by multiplying with the nominal exchange rate e_t , which denotes the Home price of Foreign currency at time t . Note that the factor $\frac{1}{1-\tau}$ is included in the range $[\frac{1}{2}, 1]$. The reason is that all prices (p_{it} , p_{it}^* , q_{it} , q_{it}^*) are f.o.b., i.e. unit prices that are charged in the country of production of each good. If a Foreign good is shipped to the Home country, only the fraction $(1-\tau)$ arrives. Therefore, the Home consumer must buy $\frac{1}{1-\tau}$ units of that good in the Foreign country so that one unit arrives in the Home country. So from a Home consumer's perspective $\frac{1}{1-\tau} p_{it}^*$ is the c.i.f. price of one unit of a Foreign pricing-to-market good, and $\frac{1}{1-\tau} e_t q_{it}^*$ is the c.i.f. price of one unit of a Foreign non-pricing-to-market good⁶. One can think of this f.o.b./c.i.f. relationship either as consumers' travelling to the other country to go shopping, thereby incurring costs that are equivalent to the trade costs, or as firms' charging an additional markup for having the purchased goods shipped over to the country of destination. The Foreign price index can be explained analogously.

The Home budget constraint at time t is given by

$$P_t C_t + M_t + d_t F_t = W_t h_t + \pi_t + M_{t-1} + Z_t + F_{t-1} \quad (5)$$

where W_t is the nominal wage rate, π_t are the firms' profits and Z_t are lump-sum transfers from the government which can also become negative. F_{t-1} represents

⁶Section 2.3 gives more details about the pricing.

the holdings of Home currency denominated nominal discount bonds that have matured in period t and d_t is the price of a discount bond F_t that will pay its face value at time $t + 1$ so that

$$d_t = \frac{1}{1 + i_t} \quad (6)$$

where i_t is the nominal interest rate in the Home country between dates t and $t + 1$. There is free trade between the Home and the Foreign country in this Home currency denominated bond.

The Home demand function⁷ is given by

$$c_{it} = \left(\frac{\xi_{it}}{P_t} \right)^{-\rho} C_t \quad (7)$$

where

$$\xi_{it} = \begin{cases} p_{it} & \text{for } 0 \leq i < \frac{1}{2} \\ \frac{1}{1-\tau} p_{it}^* & \text{for } \frac{1}{2} \leq i < \frac{1}{2} + \frac{1}{2}s \\ \frac{1}{1-\tau} e_t q_{it}^* & \text{for } \frac{1}{2} + \frac{1}{2}s \leq i \leq 1 \end{cases} \quad (8)$$

according to the three terms in the price index (3). The households maximise utility (1) subject to the budget constraint (5), resulting in an optimality condition for the labour supply

$$\frac{\eta}{1 - h_t} = \frac{W_t}{P_t C_t} \quad (9)$$

and in a money demand function

$$\frac{M_t}{P_t} = \left(\frac{\gamma C_t}{1 - d_t} \right)^{\frac{1}{\epsilon}} \quad (10)$$

Moreover, the intertemporal consumption stream can be described by

$$d_t P_{t+1} C_{t+1} = \beta P_t C_t \quad (11)$$

The corresponding equations for the Foreign country are analogous.

2.2 Government

Let the composite government consumption good, G_t , be defined like the private one in (2). Government demand is then analogous to private demand and given by

$$g_{it} = \left(\frac{\xi_{it}}{P_t} \right)^{-\rho} G_t \quad (12)$$

The Home country government budget constraint is

$$P_t G_t + Z_t = M_t - M_{t-1} \quad (13)$$

⁷The derivations are outlined in Appendix A.

So the Home government has two potential sources to finance its spending, by printing money and by receiving lump-sum transfers from its citizens (in that case $Z_t < 0$). Therefore, the government budget is always balanced and Ricardian equivalence holds. The corresponding Foreign country equations are analogous.

2.3 Firms

Each firm faces the same linear production technology so that

$$y_t = h_t \quad (14)$$

where y_t denotes aggregate Home output and h_t is the labour input used by the Home firms. Labour markets are perfectly competitive⁸ in each country but workers are internationally immobile. Total output can be divided into output destined for the Home country, denoted by x and x^* , respectively, and output destined for the Foreign country, denoted by z and z^* , respectively, so that

$$y_t = x_t + z_t \quad (15)$$

and accordingly

$$y_t^* = x_t^* + z_t^* \quad (16)$$

for the Foreign country. Note that z_t is the amount of Home output that is *shipped to* the Foreign country (and not the amount that is *demanded in* the Foreign country), meaning that due to the trade costs only the fraction $(1 - \tau)$ actually arrives at the destination. Thus, in equilibrium $(1 - \tau)z_t$ is the amount demanded in the Foreign country. The same logic applies to x_t^* .

For the time being, price flexibility is assumed in the sense that the firms set their prices after the exchange rate and the wage rate are known. Then the per capita profit function is described by

$$\pi_t = s(p_t x_t + e_t q_t z_t) + (1 - s)(p_t x_t + p_t z_t) - W_t (x_t + z_t) \quad (17)$$

The first term of the right-hand side of the aggregate profit function reflects the revenue from pricing-to-market firms. These can charge the Foreign-currency price q_t ⁹ to Foreign consumers. The second term is the revenue from non-pricing-to-market firms, which can only charge the Home-currency price p_t ⁹, and the last term constitutes the costs of production. Inserting the demand function given by (7) for x_t and $(1 - \tau)z_t$, respectively, and maximising profits yields the following price markup

$$p_t = e_t q_t = \frac{\rho}{\rho - 1} W_t \quad (18)$$

⁸In fact, W and W^* are normalised to one, i.e. in a perfectly competitive labour markets the workers are 'wage takers'. However, the real wage, $\frac{W}{P}$ and $\frac{W^*}{P^*}$, respectively, is pinned down by the parameters ρ and τ . Cf. Section 3 for more details.

⁹Since all firms within one country are symmetric, the prices they charge are the same so that $p_{it} = p_t$, $p_{it}^* = p_t^*$, $q_{it} = q_t$ and $q_{it}^* = q_t^*$.

The corresponding price markup set by Foreign firms is

$$q_t^* = \frac{p_t^*}{e_t} = \frac{\rho}{\rho - 1} W_t^* \quad (19)$$

The assumption of price flexibility makes it irrelevant whether there is pricing to market. It can be shown on the basis of the price indices that purchasing power parity (PPP) still obtains in the aggregate¹⁰ because the Home and Foreign countries are of equal size.

3 The initial equilibrium with flexible prices

As a benchmark scenario it is first assumed that prices are perfectly flexible, i.e. firms set prices after the exchange rate and the costs of production are known. It is also assumed that initially there are no transfers and neither bond holdings nor government expenditures¹¹ so that $Z = Z^* = F = F^* = G = G^* = 0$. The following markets must be cleared in an equilibrium: the world markets for each good, the world capital market, the labour markets in each country and the money markets in each country.

It turns out handy to use

$$\lambda \equiv \frac{1 - (1 - \tau)^{\rho-1}}{1 + (1 - \tau)^{\rho-1}} \quad (20)$$

as a term involving the trade costs parameter, τ . Note that $0 < \lambda < 1$ for $0 < \tau < 1$ with $\lambda = 0$ for $\tau = 0$ and $\frac{\partial \lambda}{\partial \tau} > 0$. The time index t is dropped to denote the initial equilibrium values. Appendix B outlines the derivations of this section and gives the corresponding expressions without trade costs for reference.

In equilibrium the supply of labour and thus output is the same¹² in both countries

$$h = h^* = y = y^* = \frac{\rho - 1}{\rho - 1 + \rho\eta} \quad (21)$$

Note in particular that this expression does not depend on trade costs. The intuition behind this result is that the optimal labour supply decision, which is derived from the household optimisation, is unaffected by trade costs.

However, consumption is diminished¹³ by trade costs and can be shown to be

$$C = C^* = h \left(\frac{1}{1 + \lambda} \right)^{\frac{1}{\rho-1}} \quad (22)$$

¹⁰Cf. equation (29).

¹¹A typical shortcoming of New Open Economy Macroeconomics models is the indeterminacy of a steady state in the sense that an equilibrium is not determined endogenously.

¹²The initial equilibrium is symmetric due to the equal country size.

¹³With $\tau = 0$ consumption is given by $C = C^* = h > h \left(\frac{1}{1 + \lambda} \right)^{\frac{1}{\rho-1}}$.

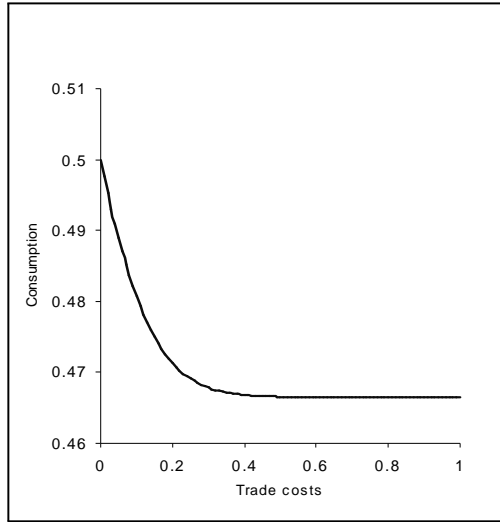


Figure 1: How consumption reacts to trade costs

The derivative of the last expression with respect to τ is negative

$$\frac{\partial C}{\partial \tau} = \frac{\partial C^*}{\partial \tau} < 0$$

i.e. the higher trade costs are, the less can be consumed by the households. This relationship is illustrated in Figure 1, where the parameter values of Section 7 are used¹⁴. Note that trade costs reduce consumption in a nonlinear fashion, i.e. once they reach a threshold value (roughly 40% in this numerical example), they have no further significant effect on consumption.

Real profits of Home firms can be shown to amount to

$$\frac{\pi}{P} = \frac{h}{\rho} \left(\frac{1}{1+\lambda} \right)^{\frac{1}{\rho-1}} \quad (23)$$

and the real wage is given by

$$\frac{W}{P} = \frac{\rho-1}{\rho} \left(\frac{1}{1+\lambda} \right)^{\frac{1}{\rho-1}} \quad (24)$$

So both real profits and the real wage decline with trade costs. More precisely,

$$\frac{\partial(\frac{\pi}{P})}{\partial \tau} = \frac{\partial(\frac{W}{P})}{\partial \tau} \frac{1}{(\rho-1+\rho\eta)} < 0 \quad (25)$$

¹⁴The chosen parameter values are $\rho = 11$, $\eta = 10/11$, $r = 0.06$, $\epsilon = 1$, implying a markup of 10% and $\beta \simeq 0.94$. See Section 7 for a more detailed discussion.

In other words, the real wage declines more quickly¹⁵ than real profits in response to rising trade costs. In that sense and also because consumption decreases with trade costs while the supply of labour remains constant, workers suffer more from trade costs than firms do. Intuitively, firms have some degree of market power due to monopolistic competition, whereas workers face perfect competition in the labour market.

Via (11) it can be seen that in equilibrium the price of a discount bond is

$$d = d^* = \beta \quad (26)$$

The equilibrium real interest rate is given by

$$r = r^* = \frac{1 - \beta}{\beta} \quad (27)$$

Finally, the equilibrium exchange rate¹⁶ is given by

$$e = \frac{M}{M^*} \left(\frac{C^*}{C} \right)^{\frac{1}{\epsilon}} \quad (28)$$

The intuition for this exchange rate equation is the same as in the monetary approach to the exchange rate. If the Home country increases its money supply, there is a depreciation (an increase in e) and vice versa. If the Home country produces higher output, then via (22) there is an increase in C , leading to an appreciation. As mentioned in Section 2.3, PPP holds under price flexibility, i.e. the real exchange rate, ψ , is equal to one

$$\psi \equiv \frac{eP^*}{P} = 1 \quad (29)$$

The price levels can be shown to be

$$P = \left(\frac{\rho}{\rho - 1} \right) W(1 + \lambda)^{\frac{1}{\rho - 1}} \quad (30)$$

$$P^* = \left(\frac{\rho}{\rho - 1} \right) W^*(1 + \lambda)^{\frac{1}{\rho - 1}} \quad (31)$$

Comparing the price indices in the presence of trade costs, given in (3) and (4), to the corresponding indices without trade costs from Appendix B makes clear that trade costs push up the price levels¹⁷ for given nominal wage rates, i.e.

$$\frac{\partial P}{\partial \tau} = \frac{\partial P^*}{\partial \tau} > 0 \quad (32)$$

¹⁵Strictly speaking, this depends on parameter values, i.e. it must be that $(\rho - 1 + \rho\eta) > 1$. However, as Section 7 shows, this is clearly the case for empirically plausible parameter values.

¹⁶Strictly speaking, the level of the initial equilibrium exchange rate is given by the relative money supply, $e = M/M^*$, since $C = C^*$. Therefore, trade costs have no influence on the equilibrium exchange rate.

¹⁷Note that the factor $\frac{1}{1 - \tau}$ enters the price indices (3) and (4).

Note that the expenditure for one unit of the composite consumption index defined in (2) does not depend on trade costs¹⁸

$$PC = \frac{\rho}{\rho - 1 + \rho\eta} W \quad (33)$$

$$P^*C^* = \frac{\rho}{\rho - 1 + \rho\eta} W^* \quad (34)$$

In total, since in the presence of trade costs consumption drops and the price level increases while labour input remains constant, trade costs make individuals worse off, i.e.

$$\frac{\partial U}{\partial \tau} = \frac{\partial U^*}{\partial \tau} < 0 \quad (35)$$

Via (10), (21), (22), (26) and (30) it can be shown that

$$W = \left(\frac{(1 - \beta)(\rho - 1 + \rho\eta)}{\gamma} \right)^{\frac{1}{c}} \frac{1}{\rho} \left((\rho - 1) \left(\frac{1}{1 + \lambda} \right)^{\frac{1}{\rho - 1}} \right)^{\frac{c-1}{c}} M \quad (36)$$

$$W^* = \left(\frac{(1 - \beta)(\rho - 1 + \rho\eta)}{\gamma} \right)^{\frac{1}{c}} \frac{1}{\rho} \left((\rho - 1) \left(\frac{1}{1 + \lambda} \right)^{\frac{1}{\rho - 1}} \right)^{\frac{c-1}{c}} M^* \quad (37)$$

In the following the monetary bases, M and M^* , respectively, are normalised as the nominal anchor of the model such that the nominal wage rates, W and W^* , respectively, are equal to one. Therefore, $P = P^*$, $M = M^*$ and $e = 1$. In particular, PPP¹⁹ and the law of one price for labour hold, i.e. $P = eP^*$ and $W = eW^*$.

4 Sticky prices and the exchange rate

Now, as a Keynesian feature of the model, it is assumed that all prices (p_t , p_t^* , q_t , q_t^*) are sticky and that therefore, they cannot immediately respond to either type of shock, a monetary shock or a government spending shock. Firms set their prices such that they are optimal in the absence of shocks. For a sufficiently small shock in period t , output is demand-determined, i.e. firms produce the post-shock market demand given their initial prices. They have an incentive to do so since they still make profits because of monopolistic competition. As there is no capital in the model, prices and all other variables reach their new equilibrium after a single period, i.e. the long-run responses are attained in period $t + 1$.

Like in the initial equilibrium those firms that engage in pricing to market set their prices in the currency of the country where the products are sold (local-currency pricing). For these firms an unanticipated movement in the exchange rate does therefore not lead to a deviation from the optimal price.

¹⁸ C is lower and P is higher for $\tau > 0$. These two effects exactly offset each other in this model's specification.

¹⁹Cf. equation (29).

The remaining firms set their prices in the currency of the country where the products are produced (producer-currency pricing). For these firms, however, an unanticipated movement in the exchange rate does lead to a deviation from the optimal price.

Regarding the methodology, log-linear approximations are taken around the initial pre-shock equilibrium of Section 3, first with flexible prices in order to solve for the long-run (i.e. period $t + 1$) response of the economy and then with sticky prices in order to identify the short-run (i.e. period t) response to shocks. As to the notation, for any variable U let

$$\widehat{U}_{t+i} \equiv \frac{U_{t+i} - U}{U}$$

be the percentage deviation at time $t + i$ for $i = 0, 1$ from the initial pre-shock equilibrium described in the preceding sections.

This section discusses the main results relating to the nominal exchange rate and introduces the ‘containment effect’ which describes the segmentation of international goods markets induced by trade costs. Section 5 focuses in detail on the two types of shocks and extends the analysis to other key variables. Appendix C outlines the derivations and gives the corresponding expressions without trade costs for reference.

4.1 The long-run response to shocks

The long-run (i.e. period $t + 1$) response of the exchange rate can be directly derived from (28)

$$\widehat{e}_{t+1} = (\widehat{M}_{t+1} - \widehat{M}_{t+1}^*) - \frac{1}{\epsilon}(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*) \quad (38)$$

where the long-run response of Home relative to Foreign consumption is given by²⁰

$$(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*) = \frac{1}{\sigma} \frac{\rho}{(\rho - \lambda)} \frac{2(1 - \beta) dF_t}{PC^w} - \frac{1}{\sigma} \frac{(\rho - \sigma\lambda)}{(\rho - \lambda)} \frac{(dG_{t+1} - dG_{t+1}^*)}{C^w} \quad (39)$$

with

$$\sigma \equiv \frac{\rho - 1 + \rho\eta}{\rho - 1 + \eta} \quad (40)$$

and $\rho > \sigma > 1$. P denotes the initial price level and C^w is the initial value of average world consumption, i.e. $C^w = \frac{1}{2}(C + C^*)$. Thus, for a given government spending pattern, if the Home country has a trade balance surplus in period t , implying $dF_t > 0$, Home consumption is raised permanently relative to Foreign consumption. Note that since the steady state response to a shock is attained in period $t + 1$ already, it must be that $dF_{t+1} = dF_t$ so that the asset positions

²⁰The derivations are outlined in Appendix C.

induced by the initial trade surplus are carried over to the new equilibrium²¹. Intuitively, there is a permanent positive wealth effect for the Home country due to the interest payments received from the Foreign country. This helps Home individuals to finance their relative consumption rise. All being equal, an increase in Home government spending has the opposite effect, driving out Home consumption and decreasing the relative Home consumption rise. Higher trade costs, reflected in a higher λ , lead to a more pronounced relative consumption rise²². Intuitively, trade costs make Home goods relatively cheaper compared to Foreign goods so that the additional wealth from the trade balance surplus tends to be spent more on Home goods. Similarly, for given dF_t , the Home government spends a greater fraction of its budget on Home goods in the presence of trade costs. This additional income in turn allows Home individuals to increase their consumption more than Foreign individuals. In other words, trade costs act like a buffer between the Home and the Foreign country and segment the international goods markets into a Home and a Foreign one, i.e. they tend to contain spending in its country of origin²³. This form of imperfect international goods market integration is subsequently referred to as the ‘*containment effect*’ of trade costs.

4.2 The short-run response to shocks

When a shock occurs in time period t , the responses of the price indices (3) and (4) are given by

$$\widehat{P}_t = \frac{1}{2}(1 - \lambda)(1 - s)\widehat{e}_t \quad (41)$$

and

$$\widehat{P}_t^* = -\frac{1}{2}(1 - \lambda)(1 - s)\widehat{e}_t \quad (42)$$

Since $\lambda = 0$ for $\tau = 0$ and $0 < \lambda < 1$ for $0 < \tau < 1$ it follows that for a given exchange rate depreciation ($\widehat{e}_t > 0$) the price indices are less volatile in the presence of trade costs than without trade costs unless there is full pricing to market. In the latter case the price indices are insulated from nominal exchange rate fluctuations since all firms set local-currency prices. Hence

$$0 < \widehat{P}_t < \widehat{P}_{t,\tau=0} < \widehat{e}_t \text{ if } s < 1 \quad (43)$$

$$\widehat{e}_t > 0 > \widehat{P}_t^* > \widehat{P}_{t,\tau=0}^* \text{ if } s < 1$$

$$\widehat{P}_t = \widehat{P}_{t,\tau=0} = \widehat{P}_t^* = \widehat{P}_{t,\tau=0}^* = 0 \text{ if } s = 1 \quad (44)$$

²¹In the new equilibrium the trade balance is zero, i.e. neither does the Foreign country incur any further debts nor are any principal payments made. Hence, the Foreign country pays a constant sum of interest to the Home country in every period of the new equilibrium.

²²For the government spending term in (39) note that $\sigma > 1$. For the first term on the right-hand side of (39) note that $\frac{\partial\left(\frac{1}{\sigma}\frac{\rho}{\rho-\lambda}\right)}{\partial\lambda} > 0$, for the second $\frac{\partial\left(-\frac{1}{\sigma}\frac{(\rho-\sigma\lambda)}{\rho-\lambda}\right)}{\partial\lambda} > 0$.

²³This qualitative result is in line with the ‘Home Bias in Trade Puzzle’ in Obstfeld and Rogoff (2000).

Intuitively, with some extent of producer-currency pricing, a nominal exchange rate depreciation makes Foreign goods more expensive in the Home country and Home goods cheaper in the Foreign country, leading to inflation in the former and deflation in the latter. However, this change in relative price levels is weakened by trade costs since they make all imports less attractive in the first place. In other words, trade costs act like a buffer that impairs the integration of international goods markets.

The relationship between long-run relative consumption and short-run relative consumption can be expressed as

$$\widehat{C}_{t+1} - \widehat{C}_{t+1}^* = \widehat{C}_t - \widehat{C}_t^* - (s + \lambda(1 - s))\widehat{e}_t \quad (45)$$

With no trade costs ($\lambda = 0$) and producer-currency pricing ($s = 0$) the short-run change in relative consumption is carried over into the long run. With full pricing to market ($s = 1$) and for a given exchange rate movement, the effect on relative long-run consumption is the same in the presence and in the absence of trade costs.

Taking log-linear approximations of the period t goods market and labour market clearing conditions as well as of the countries' budget constraints²⁴ leads to an expression involving the short-run exchange rate response and the movement of short-run relative consumption

$$\widehat{e}_t = \frac{(\widehat{C}_t - \widehat{C}_t^*) + \frac{1}{(1-\lambda)} \frac{2\beta}{PC^w} dF_t + \frac{(dG_t - dG_t^*)}{C^w}}{(1-s)(\rho - 1 + \rho\lambda) + s} \quad (46)$$

To get the intuition behind this expression, look at its denominator and remember from (44) that with local-currency pricing, the price indices are unaffected by exchange rate movements. In that case an exchange rate depreciation does not change the relative prices of goods and hence demand is unaffected for given dF_t and $(dG_t - dG_t^*)$. However, there is an improvement in the Home firms' profits²⁵ and therefore in the Home country's income. The reason is that due to the depreciation the revenue of Home firms that is generated in Foreign currency in the Foreign country has increased when measured in Home currency. Similarly, there is a deterioration in the Foreign country's income. Hence, this is referred to as the '*income-switching effect*'. As a consequence the boost in Home income increases relative short-run Home consumption.

With producer-currency pricing, relative prices do change and demand is shifted to Home goods in the course of a nominal exchange rate depreciation. Subsequently this is referred to as the '*demand-switching effect*'. Then, again for given dF_t and $(dG_t - dG_t^*)$, relative Home consumption also increases due to the increase in Home production and thereby Home income, but now this increase is dependent on the magnitude of ρ , which is the elasticity of substitution between goods. So for a given nominal exchange rate depreciation, a higher ρ means a stronger demand increase for Home goods and thus a bigger rise in Home relative to Foreign consumption.

²⁴Cf. Appendix C for details.

²⁵Cf. equation (17).

Inserting (39) and (45) into (46) eliminates the trade balance variable $d F_t$, so

$$\widehat{e}_t = \frac{\left\{ \begin{aligned} &\left(1 + \frac{\sigma}{r} - \lambda \left(1 + \frac{\sigma}{r\rho}\right)\right) (\widehat{C}_t - \widehat{C}_t^*) + (1 - \lambda) \frac{(d G_t - d G_t^*)}{C^w} \\ &+ \left(\frac{1}{r} - \frac{\sigma\lambda}{r\rho}\right) \frac{(d G_{t+1} - d G_{t+1}^*)}{C^w} \end{aligned} \right\}}{\left\{ \begin{aligned} &(1 - s) \left[(\rho - 1) + \lambda \left(1 + \frac{\sigma}{r}\right) - \lambda^2 \left(\rho + \frac{\sigma}{r\rho}\right) \right] \\ &+ s \left[1 + \frac{\sigma}{r} - \lambda \left(1 + \frac{\sigma}{r\rho}\right) \right] \end{aligned} \right\}} \quad (47)$$

Finally, using money market clearing conditions as well as (41), (42) and (45) and assuming that money shocks are permanent, i.e. $(\widehat{M}_t - \widehat{M}_t^*) = (\widehat{M}_{t+1} - \widehat{M}_{t+1}^*)$, one can show how nominal exchange rate movements are dependent on monetary shocks and government spending shocks

$$\widehat{e}_t = \frac{\left\{ \begin{aligned} &\left(1 + \frac{\sigma}{r} - \lambda \left(1 + \frac{\sigma}{r\rho}\right)\right) \epsilon (\widehat{M}_t - \widehat{M}_t^*) + (1 - \lambda) \frac{(d G_t - d G_t^*)}{C^w} \\ &+ \frac{1}{r} \left(1 - \frac{\lambda\sigma}{\rho}\right) \frac{(d G_{t+1} - d G_{t+1}^*)}{C^w} \end{aligned} \right\}}{\left\{ \begin{aligned} &(1 - s) \left[\begin{aligned} &\epsilon \left(1 + \frac{\sigma}{r}\right) + (\rho - 1) - \epsilon\lambda \left(\left(1 + \frac{\sigma}{r}\right) \frac{(\epsilon - 1)r}{(\epsilon r + 1)} + \left(1 + \frac{\sigma}{r\rho}\right) \right) \\ &+ \lambda^2 \left(\left(1 + \frac{\sigma}{r\rho}\right) \frac{(\epsilon^2 r + 1)}{(\epsilon r + 1)} - \left(\rho + \frac{\sigma}{r\rho}\right) \right) \end{aligned} \right] \\ &+ s \left[\left(1 + \frac{\sigma}{r}\right) - \lambda \left(1 + \frac{\sigma}{r\rho}\right) \right] \frac{\epsilon(1+r)}{(\epsilon r + 1)} \end{aligned} \right\}} \quad (48)$$

With full pricing to market both a positive Home monetary shock and a positive Home government spending shock, either in period t or in period $t + 1$, lead to a nominal exchange rate depreciation, and vice versa for positive Foreign shocks. With producer-currency pricing, however, the effect of shocks is in principle ambiguous²⁶ and depends on parameter values but will generally also lead to a depreciation. Note that this ambiguity does not arise without trade costs. These features will be discussed in more detail in the following section.

5 Sticky prices and other key variables

Apart from the effects of shocks on the nominal exchange rate the reactions of other key economic variables are examined in this section, both for the short and the long run. In particular, the focus is on the two extreme cases of full pricing to market, i.e. local-currency pricing ($s = 1$), and of no pricing to market, i.e. producer-currency pricing ($s = 0$). In the latter case, there is complete pass-through of the exchange rate to consumer prices.

Due to the log-linear approximation methodology monetary shocks and government spending shocks are additive and thus can be analysed separately. The reactions of the nominal exchange rate and other key variables to the two types of shocks in the presence of trade costs are then compared to the ones without trade costs. Derivations can be found in Appendix C. A numerical evaluation of the effects is provided in Section 7.

²⁶The ambiguity critically hinges on the magnitudes of ϵ and τ .

5.1 The long-run response to shocks

It can be shown²⁷ that

$$\widehat{C}_{t+1}^w = -\frac{1}{2} \left(\frac{\rho-1}{\rho-1+\rho\eta} \right) \left(\frac{dG_{t+1}}{C} + \frac{dG_{t+1}^*}{C^*} \right) \quad (49)$$

where $\widehat{C}_{t+1}^w = \frac{1}{2}(\widehat{C}_{t+1} + \widehat{C}_{t+1}^*)$. Thus, the average change in world consumption is not dependent on monetary shocks.

$$\widehat{C}_{t+1} = \frac{\rho}{(\rho-\lambda)} \left[\begin{array}{l} + \left\{ -\left(\frac{\rho-1+\frac{1}{2}\eta}{\rho-1+\rho\eta} \right) + \frac{1}{2} \frac{\lambda}{\rho} \left(\frac{2(\rho-1)+\rho\eta}{\rho-1+\rho\eta} \right) \right\} \frac{dG_{t+1}}{C} \\ + \left\{ \left(\frac{\frac{1}{2}\eta}{\rho-1+\rho\eta} \right) - \frac{1}{2} \frac{\lambda}{\rho} \left(\frac{\rho\eta}{\rho-1+\rho\eta} \right) \right\} \frac{dG_{t+1}^*}{C^*} \end{array} \right] \quad (50)$$

$$\widehat{C}_{t+1}^* = \frac{\rho}{(\rho-\lambda)} \left[\begin{array}{l} + \left\{ \left(\frac{\frac{1}{2}\eta}{\rho-1+\rho\eta} \right) - \frac{1}{2} \frac{\lambda}{\rho} \left(\frac{\rho\eta}{\rho-1+\rho\eta} \right) \right\} \frac{dG_{t+1}}{C} \\ + \left\{ -\left(\frac{\rho-1+\frac{1}{2}\eta}{\rho-1+\rho\eta} \right) + \frac{1}{2} \frac{\lambda}{\rho} \left(\frac{2(\rho-1)+\rho\eta}{\rho-1+\rho\eta} \right) \right\} \frac{dG_{t+1}^*}{C^*} \end{array} \right] \quad (51)$$

$$\widehat{h}_{t+1} = \left(\frac{\rho\eta}{\rho-1+\rho\eta} \right) \left(-r \frac{\beta dF_t}{PC} + \frac{dG_{t+1}}{C} \right) \quad (52)$$

$$\widehat{h}_{t+1}^* = \left(\frac{\rho\eta}{\rho-1+\rho\eta} \right) \left(r \frac{\beta dF_t}{PC} + \frac{dG_{t+1}^*}{C^*} \right) \quad (53)$$

The steady state interest rate $r = \frac{1-\beta}{\beta}$ does not change in the long run, i.e.

$$\widehat{r}_{t+1} = 0 \quad (54)$$

5.2 Monetary shocks

This section focuses on monetary shocks, i.e. government spending is set to zero $dG_t = dG_t^* = dG_{t+1} = dG_{t+1}^* = 0$. As it is obvious from (48), if both countries face monetary shocks of the same extent ($\widehat{M}_t = \widehat{M}_t^* \neq 0$), these two effects exactly offset each other. Since in this special case real variables remain unaffected, it is not considered hereafter. Instead, it is assumed for simplicity that there is a permanent Home monetary shock only ($\widehat{M}_t > 0$, $\widehat{M}_t^* = 0$).

Assuming full pricing to market, i.e. $s = 1$, the expression for the exchange rate movement given in (48) simplifies to

$$\widehat{e}_t = \widehat{e}_{t,\tau=0} = \left(\frac{\epsilon + 1/r}{1 + 1/r} \right) \widehat{M}_t \quad (55)$$

for any τ . Hence, with full pricing to market, it does not matter for the behaviour of the nominal exchange rate whether there are trade costs.

²⁷The derivations are outlined in Appendix C.

In the case of producer-currency pricing, i.e. $s = 0$, the effect of an expansionary Home monetary shock on the nominal exchange rate is

$$\widehat{e}_t = \begin{bmatrix} \left(1 + \frac{\sigma}{r} - \lambda \left(1 + \frac{\sigma}{r\rho}\right)\right) \epsilon \widehat{M}_t / \\ \epsilon \left(1 + \frac{\sigma}{r}\right) + (\rho - 1) - \epsilon \lambda \left(\left(1 + \frac{\sigma}{r}\right) \frac{(\epsilon-1)r}{(\epsilon r+1)} + \left(1 + \frac{\sigma}{r\rho}\right) \right) \\ - \lambda^2 \left(\left(\rho + \frac{\sigma}{r\rho}\right) - \left(1 + \frac{\sigma}{r\rho}\right) \frac{(\epsilon^2 r+1)}{(\epsilon r+1)} \right) \end{bmatrix} \quad (56)$$

which is in principle ambiguous in the presence of trade costs. However, as described in Section 7, within the range of empirically plausible parameter values, a Home monetary expansion clearly leads to a depreciation. Moreover, this depreciation appears more pronounced with trade costs, i.e. $\widehat{e}_t > \widehat{e}_{t,\tau=0}$.

Intuitively, an increase in the level of Home money supply depreciates the Home currency. This depreciation leads to the demand-switching effect mentioned in relation to (46) because it decreases the relative price of Home goods in both countries. Now due to containment²⁸ trade costs diminish this demand-switching effect in the Foreign country since they make Home goods less attractive there, whereas they amplify the demand-switching effect in the Home country. In total, the containment effect reinforces²⁹ the demand-switching effect because the level of demand for Home goods is lower in the Foreign country than in the Home country. Hence, in the presence of trade costs a Home monetary expansion leads to a larger depreciation.

In the case of full pricing to market the income-switching effect of the depreciation is weaker due to the containment effect. However, also due to the containment effect, the increased income is then predominantly spent on Home goods so that the two offsetting effects exactly cancel each other.

Combining (41) and (42) yields the real exchange movement

$$\widehat{\psi}_t = \widehat{e}_t + \widehat{P}_t^* - \widehat{P}_t = (s + \lambda(1-s))\widehat{e}_t \quad (57)$$

In the case of a Home monetary expansion the real and nominal exchange rates depreciate in lockstep for any τ under full pricing to market ($s = 1$) since the price indices are unaffected by shocks. For producer currency pricing ($s = 0$), however, the real exchange rate is stable with no trade costs but still depreciates in the presence of trade costs.

Similarly, it can be shown that in the presence of trade costs there is always exchange rate overshooting for any degree of pricing to market as long as $\epsilon > 1$, i.e. the consumption elasticity of money demand, represented by $1/\epsilon$, is smaller than one

$$\widehat{e}_{t+1} - \widehat{e}_t = -\frac{1}{\epsilon} \left[s \left(\frac{\epsilon-1}{1+1/\epsilon r} \right) + \lambda(1-s) \left(\frac{\epsilon-1}{1+1/\epsilon r} \right) \right] \widehat{e}_t \quad (58)$$

The underlying intuition is the same as in the famous Dornbusch (1976) overshooting model, namely that overshooting occurs when money demand is sufficiently inelastic with respect to output and consumption. However, without

²⁸Cf. Section 4.1 for an explanation of the containment effect.

²⁹This is true for empirically plausible parameter values, cf. Table 2 in Section 7.

trade costs a monetary shock cannot display exchange rate overshooting if $s = 0$, whereas this does happen with trade costs due to the reinforcing influence of containment on the demand-switching effect.

Since interest parity must obtain in this model, overshooting implies that the ensuing appreciation must be compensated by a relatively higher interest rate in the Foreign country. Real interest rates move according to

$$\hat{r}_t = -\frac{(1-r)}{r} \left(\hat{C}_t^w + \frac{1}{2}(s + \lambda(1-s))\hat{e}_t \right) = -\frac{(1-r)}{r} \left(\hat{C}_t^w + \frac{1}{2}\hat{\psi}_t \right) \quad (59)$$

$$\hat{r}_t^* = -\frac{(1-r)}{r} \left(\hat{C}_t^w - \frac{1}{2}(s + \lambda(1-s))\hat{e}_t \right) = -\frac{(1-r)}{r} \left(\hat{C}_t^w - \frac{1}{2}\hat{\psi}_t \right) \quad (60)$$

where

$$\hat{C}_t^w = \hat{C}_{t,\tau=0}^w = \frac{1}{2} \left(\frac{\epsilon + 1/r}{1 + 1/r} \right) \hat{M}_t \quad (61)$$

denotes the average change in world consumption in period t , i.e. $\hat{C}_t^w = \frac{1}{2}(\hat{C}_t + \hat{C}_t^*)$. Thus, a Home monetary expansion ($\hat{M}_t > 0$, $\hat{M}_t^* = 0$) always³⁰ reduces the Home real interest rate, whereas the effect on the Foreign real interest rate is ambiguous, i.e. $\hat{r}_t < 0$ and $\hat{r}_t^* \stackrel{\leq}{\geq} 0$ with $\hat{r}_t < \hat{r}_t^*$. For $s = 1$ it follows that $\hat{r}_t = \hat{r}_{t,\tau=0} < 0$ and $\hat{r}_t^* = \hat{r}_{t,\tau=0}^* = 0$, i.e. under full pricing to market the Home real interest rate declines and the Foreign real interest rate is left unaffected irrespective of trade costs. For $s = 0$ it follows that $\hat{r}_{t,\tau=0} = \hat{r}_{t,\tau=0}^* < 0$ and $\hat{r}_t < \hat{r}_{t,\tau=0} < \hat{r}_t^* < 0$. So without trade costs producer-currency pricing leads to the same decline in real interest rates for both countries, but in the presence of trade costs the decline in the Home real interest rate is exacerbated whereas the decline in the Foreign real interest rate is mitigated. Overall it can therefore be concluded that trade costs introduce an asymmetry to real interest rates across countries in response to a monetary shock.

Consumption can be shown to behave according to

$$\hat{C}_t = \hat{C}_t^w + \frac{1}{2} \left((1-s) \frac{(\rho-1) + \lambda \left(1 + \frac{\sigma}{r}\right) - \lambda^2 \left(\rho + \frac{\sigma}{r\rho}\right)}{\left(1 + \frac{\sigma}{r}\right) - \lambda \left(\rho + \frac{\sigma}{r\rho}\right)} + s \right) \hat{e}_t \quad (62)$$

$$\hat{C}_t^* = \hat{C}_t^w - \frac{1}{2} \left((1-s) \frac{(\rho-1) + \lambda \left(1 + \frac{\sigma}{r}\right) - \lambda^2 \left(\rho + \frac{\sigma}{r\rho}\right)}{\left(1 + \frac{\sigma}{r}\right) - \lambda \left(\rho + \frac{\sigma}{r\rho}\right)} + s \right) \hat{e}_t \quad (63)$$

For empirically plausible parameter values³¹ Home consumption always goes up in response to a Home monetary expansion ($\hat{M}_t > 0$, $\hat{M}_t^* = 0$). The effect on Foreign consumption is ambiguous, but $\hat{C}_t > \hat{C}_t^*$ always holds for empirically plausible magnitudes³¹ irrespective of trade costs.

³⁰Recall the assumption $\beta > \frac{1}{2}$.

³¹Cf. Section 7 for a discussion of the chosen parameter values.

In the special case of $s = 1$ when $\widehat{e}_t = \widehat{e}_{t,\tau=0}$ holds it follows that

$$\widehat{C}_t = \widehat{C}_{t,\tau=0} = \left(\frac{\epsilon + 1/r}{1 + 1/r} \right) \widehat{M}_t \quad (64)$$

$$\widehat{C}_t^* = \widehat{C}_{t,\tau=0}^* = 0 \quad (65)$$

The intuition is that with full pricing to market, there are two effects of a Home monetary expansion in the Foreign country that exactly offset each other. As a direct effect the income-switching reduces Foreign firms' profits. As an indirect effect there is an increase in Home spending on Foreign goods as a consequence of the income-switching, which pushes Foreign firms' profits up again. Hence, in terms of consumption, under full pricing to market the Foreign country is insulated from events in the Home money market and there is no spillover effect on Foreign consumption³². However, if $s = 0$ and also if $\epsilon = 1$ it can be shown that a Home monetary expansion raises consumption in both countries for empirically plausible parameter values³¹, both with and without trade costs, so that there is a spillover effect.

Output moves according to

$$\begin{aligned} \widehat{h}_t = & \widehat{C}_t^w + \frac{1}{2}(1-s)\rho\widehat{e}_t \\ & + \frac{1}{2}\lambda \left[(1-s) \left(\frac{(\rho-1)+\lambda(1+\frac{\sigma}{r})-\lambda^2(\frac{\rho+\frac{\sigma}{r\rho}})}{(1+\frac{\sigma}{r})-\lambda(\frac{\rho+\frac{\sigma}{r\rho}})} - \lambda\rho \right) + s \right] \widehat{e}_t \end{aligned} \quad (66)$$

$$\begin{aligned} \widehat{h}_t^* = & \widehat{C}_t^w - \frac{1}{2}(1-s)\rho\widehat{e}_t \\ & - \frac{1}{2}\lambda \left[(1-s) \left(\frac{(\rho-1)+\lambda(1+\frac{\sigma}{r})-\lambda^2(\frac{\rho+\frac{\sigma}{r\rho}})}{(1+\frac{\sigma}{r})-\lambda(\frac{\rho+\frac{\sigma}{r\rho}})} - \lambda\rho \right) + s \right] \widehat{e}_t \end{aligned} \quad (67)$$

With $s = 1$ a Home monetary expansion leads to

$$0 < \widehat{h}_t = \widehat{C}_t^w + \frac{1}{2}\lambda\widehat{e}_t = (1+\lambda)\frac{1}{2} \left(\frac{\epsilon + 1/r}{1 + 1/r} \right) \widehat{M}_t > \widehat{C}_t^w = \widehat{h}_{t,\tau=0} > 0 \quad (68)$$

$$0 < \widehat{h}_t^* = \widehat{C}_t^w - \frac{1}{2}\lambda\widehat{e}_t = (1-\lambda)\frac{1}{2} \left(\frac{\epsilon + 1/r}{1 + 1/r} \right) \widehat{M}_t < \widehat{C}_t^w = \widehat{h}_{t,\tau=0}^* > 0 \quad (69)$$

This is a direct consequence of the income-switching effect and the containment effect. The Home monetary expansion and the ensuing depreciation boost Home income so that Home spending increases. This additional spending is evenly spread over the two countries in the absence of trade costs, leading to the same output rise in both countries. However, the containment effect of trade costs channels a greater share of that spending towards Home goods so that the symmetry vanishes, i.e. $\widehat{h}_t > \widehat{h}_t^*$. If $s = 0$ the results again depend on parameter

³²In the case of $s = 1$ note that a higher ϵ leads to a higher \widehat{C}_t^w and \widehat{e}_t and thus a higher \widehat{C}_t . Intuitively, the reason for the rise is the consumption elasticity of money demand, given by $1/\epsilon$ in (10). Home consumption must increase more strongly in response to a Home monetary expansion when ϵ is higher. As can be seen from (55), this is achieved with a stronger depreciation of the Home currency, leading to higher Home income and thus higher Home consumption.

values but $\widehat{h}_t > \widehat{h}_t^*$ always holds for empirically plausible magnitudes³¹, both with and without trade costs.

As discussed with (39), the asset positions induced by a period t change in the trade balance are carried over to the long-run equilibrium, i.e. $dF_{t+1} = dF_t$.

$$\frac{\beta dF_t}{PC^w} = \frac{1}{2} \frac{\sigma}{r} \frac{(\rho - \lambda)}{\rho} \left((1 - s) \frac{(\rho - 1)(1 - \lambda^2)}{(1 + \frac{\sigma}{r}) - \lambda(1 + \frac{\sigma}{r\rho})} \right) \widehat{e}_t \quad (70)$$

With full pricing to market there is no effect of a Home monetary expansion on the Home trade balance and therefore, via (39), $(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*) = 0$, i.e. in the long run consumption reverts to the initial equilibrium level in both countries. Intuitively, as mentioned with (64) and (65), the income-switching reduces Foreign firms' profits, but this is exactly offset by the increase in Home spending on Foreign goods and the ensuing increase in Foreign production in (69). Thus, from (60) and (65) the Foreign real interest rate and Foreign consumption do not change so that in total the Foreign country does not need to borrow from the Home country in order to smooth its consumption path. However, from (59) the Home real interest rate drops, giving Home consumers an incentive to consume more in the short run. This can be seen from (45) by setting $s = 1$

$$\widehat{C}_{t+1} - \widehat{C}_{t+1}^* = 0 = \widehat{C}_t - \widehat{e}_t$$

Hence, the short-run Home consumption rise is proportional to the depreciation caused by a Home monetary expansion. Yet in the long run this increase in consumption cannot be sustained³³ since there is no permanent wealth effect from the Home monetary expansion, or - as Betts and Devereux (2000) put it - "after the first period there is full money neutrality". This result holds both with and without trade costs.

For $s < 1$ the right-hand side of expression (70) is unambiguously positive in the case of an exchange rate depreciation and therefore leads to higher relative long-run consumption for the Home country, i.e. $(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*) > 0$. The extent of this effect is in general different with trade costs, depending on parameter values, but not significantly so.

Table 1 summarises the above discussion. In combination with monetary shocks trade costs tend to increase the variability of the nominal and real exchange rates. Moreover, they make overshooting of the nominal exchange rate more likely and introduce asymmetries to real interest rates and output across countries. The effects on short and long-run consumption variability and on the trade balance are not significantly altered by trade costs.

³³See equation (50).

Table 1 The effect of a permanent Home monetary expansion

$\widehat{M} > 0, \widehat{M}^* = 0$	Full PTM ($s = 1$)	No PTM ($s = 0$)
Nom. exch. rate	$\widehat{e}_t = \widehat{e}_{t,\tau=0} > 0$	$\widehat{e}_t > \widehat{e}_{t,\tau=0} > 0^*$
Real exch. rate	$\widehat{\psi}_t = \widehat{\psi}_{t,\tau=0} > 0$	$\widehat{\psi}_t > \widehat{\psi}_{t,\tau=0} = 0$
Overshooting (\sim)	$(\sim) = (\sim)_{\tau=0} > 0^*$	$(\sim) > (\sim)_{\tau=0} = 0^*$
Real interest rates	$\widehat{r}_t = \widehat{r}_{t,\tau=0} < 0$ $\widehat{r}_t^* = \widehat{r}_{t,\tau=0}^* = 0$	$\widehat{r}_t < \widehat{r}_{t,\tau=0} < \widehat{r}_t^* < 0$ $\widehat{r}_{t,\tau=0} = \widehat{r}_{t,\tau=0}^* < 0$
World cons.	$\widehat{C}_t^w = \widehat{C}_{t,\tau=0}^w > 0$	$\widehat{C}_t^w = \widehat{C}_{t,\tau=0}^w > 0$
Home cons.	$\widehat{C}_t = \widehat{C}_{t,\tau=0} > 0$	$\widehat{C}_t > 0, \widehat{C}_{t,\tau=0} > 0^*$
Foreign cons.	$\widehat{C}_t^* = \widehat{C}_{t,\tau=0}^* = 0$	$\widehat{C}_t^* > 0, \widehat{C}_{t,\tau=0}^* > 0^*$
Output	$\widehat{h}_t > \widehat{h}_t^*, \widehat{h}_t > 0$ $\widehat{h}_{t,\tau=0} = \widehat{h}_{t,\tau=0}^* > 0$	$\widehat{h}_t > \widehat{h}_t^*$ $\widehat{h}_{t,\tau=0} > \widehat{h}_{t,\tau=0}^*$
Home trade bal.	$dF_t = dF_{t,\tau=0} = 0$	$dF_t > 0, dF_{t,\tau=0} > 0$
Long-run rel. cons.	$(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*) = 0$	$(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*) > 0$

*For empirically empirically plausible parameter values³¹.

5.3 Government spending shocks

This section focuses on government spending shocks and rules out monetary shocks ($\widehat{M}_t = \widehat{M}_t^* = 0$). As it is obvious from (48), if both governments increase their spending by the same extent at any time ($dG_t = dG_t^* \neq 0$ and/or $dG_{t+1} = dG_{t+1}^* \neq 0$), these two effects exactly offset each other. It is therefore assumed that there is no government spending in the Foreign country ($dG_t^* = dG_{t+1}^* = 0$). Due to the symmetry of the model it is straightforward to derive the effects of the opposite scenario.

Furthermore, there are three different scenarios with government spending - transitory government spending shocks ($dG_t > 0, dG_{t+1} = 0$), anticipated government spending shocks ($dG_t = 0, dG_{t+1} > 0$) and permanent government spending shocks ($dG_t = dG_{t+1} > 0$), which are the sum of transitory and anticipated government spending shocks³⁴. Note that since in period $t + 1$ the new long-run equilibrium is reached, a government spending shock occurring in that period by definition means that it persists in all subsequent periods in the absence of further shocks³⁵.

Turning to transitory government spending shocks first, from (48) full pricing to market ($s = 1$) brings about

$$\widehat{e}_t = \frac{(1 - \lambda) \frac{dG_t}{C_t^w}}{\left(\left(1 + \frac{\sigma}{r}\right) - \lambda \left(1 + \frac{\sigma}{r\rho}\right) \right) \left(\frac{1+1/r}{1+1/\epsilon r} \right)} \quad (71)$$

³⁴Remember that due to the log-linear approximation methodology all shocks are additive and thus can be analysed separately.

³⁵In that respect an anticipated government spending shock can be regarded as 'permanent' in the sense of 'persistent' as well. However, in this paper's terminology 'permanent' means that a government spending shock already occurs in period t and then in all subsequent periods, not only in period $t + 1$ and all subsequent periods.

Since in the denominator $\left[\left(1 + \frac{\sigma}{r}\right) - \lambda \left(1 + \frac{\sigma}{r\rho}\right) \right] > \left(1 + \frac{\sigma}{r}\right) (1 - \lambda)$ it follows that $\hat{e}_t < \hat{e}_{t,\tau=0}$. As the price indices are unaffected by shocks with full pricing to market, due to (57) this result means that both the nominal and the real exchange rate are less volatile with trade costs. The intuition is that with trade costs the containment effect tends to keep Home government spending in the Home country. Therefore, with an increase in Home government spending ($dG_t > 0$), the resulting relative demand for Home goods is higher in the presence of trade costs such that Home income is boosted relative to Foreign income. Thus, the required income-switching effect is smaller than in the absence of trade costs and can be achieved with a more moderate exchange rate depreciation. Hence, in the presence of trade costs and full pricing to market exchange rate adjustments caused by transitory government spending are less pronounced³⁶.

An anticipated government spending shock in the case of full pricing to market also causes an exchange rate depreciation

$$\hat{e}_t = \frac{\frac{1}{r} \left(1 - \frac{\lambda\sigma}{\rho}\right) \frac{dG_{t+1}}{C^w}}{\left[\left(1 + \frac{\sigma}{r}\right) - \lambda \left(1 + \frac{\sigma}{r\rho}\right) \right] \left(\frac{1+1/r}{1+1/\epsilon r} \right)} \quad (72)$$

Again this depreciation is less pronounced in the presence of trade costs³⁶ ($\hat{e}_t < \hat{e}_{t,\tau=0}$) since in the denominator $\left[\left(1 + \frac{\sigma}{r}\right) - \lambda \left(1 + \frac{\sigma}{r\rho}\right) \right] > \left(1 + \frac{\sigma}{r}\right) (1 - \lambda)$ and, comparing this with the numerator, $(1 - \lambda) < \left(1 - \frac{\lambda\sigma}{\rho}\right)$. The intuition for an anticipated government spending shock is the same as with a transitory shock. However, as Section 7 shows numerically, the exchange rate depreciation is much stronger in the case of an anticipated Home fiscal expansion. The reason is that in period $t+1$ the new long-run equilibrium is reached, which means that the Home government maintains its higher spending in all subsequent periods. Accordingly, Home individuals anticipate the future higher burden on their budget constraints via (5) and (13) and a more vigorous income-switching effect is needed compared to a transitory government spending shock.

Since the effect of a permanent government spending shock is simply the sum of the transitory and the anticipated shock, the above results go through for a permanent shock. Hence, when there is full pricing to market, trade costs unambiguously³⁶ reduce the variability of the nominal exchange rate for any type of government spending shock since they act like a buffer that contains spending in its country of origin. As a consequence of that containment effect the required income-switching effect is smaller and exchange rate movements are toned down.

The effects of government spending shocks with no pricing to market ($s = 0$) are more intricate. From (48) the exchange rate movement caused by a

³⁶Note that this conclusion does not depend on parameter values.

transitory fiscal expansion equals

$$\widehat{e}_t = \frac{(1 - \lambda) \frac{dG_t}{C^w}}{\left[\begin{aligned} &\epsilon \left(1 + \frac{\sigma}{r}\right) + (\rho - 1) - \lambda \epsilon \left(\left(1 + \frac{\sigma}{r}\right) \frac{(\epsilon - 1)r}{(\epsilon r + 1)} + \left(1 + \frac{\sigma}{r\rho}\right) \right) \\ &+ \lambda^2 \left(\left(1 + \frac{\sigma}{r\rho}\right) \frac{(\epsilon^2 r + 1)}{(\epsilon r + 1)} - \left(\rho + \frac{\sigma}{r\rho}\right) \right) \end{aligned} \right]} \quad (73)$$

The sign of this expression is in principle ambiguous in the presence of trade costs. However, as described in Section 7, within the range of empirically plausible parameter values³¹, a transitory Home fiscal expansion leads to a depreciation and, like with full pricing to market, is considerably less pronounced with trade costs, i.e. $\widehat{e}_t < \widehat{e}_{t,\tau=0}$. However, the underlying intuition³⁷ is different. The depreciation decreases the relative price of Home goods in both countries, leading to the demand-switching effect mentioned in relation to (46). The containment effect of trade costs amplifies the demand-switching effect in the Home country, whereas it diminishes this demand-switching effect in the Foreign country since it makes Home goods less attractive there. But the level of demand for Home goods is lower in the Foreign country than in the Home country so that in total, the containment effect reinforces the demand-switching effect. Therefore, a comparatively small depreciation is enough to boost the Home economy and finance the government spending.

An anticipated Home fiscal expansion with no pricing to market can in principle also lead to an ambiguous exchange rate movement

$$\widehat{e}_t = \frac{\frac{1}{r} \left(1 - \frac{\lambda\sigma}{\rho}\right) \frac{dG_{t+1}}{C^w}}{\left[\begin{aligned} &\epsilon \left(1 + \frac{\sigma}{r}\right) + (\rho - 1) - \lambda \epsilon \left(\left(1 + \frac{\sigma}{r}\right) \frac{(\epsilon - 1)r}{(\epsilon r + 1)} + \left(1 + \frac{\sigma}{r\rho}\right) \right) \\ &+ \lambda^2 \left(\left(1 + \frac{\sigma}{r\rho}\right) \frac{(\epsilon^2 r + 1)}{(\epsilon r + 1)} - \left(\rho + \frac{\sigma}{r\rho}\right) \right) \end{aligned} \right]} \quad (74)$$

But within the range of empirically plausible parameter values³¹ it clearly leads to a depreciation. However, it is likely to be larger than without trade costs, i.e. $\widehat{e}_t > \widehat{e}_{t,\tau=0}$.³⁷ Like with full pricing to market the exchange rate depreciation is much stronger numerically in the case of an anticipated Home fiscal expansion due to the fact that period $t + 1$ government spending is maintained through all subsequent periods. The permanent government spending shock as a combination of transitory and anticipated fiscal policy is therefore dominated by the latter.

A government spending shock can only have an impact on period t world average consumption if the shock is permanent

$$\widehat{C}_t^w = \widehat{C}_{t,\tau=0}^w = - \frac{\epsilon - 1}{\epsilon(1 + r)} \frac{\rho - 1}{\rho - 1 + \rho\eta} \frac{1}{2} \frac{dG_{t+1}}{C^w} \quad (75)$$

Also note that there is no effect unless $\epsilon \neq 1$. Intuitively, it can be seen from (10) that a higher price level, caused by the depreciation, reduces the real monetary

³⁷Note that this intuition is also dependent on the parameters and is particularly sensitive to the effects of a depreciation on consumption and to the consumption elasticity of money demand, represented by $1/\epsilon$.

base of the Home economy. The bigger ϵ is, i.e. the more inelastic money demand is with respect to consumption, the bigger is the reduction in consumption that is necessary for a money market equilibrium³⁸.

$$\widehat{C}_t = \widehat{C}_t^w + \frac{1}{2} \left[\begin{aligned} & \left((1-s) \frac{(\rho-1) + \lambda(1+\frac{\sigma}{r}) - \lambda^2(\rho + \frac{\sigma}{r\rho})}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} + s \right) \widehat{e}_t \\ & - \frac{(1-\lambda) \frac{dG_t}{C^w} + \frac{1}{r} (1 - \frac{\lambda\sigma}{\rho}) \frac{dG_{t+1}}{C^w}}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} \end{aligned} \right] \quad (76)$$

$$\widehat{C}_t^* = \widehat{C}_t^w - \frac{1}{2} \left[\begin{aligned} & \left((1-s) \frac{(\rho-1) + \lambda(1+\frac{\sigma}{r}) - \lambda^2(\rho + \frac{\sigma}{r\rho})}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} + s \right) \widehat{e}_t \\ & - \frac{(1-\lambda) \frac{dG_t}{C^w} + \frac{1}{r} (1 - \frac{\lambda\sigma}{\rho}) \frac{dG_{t+1}}{C^w}}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} \end{aligned} \right] \quad (77)$$

$$\begin{aligned} \widehat{h}_t &= \widehat{C}_t^w + \frac{1}{2}(1-s)\rho\widehat{e}_t + \frac{1}{2} \frac{dG_t}{C^w} \\ &+ \frac{1}{2}\lambda \left[(1-s) \left(\frac{(\rho-1) + \lambda(1+\frac{\sigma}{r}) - \lambda^2(\rho + \frac{\sigma}{r\rho})}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} - \lambda\rho \right) + s \right] \widehat{e}_t \\ &- \frac{1}{2}\lambda \left[\frac{\frac{1}{r}(\frac{\lambda\sigma}{\rho} - \sigma) \frac{dG_t}{C^w} + \frac{1}{r} (1 - \frac{\lambda\sigma}{\rho}) \frac{dG_{t+1}}{C^w}}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right] \end{aligned} \quad (78)$$

$$\begin{aligned} \widehat{h}_t^* &= \widehat{C}_t^w - \frac{1}{2}(1-s)\rho\widehat{e}_t + \frac{1}{2} \frac{dG_t}{C^w} \\ &- \frac{1}{2}\lambda \left[(1-s) \left(\frac{(\rho-1) + \lambda(1+\frac{\sigma}{r}) - \lambda^2(\rho + \frac{\sigma}{r\rho})}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} - \lambda\rho \right) + s \right] \widehat{e}_t \\ &+ \frac{1}{2}\lambda \left[\frac{\frac{1}{r}(\frac{\lambda\sigma}{\rho} - \sigma) \frac{dG_t}{C^w} + \frac{1}{r} (1 - \frac{\lambda\sigma}{\rho}) \frac{dG_{t+1}}{C^w}}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right] \end{aligned} \quad (79)$$

$$\frac{\beta dF_t}{PC^w} = \frac{1}{2} \frac{1}{\left(\frac{1-\lambda}{1-\frac{\lambda}{\rho}} \right) + \frac{\sigma}{r}} \left(\begin{aligned} & (1-s) \frac{\sigma}{r} (\rho-1) (1-\lambda^2) \widehat{e}_t \\ & - \frac{\sigma}{r} (1-\lambda) \frac{dG_t}{C^w} + \frac{(1-\lambda)}{r} \left(\frac{1-\frac{\lambda\sigma}{\rho}}{1-\frac{\lambda}{\rho}} \right) \frac{dG_{t+1}}{C^w} \end{aligned} \right) \quad (80)$$

6 Welfare analysis

The advantage of this model's utility-based approach is that it allows for a rigorous welfare analysis. Thus, the international welfare spillover effects of shocks can be examined. The methodology of Obstfeld and Rogoff (1995) is adopted such that the utility function (1) is decomposed as $U_t = U_t^R + U_t^M$, where U_t^R consists of the terms depending on consumption and labour and U_t^M

³⁸The effect on the Foreign price level and hence the Foreign real monetary base is the opposite. However, in terms of world average consumption these two effects do not offset each other because only the Home individuals have to finance the Home government's spending increase via the payments made to the government in (5), where Z_t becomes negative. This further reduces Home consumption and therefore dominates the world average.

incorporates real money balances

$$U_t^R = \sum_{v=t}^{\infty} \beta^{v-t} (\log C_v + \eta \log(1 - h_v)) \quad (81)$$

$$U_t^M = \sum_{v=t}^{\infty} \beta^{v-t} \left(\frac{\gamma}{1-\epsilon} \left(\frac{M_v}{P_v} \right)^{1-\epsilon} \right) \quad (82)$$

As Obstfeld and Rogoff (2000) argue, unless real money balances are assigned an implausibly large weight, γ , the overall welfare effect of a Home shock is dominated by U_t^R . Therefore, U_t^M is neglected in the following. The notation for the Foreign country is analogous. In the following the focus is on the two extreme cases of local-currency pricing ($s = 1$) and producer-currency pricing ($s = 0$). The equations for the intermediate case of $0 < s < 1$ and the derivations of the results can be found in Appendix D.

6.1 Monetary shocks

The welfare impact of a permanent Home monetary expansionary shock is examined³⁹, i.e. $\widehat{M}_t > 0$ and $\widehat{M}_t^* = 0$. For local-currency pricing (full pricing to market) it can be shown that

$$dU_t^R = \frac{\widehat{C}_t^w}{\rho} + \frac{1}{2} \left(1 - \frac{\rho-1}{\rho} \lambda \right) \widehat{e}_t > 0 \quad (83)$$

$$dU_t^{*R} = \frac{\widehat{C}_t^w}{\rho} - \frac{1}{2} \left(1 - \frac{\rho-1}{\rho} \lambda \right) \widehat{e}_t \quad (84)$$

$$= 2 \frac{\widehat{C}_t^w}{\rho} - dU_t^R < 0 \quad (85)$$

This result is in line with Betts and Devereux (2000) in the sense that pricing to market leads to negative international welfare spillover effects, making the Home country better off, whereas the effect on Foreign welfare is negative⁴⁰. Hence, an unexpected Home monetary expansion is a “beggar-thy-neighbour” instrument. However, since $0 < \frac{\rho-1}{\rho} \lambda < 1$ trade costs diminish the asymmetry of the shock⁴¹, reducing the positive effect for the Home country and decreasing the welfare loss of the Foreign country. Only in the limit, with λ close to one, is the welfare effect neutral⁴⁰ for the Foreign country since in that case, trade does not take place and the Foreign country is insulated from the Home money market

$$\lim_{\lambda \rightarrow 1} dU_t^R = \frac{\widehat{C}_t^w}{\rho} + \frac{1}{2\rho} \widehat{e}_t = 2 \frac{\widehat{C}_t^w}{\rho} \quad (86)$$

$$\lim_{\lambda \rightarrow 1} dU_t^{*R} = 0 \quad (87)$$

³⁹Due to the symmetry of the model the impact of a Foreign monetary expansion is analogous.

⁴⁰See Appendix D for the derivation of this result.

⁴¹Recall from (55) and (61) that neither \widehat{e}_t nor \widehat{C}_t^w depend on trade costs.

Intuitively, as can be seen from (68) and (69), trade costs induce Home individuals to work harder than Foreign individuals in response to a Home monetary shock because of the income-switching and containment effects. But as (61), (64) and (65) show, \widehat{C}_t^w and therefore \widehat{C}_t and \widehat{C}_t^* do not depend on trade costs.

Like in the previous sections the analysis under producer-currency pricing depends on parameter values. The changes in utility induced by a permanent Home monetary shock can be expressed as

$$\begin{aligned} dU_t^R &= \frac{\widehat{C}_t^w}{\rho} - \frac{1}{2} \left(\frac{(\rho-1)\frac{\sigma}{r} - \lambda(1+\frac{\sigma}{r}) - ((\rho-1)\lambda - \lambda^2)(\rho + \frac{\sigma}{r\rho})}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} \right) \widehat{e}_t \\ &\quad - \frac{1}{2} \frac{(\rho-1)^2}{\rho} \lambda \left(\frac{1+(1-\lambda)(1+\frac{\sigma}{r})}{(1+\frac{\sigma}{r}) - \lambda^2(\rho + \frac{\sigma}{r\rho})} - 1 \right) \widehat{e}_t \\ &\quad + \frac{1}{r} \frac{1}{2} \left(\sigma - \lambda \frac{(\rho-1)\eta}{\rho(\rho+\eta-1)} \right) \left(\frac{(\rho-1)(1-\lambda^2)}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right) \widehat{e}_t \end{aligned} \quad (88)$$

$$\begin{aligned} dU_t^{*R} &= \frac{\widehat{C}_t^w}{\rho} + \frac{1}{2} \left(\frac{(\rho-1)\frac{\sigma}{r} - \lambda(1+\frac{\sigma}{r}) - ((\rho-1)\lambda - \lambda^2)(\rho + \frac{\sigma}{r\rho})}{(1+\frac{\sigma}{r}) - \lambda(\rho + \frac{\sigma}{r\rho})} \right) \widehat{e}_t \\ &\quad + \frac{1}{2} \frac{(\rho-1)^2}{\rho} \lambda \left(\frac{1+(1-\lambda)(1+\frac{\sigma}{r})}{(1+\frac{\sigma}{r}) - \lambda^2(\rho + \frac{\sigma}{r\rho})} - 1 \right) \widehat{e}_t \\ &\quad - \frac{1}{r} \frac{1}{2} \left(\sigma - \lambda \frac{(\rho-1)\eta}{\rho(\rho+\eta-1)} \right) \left(\frac{(\rho-1)(1-\lambda^2)}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right) \widehat{e}_t \\ &= 2 \frac{\widehat{C}_t^w}{\rho} - dU_t^R \end{aligned} \quad (89)$$

Thus, with producer-currency pricing trade costs lead to asymmetric international welfare spillover effects. Without trade costs these expressions become unambiguously positive and symmetric

$$dU_t^R = dU_t^{*R} = \frac{\widehat{C}_t^w}{\rho} \quad (90)$$

The symmetry of this special case occurs because monopolistic competition, through which the price of a good exceeds its marginal cost of production, is the only source of distortion in the two economies. Due to the Home monetary expansion and the ensuing depreciation Home goods get relatively cheaper under sticky prices. Hence, Home individuals work harder and produce more. In the initial equilibrium with monopolistic competition this decrease in the price and the subsequent increase in production lead to higher revenue. In terms of utility the marginal effects of higher revenue and higher work effort cancel due to the envelope theorem. Vice versa, Foreign individuals get less revenue but work less and the utility effects also cancel. Thus, the first-order effect of a Home monetary expansion is only to raise world demand and output. The associated demand-switching effects are second-order⁴².

The results of this welfare analysis differ from the findings of Obstfeld and Rogoff (1995) and Betts and Devereux (2000). These authors conclude that

⁴²For more details on the intuition cf. Obstfeld and Rogoff (1995), pp. 647-649.

with producer-currency pricing⁴³ there are positive and symmetric international welfare spillover effects in the presence of monetary shocks like in (90). However, as can be seen from (89) trade costs introduce a welfare asymmetry that depends on parameter values. The reason is that apart from monopolistic competition trade costs introduce a second type of distortion such that they directly influence the amount of labour worked, which can be seen in (66) and (67). As Obstfeld and Rogoff (1995) postulate, it is a matter of empirical analysis to find out which type of distortion dominates.

Furthermore, in their pricing-to-market extension Betts and Devereux (2000) find that local-currency pricing introduces asymmetries to the international welfare spillover effects, which can be seen by setting $\lambda = 0$ in (83) and (84). While this property still holds in general, trade costs diminish the extent of the asymmetry due to the containment effect. Therefore, in the presence of trade costs the consequences of monetary shocks in terms of welfare are distinct from a world with perfect international goods market integration.

6.2 Government spending shocks

For the analysis of a government spending shock it is assumed that there is no government spending in the Foreign country ($dG_t^* = dG_{t+1}^* = 0$). Furthermore, the Home government spending shock is permanent ($dG_t = dG_{t+1} > 0$). For local-currency pricing (full pricing to market) it can be shown that

$$dU_t^R = \frac{\hat{C}_t^w}{\rho} + \frac{1}{2} \left(1 - \frac{\rho-1}{\rho} \lambda\right) \hat{e}_t - \frac{1}{2} \left[\frac{\rho-1}{\rho} + \frac{1}{r\rho} \left(\frac{\rho(1+r) + \lambda(\rho(1-\sigma-r)-1)}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right) \right] \frac{dG_t}{C^w} + \frac{1}{r} \left[\frac{\frac{1}{2}(1-\sigma)}{1+\frac{\sigma}{r}} + \frac{\frac{1}{2}\eta}{\rho-1+\rho\eta} - 1 + \frac{\lambda}{\rho-\lambda} \left(\frac{\frac{1}{2}(1-\sigma)}{(1+\frac{\sigma}{r})\sigma} + \frac{1}{2} \frac{\rho\eta-\eta}{\rho-1+\rho\eta} \right) \right] \frac{dG_t}{C^w} \quad (91)$$

$$dU_t^{*R} = \frac{\hat{C}_t^w}{\rho} - \frac{1}{2} \left(1 - \frac{\rho-1}{\rho} \lambda\right) \hat{e}_t + \frac{1}{2} \left[- \left(\frac{\rho-1}{\rho} \right) + \frac{1}{r\rho} \left(\frac{\rho(1+r) + \lambda(\rho(1-\sigma-r)-1)}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right) \right] \frac{dG_t}{C^w} + \frac{1}{r} \left[- \frac{\frac{1}{2}(1-\sigma)}{1+\frac{\sigma}{r}} + \left(\frac{\rho}{\rho-\lambda} \right) \left(\frac{\frac{1}{2}\eta}{\rho-1+\rho\eta} \right) - \frac{\lambda}{\rho-\lambda} \left(\frac{\frac{1}{2}(1-\sigma)}{(1+\frac{\sigma}{r})\sigma} + \frac{1}{2} \frac{\rho\eta-\eta}{\rho-1+\rho\eta} \right) \right] \frac{dG_t}{C^w} \quad (92)$$

For producer-currency pricing welfare changes are given by

$$dU_t^R = \frac{\hat{C}_t^w}{\rho} + \frac{1}{2} \left[- (1 - \lambda^2) (\rho - 1) + \left(1 - \frac{\rho-1}{\rho} \lambda\right) \left(\frac{(\rho-1) + \lambda(1+\frac{\sigma}{r}) - \lambda^2(\rho+\frac{\sigma}{r\rho})}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right) \right] \hat{e}_t - \frac{1}{2} \left[\frac{\rho-1}{\rho} + \frac{1}{r\rho} \left(\frac{\rho(1+r) + \lambda(\rho(1-\sigma-r)-1)}{(1+\frac{\sigma}{r}) - \lambda(1+\frac{\sigma}{r\rho})} \right) \right] \frac{dG_t}{C^w} + \left(\frac{\lambda}{\rho-\lambda} \frac{1}{\sigma} + 1 \right) \left(\frac{\frac{1}{2}\sigma}{1+\frac{\sigma}{r}} \right) \frac{\sigma}{r} (\rho - 1) \hat{e}_t + \frac{1}{r} \left[\frac{\frac{1}{2}(1-\sigma)}{1+\frac{\sigma}{r}} + \frac{\frac{1}{2}\eta}{\rho-1+\rho\eta} - 1 + \frac{\lambda}{\rho-\lambda} \left(\frac{\frac{1}{2}(1-\sigma)}{(1+\frac{\sigma}{r})\sigma} + \frac{1}{2} \frac{\rho\eta-\eta}{\rho-1+\rho\eta} \right) \right] \frac{dG_t}{C^w} \quad (93)$$

⁴³Producer-currency pricing ($s = 0$) is an implicit assumption of the Obstfeld and Rogoff (1995) paper.

$$\begin{aligned}
dU_t^{*R} = & \frac{\hat{C}_t^w}{\rho} - \frac{1}{2} \left[-(1 - \lambda^2) (\rho - 1) + \left(1 - \frac{\rho-1}{\rho} \lambda \right) \left(\frac{(\rho-1) + \lambda(1 + \frac{\sigma}{r}) - \lambda^2(\rho + \frac{\sigma}{r\rho})}{(1 + \frac{\sigma}{r}) - \lambda(1 + \frac{\sigma}{r\rho})} \right) \right] \hat{e}_t \\
+ \frac{1}{2} \left[- \left(\frac{\rho-1}{\rho} \right) + \frac{1}{r\rho} \left(\frac{\rho(1+r) + \lambda(\rho(1-\sigma-r) - 1)}{(1 + \frac{\sigma}{r}) - \lambda(1 + \frac{\sigma}{r\rho})} \right) \right] \frac{dG_t}{C_t^w} - \left(\frac{\lambda}{\rho-\lambda} \frac{1}{\sigma} + 1 \right) \left(\frac{\frac{1}{2}}{1 + \frac{\sigma}{r}} \right) \frac{\sigma}{r} (\rho - 1) \hat{e}_t \\
+ \frac{1}{r} \left[- \frac{\frac{1}{2}(1-\sigma)}{1 + \frac{\sigma}{r}} + \left(\frac{\rho}{\rho-\lambda} \right) \left(\frac{\frac{1}{2}\eta}{\rho-1+\rho\eta} \right) - \frac{\lambda}{\rho-\lambda} \left(\frac{\frac{1}{2}(1-\sigma)}{(1 + \frac{\sigma}{r})\sigma} + \frac{1}{2} \frac{\rho\eta}{\rho-1+\rho\eta} \right) \right] \frac{dG_t}{C_t^w}
\end{aligned} \tag{94}$$

All above expressions depend on parameter values and hence have to be evaluated numerically.

7 A numerical evaluation

Key results from the above discussion are that trade costs tend to increase the variability of the nominal exchange rate in response to monetary shocks, whereas they tend to decrease it in response to government spending shocks when there is full pricing to market. The effects of shocks on the exchange rate with no pricing to market are less clear-cut and rather depend on parameter values. This section numerically evaluates the effects of shocks on the exchange rate and other variables.

Following Betts and Devereux (2000), the parameter values are chosen as

$$\begin{aligned}
\rho &= 11 \\
\eta &= 10/11 \\
r &= 0.06 \\
\epsilon &= 1
\end{aligned}$$

From (18) the price markup of the monopolistic firms is $\frac{\rho}{\rho-1}$. Thus, the chosen parameter value of ρ implies a markup of 10%. The real interest rate of 6% implies $\beta \simeq 0.94$. As to the magnitude of trade costs, Obstfeld and Rogoff (2000) suggest a value of $\tau = 0.25$. However, a slightly lower value of $\tau = 0.2$ is chosen here due to model consistency considerations. The reason is that in this model the share of real output⁴⁴ that is produced domestically and exported abroad is given by

$$\text{Export share of output} = \frac{1}{2}(1 - \lambda) \tag{95}$$

With no trade costs ($\lambda = 0$), each country exports exactly half of its output due to the equal country size. Both for the two large economies of the United States and Japan the export share of GDP was close to 10.5% in each 1999, 2000 and 2001, implying a magnitude of trade costs of about 20%. For small economies whose export shares are bigger a lower value than 20% would be more appropriate.

⁴⁴As a clarification this share is the gross share of real output that is loaded onto ships in the country of production. During the transportation process a fraction τ of this share gets lost because of the iceberg trade costs. Hence, a fraction $(1 - \tau)$ of this share is the net share that actually arrives in the destination country.

Table 2 reports the effects of expansionary Home shocks on the nominal exchange rate and Table 3 reports the effects of an expansionary Home monetary shock other variables. The effects of Foreign shocks would be analogous due to the model's symmetry. The effects are computed by evaluating (48) and, using the chosen parameter values and a 1% shock. All reported numbers are very robust such that their magnitudes do not change significantly when the parameters are decreased or increased by a quarter of the above chosen values.

Table 2 The effect of Home shocks on the exchange rate

	Full PTM ($s = 1$)		No PTM ($s = 0$)	
	$\tau > 0$	$\tau = 0$	$\tau > 0$	$\tau = 0$
$\widehat{M} > 0$	1.000	1.000	0.965	0.891
$dG_t > 0, dG_{t+1} = 0$	0.007	0.032	0.006	0.024
$dG_t = 0, dG_{t+1} > 0$	0.506	0.528	0.451	0.401
$dG_t = dG_{t+1} > 0$	0.513	0.560	0.457	0.425

Percentage deviations from initial equilibrium due to a 1% shock.

Table 3 The effect of a Home monetary shock

Effect on	Full PTM ($s = 1$)		No PTM ($s = 0$)	
	$\tau > 0$	$\tau = 0$	$\tau > 0$	$\tau = 0$
Short run				
\widehat{P}_t	0.000	0.000	0.086	0.380
\widehat{P}_t^*	0.000	0.000	-0.086	-0.380
$\widehat{\psi}_t$	1.000	1.000	0.718	0.000
Overshooting (\sim)	0.000	0.000	0.000	0.000
\widehat{r}_t	-15.667	-15.667	-13.457	-7.833
\widehat{r}_t^*	0.000	0.000	-2.210	-7.833
\widehat{C}_t^w	0.500	0.500	0.500	0.500
\widehat{C}_t	1.000	1.000	1.077	0.620
\widehat{C}_t^*	0.000	0.000	-0.077	0.380
\widehat{h}_t	0.903	0.500	2.680	4.676
\widehat{h}_t^*	0.097	0.500	-1.680	-3.676
Wealth effect				
$\frac{\beta d F_t}{P C^w}$	0.000	0.000	1.321	3.676
Long run				
\widehat{C}_{t+1}^w	0.000	0.000	0.000	0.000
\widehat{C}_{t+1}	0.000	0.000	0.047	0.120
\widehat{C}_{t+1}^*	0.000	0.000	-0.047	-0.120
\widehat{h}_{t+1}	0.000	0.000	-0.040	-0.110
\widehat{h}_{t+1}^*	0.000	0.000	0.040	0.110
\widehat{r}_{t+1}	0.000	0.000	0.000	0.000
Welfare				
$d U_t^R$	0.179	0.545	0.020	0.045
$d U_t^{*R}$	-0.088	-0.454	0.071	0.045

Percentage deviations from initial equilibrium due to a 1% shock.

Recall that all monetary shocks are assumed to be permanent, i.e. $\widehat{M}_t = \widehat{M}_{t+1} = \widehat{M}$ and that the effect of a permanent government spending shock is the sum of the effects of a transitory and an anticipated government spending shock.

The anticipated government spending shock ($d G_{t+1}$) outweighs the transitory government spending shock ($d G_t$) due to its prolonged nature as a new long-run equilibrium value. As discussed intuitively in Section 5, a monetary shock has the same effect on the exchange rate under full pricing to market independent of trade costs but it increases the exchange rate variability in the absence of pricing to market. In contrast, any government spending shock decreases the variability of the exchange rate under full pricing to market. In the case of no pricing to market the results are less clear-cut for government spending. A general caveat in that case is the analytical ambiguity in the form of a dependence on parameter values, whereas the results for $s = 1$ hold independently of any specific values.

8 Rebating trade costs

The iceberg trade costs of this model have the unrealistic feature that a substantial amount of the economy's resources disappears without benefitting anyone. Although a certain dead-weight loss associated with trade is conceivable in the form of red tape, language barriers and the dead-weight loss of tariffs, there are likely to be sectors in the economy that absorb trade costs, for example transportation companies. A recent strand of literature has incorporated a distributional sector into trade models, e.g. Bacchetta and van Wincoop (2002). This idea is also pursued in this section but in a different fashion.

It is assumed that by holding a monopoly on the shipping of imported goods the governments are able to recuperate a varying share of the resources in the economy that disappear in the trading process. This revenue is then rebated to consumers in a lump-sum fashion⁴⁵. As a motivation, the situation where a government holds a monopoly on importing goods and rebates the revenue from that monopoly to consumers is comparable to an import tariff and a lump-sum transfer of the tariff revenue. A government might want to pursue such a policy to protect its domestic economy from foreign competition.

The export share of real output for the Foreign country is denoted by x_t^* ⁴⁶ so that the real quantity of iceberg trade costs incurred by the Home country is τx_t^* .⁴⁷ Note that with $\tau = 0$ this quantity reduces to zero. Let δ denote the share of the iceberg trade costs that the government is able to recuperate. The Home government budget constraint (13) now becomes⁴⁸

$$P_t G_t + Z_t = M_t - M_{t-1} + \delta \tau [s p_t^* + (1-s) e_t q_t^*] x_t^* \quad (96)$$

Note that the amount of trade costs recuperated by the Home government depends on a part of Foreign output, x_t^* , which in turn depends on Home demand. With $\delta = 1$ the government can recuperate all trade costs so that τ is equivalent to an import tariff. With $\delta = 0$ the model collapses to the one of the preceding sections. If $0 < \delta < 1$ there is some dead-weight loss associated with trade.

8.1 The initial equilibrium with flexible prices

Analogous to Section 3 an initial equilibrium where prices are perfectly flexible can be computed. It is assumed that $F = F^* = G = G^* = 0$. In this equilibrium the supply of labour and thus output is reduced with rebating.

$$h = h^* = y = y^* = \frac{\rho - 1}{\rho - 1 + \rho \eta [1 + \delta \tau \frac{1}{2} (1 - \lambda)]} \quad (97)$$

⁴⁵If the governments consume imported goods themselves, they rebate the associated trade costs to consumers, too.

⁴⁶This notation was introduced in Section 2.3.

⁴⁷In (95) and in Appendix C the export share of real output for each country is given as $\frac{1}{2}(1 - \lambda)$ so that in equilibrium the real quantity of goods imported to the Home country is $x^* = \frac{1}{2}(1 - \lambda)y^*$, where $y^* = \frac{\rho - 1}{\rho - 1 + \rho \eta}$ from (21).

⁴⁸The aggregate budget constraint (5) does not change.

Note that for $\delta = 0$ this expression simplifies to (21). P and $\frac{W}{P}$ are unaffected. PPP still holds.

$$C = C^* = \frac{(\rho - 1) [1 + \delta\tau\frac{1}{2}(1 - \lambda)]}{\rho - 1 + \rho\eta [1 + \delta\tau\frac{1}{2}(1 - \lambda)]} \left(\frac{1}{1 + \lambda} \right)^{\frac{1}{\rho-1}} \quad (98)$$

is bigger than (22). Therefore, it must be that

$$\frac{\partial U}{\partial \delta} = \frac{\partial U^*}{\partial \delta} > 0 \quad (99)$$

Real profits drop

$$\frac{\pi}{P} = \frac{\frac{\rho-1}{\rho}}{\rho - 1 + \rho\eta [1 + \delta\tau\frac{1}{2}(1 - \lambda)]} \left(\frac{1}{1 + \lambda} \right)^{\frac{1}{\rho-1}} \quad (100)$$

Nominal profits drop as well.

8.2 The long run

$$(\hat{C}_{t+1} - \hat{C}_{t+1}^*) = \frac{1}{\tilde{\sigma}} \frac{\rho}{(\rho - \lambda)} \frac{2(1 - \beta) dF_t}{PC^w} - \frac{1}{\tilde{\sigma}} \frac{(\rho - \tilde{\sigma}\lambda)}{(\rho - \lambda)} \frac{(dG_{t+1} - dG_{t+1}^*)}{C^w} \quad (101)$$

where

$$\tilde{\sigma} \equiv \frac{\rho - 1 + \rho\eta [1 + \delta\tau\frac{1}{2}(1 - \lambda)]}{\rho - 1 + \eta [1 + \delta\tau\frac{1}{2}(1 - \lambda)]} \quad (102)$$

For $\delta = 0$ it follows $\tilde{\sigma} = \sigma$ where σ is defined in (40) and (101) reduces to (39). For $0 < \delta < 1$ it follows $\tilde{\sigma} < \sigma$ since $\frac{\partial \tilde{\sigma}}{\partial \delta} < 0$. Hence, qualitatively rebating does not change the result for the long-run consumption differential.

9 Varying country size

It is desirable to find out whether the results of the previous sections only obtain for two countries of equal size or whether they uphold qualitatively for a varying country size⁴⁹. In particular, the limiting case of a small open economy is a point of interest.

Households still choose among a range $[0, 1]$ of differentiated tradable goods, the fraction n of which is now produced in the Home country and the fraction $1 - n$ of which is produced in the Foreign country with $0 < n < 1$. Accordingly, the price indices now become

$$P_t = \left[\int_0^n p_{it}^{1-\rho} di + \int_n^{n+(1-n)s} \left(\frac{1}{1-\tau} p_{it}^* \right)^{1-\rho} di + \int_{n+(1-n)s}^1 \left(\frac{1}{1-\tau} e_t q_{it}^* \right)^{1-\rho} di \right]^{\frac{1}{1-\rho}} \quad (103)$$

⁴⁹Both Obstfeld and Rogoff (1995) and Betts and Devereux (2000) also allow for a varying country size.

and

$$P_t^* = \left[\int_0^{ns} \left(\frac{1}{1-\tau} q_{it} \right)^{1-\rho} di + \int_{ns}^n \left(\frac{1}{1-\tau} \frac{1}{e_t} p_{it} \right)^{1-\rho} di + \int_n^1 q_{it}^*{}^{1-\rho} di \right]^{\frac{1}{1-\rho}} \quad (104)$$

They are completely analogous to (3) and (4).

9.1 The initial equilibrium with flexible prices

As in Section 2.3 price flexibility is assumed for the initial equilibrium in the sense that the firms set their prices after the exchange rate and the wage rate are known. Then the aggregate profit function is described by (17). The price markups turn out to be the same as in (18) and (19). Furthermore, it is still assumed that $Z = Z^* = F = F^* = G = G^* = 0$ in the initial equilibrium. The expressions

$$\theta \equiv n + (1-n)(1-\tau)^{\rho-1} \quad (105)$$

$$\theta^* \equiv (1-n) + n(1-\tau)^{\rho-1} \quad (106)$$

$$\kappa \equiv \frac{1 - (1-\tau)^{\rho-1} \left(\frac{1-n}{n} \right) \frac{\theta}{\theta^*}}{1 + (1-\tau)^{\rho-1} \left(\frac{1-n}{n} \right) \frac{\theta}{\theta^*}} \quad (107)$$

$$\kappa^* \equiv \frac{1 - (1-\tau)^{\rho-1} \left(\frac{n}{1-n} \right) \frac{\theta^*}{\theta}}{1 + (1-\tau)^{\rho-1} \left(\frac{n}{1-n} \right) \frac{\theta^*}{\theta}} \quad (108)$$

$$\chi \equiv 1 - \left(\frac{n}{\theta^*} + \frac{(1-n)}{\theta} \right) (1-\tau)^{\rho-1} \quad (109)$$

are used in the following as terms involving the trade costs parameter τ and the country size parameter n . Note that for $n = 1 - n$ it follows that $\theta = \theta^*$ and $\kappa = \kappa^* = \chi = \lambda$ as given in (20) so that λ is a special subcase of κ and κ^* . For $n > 1 - n$ it follows $\theta = \theta^*$. Also note that $-1 < \kappa < 1$ ⁵⁰ for $0 < \tau < 1$ and $0 < n < 1$. For $\tau = 0$ it follows that $\theta = \theta^* = 1$, $\kappa = 2n - 1$, $\kappa^* = 1 - 2n$ and $\chi = 0$. The time index t is dropped to denote the initial equilibrium values.

In the literature models without trade costs normally use the law of one price for labour ($W = eW^*$) as a nominal anchor, which then automatically leads to PPP ($P = eP^*$). However, in a model with trade costs in combination with a differing country size these two cannot hold at the same time. The law of one price for labour is chosen in the following as the nominal anchor of the model⁵¹. As a consequence, the law of one price holds for the f.o.b. prices of

⁵⁰ κ becomes negative for a combination of very small τ and very small n .

⁵¹ Alternatively, PPP could be chosen as the nominal anchor of the model. Then, however, the law of one price would no longer hold for the f.o.b. prices of all goods. In the special case of symmetry (equal country size) or if there were no trade costs, there would be no deviation from PPP in the initial equilibrium, cf. Obstfeld and Rogoff (1995) and Betts and Devereux (2000).

all goods. This normalisation allows for the analysis of PPP deviations that are due to trade costs.

Output is still of the same magnitude as with equal country size, given in (21)⁵². However, the equilibrium levels of consumption, real profits, real wages and prices are influenced by the country size

$$C = h\theta^{\frac{1}{\rho-1}} \quad (110)$$

$$C^* = h^*\theta^{*\frac{1}{\rho-1}} \quad (111)$$

$$\frac{\pi}{P} = \frac{h}{\rho}\theta^{\frac{1}{\rho-1}} \quad (112)$$

$$\frac{\pi^*}{P^*} = \frac{h^*}{\rho}\theta^{*\frac{1}{\rho-1}} \quad (113)$$

$$\frac{W}{P} = \frac{\rho-1}{\rho}\theta^{\frac{1}{\rho-1}} \quad (114)$$

$$\frac{W^*}{P^*} = \frac{\rho-1}{\rho}\theta^{*\frac{1}{\rho-1}} \quad (115)$$

Remember that C , C^* , π and π^* are per capita variables⁵³. The nominal and real interest rates of the initial equilibrium are the same as with equal country size, given in (26) and (27). The equilibrium nominal and real exchange rates are given by

$$e = \frac{M}{M^*} \left(\frac{C^*}{C} \right)^{\frac{1}{\epsilon}} \left(\frac{\theta}{\theta^*} \right)^{\frac{1}{\rho-1}} \quad (116)$$

$$\psi \equiv \frac{eP^*}{P} = \left(\frac{\theta}{\theta^*} \right)^{\frac{1}{\rho-1}} \quad (117)$$

The price levels can be shown to be

$$P = \theta^{\frac{1}{1-\rho}} \frac{\rho}{\rho-1} W \quad (118)$$

$$P^* = \theta^{*\frac{1}{1-\rho}} \frac{\rho}{\rho-1} W^* \quad (119)$$

As in Section 3 M and M^* are normalised such that $W = W^* = 1$. It therefore follows that $e = 1$ in the initial equilibrium. Note that the law of one price for labour holds, i.e. $W = eW^*$, but PPP does not hold in the initial equilibrium since $\psi \neq 1$.

⁵²Note that h and h^* denote output per Home and Foreign household, respectively. nh and $(1-n)h^*$ denote the aggregate quantities.

⁵³The corresponding aggregate quantities would be nC and $n\pi$ for the Home country and $(1-n)C^*$ and $(1-n)\pi^*$ for the Foreign country, respectively.

Relative consumption can be expressed as

$$\frac{C}{C^*} = \left(\frac{\theta}{\theta^*} \right)^{\frac{1}{\rho-1}} \quad (120)$$

Note that for $n = 1 - n$, $\frac{C}{C^*} = 1$, for $n > 1 - n$, $\frac{C}{C^*} > 1$ and for $n < 1 - n$, $\frac{C}{C^*} < 1$. There are similar effects for the other above variables. Thus, a small country suffers more from trade relative to a big country in terms of lower consumption, a higher price level, a lower real wage and lower real profits. In other words, the bigger a country is, the more it is insulated from trade costs. Intuitively, since all goods are equally desired by the consumers in the composite consumption index (2), a small country has to rely more on imports than a big country and hence is more exposed to trade costs. Since imports are more expensive than domestically produced goods due to trade costs, the composition of consumption in (110) and (111) is biased towards domestically produced goods. In other words, there is a ‘Home Bias’ in consumption in each country, which is consistent with the ‘Home Bias in Trade Puzzle’ pointed out by McCallum (1995) and others.

Obviously, this result crucially depends on the symmetric preference specification in (2). Abandoning this symmetry and introducing a home bias in preferences would mitigate the negative effects for the small country. However, Evans (2001) finds that empirically locational factors arising due to geographic distance and legal regulations - and not consumer preferences - are the only significant reason for the tendency of consumers to purchase domestic goods⁵⁴. Her findings are therefore consistent with the preference structure of (2) and the trade cost specification of this model.

9.2 The long run

(38) still holds and (39) becomes

$$(\hat{C}_{t+1} - \hat{C}_{t+1}^*) = \frac{1}{\sigma} \frac{\rho}{\left(\rho - \frac{1}{2}(\kappa + \kappa^*)\right)} \frac{(1 - \beta) dF_t}{PC(1 - n)} - \frac{1}{\sigma} \frac{(\rho - \sigma \frac{1}{2}(\kappa + \kappa^*))}{\left(\rho - \frac{1}{2}(\kappa + \kappa^*)\right)} \left(\frac{dG_{t+1}}{C} - \frac{dG_{t+1}^*}{C^*} \right) \quad (121)$$

This means that the result from the section with equal country size goes through qualitatively. The λ in (39) has effectively been replaced by $\frac{1}{2}(\kappa + \kappa^*)$ and C^w has been replaced by C and C^* , respectively. Note that for $\tau = 0$, $\frac{1}{2}(\kappa + \kappa^*) = 0$.

⁵⁴Evans (2001) compares prices and quantities of imported goods, of goods produced by American firms for domestic sale and of the same goods produced by foreign affiliates of these American firms for local sale. Her data set encompasses seven industries, ranging from transportation equipment to food products, and nine OECD countries over the period 1985-1994. She finds that the ad-valorem tariff-equivalent of producing domestically and shipping abroad ranges between 51 and 105 percent across industries, which considerably reduces the attractiveness of the foreign goods for domestic consumers. Establishing and selling from an affiliate, however, does not lead to any negative effect on sales of these foreign products when compared to sales of domestic goods. In other words, French consumers do not intrinsically prefer French to American beer, only if it is cheaper.

9.3 The short run

From (103) and (104) it can be derived that

$$\widehat{P}_t = (1-n)(1-s)\frac{1}{\theta}(1-\tau)^{\rho-1}\widehat{e}_t \quad (122)$$

$$\widehat{P}_t^* = -n(1-s)\frac{1}{\theta^*}(1-\tau)^{\rho-1}\widehat{e}_t \quad (123)$$

Note that

$$\lim_{n \rightarrow 1} \widehat{P}_t = 0 \quad (124)$$

$$\lim_{n \rightarrow 1} \widehat{P}_t^* = -(1-s)\widehat{e}_t \quad (125)$$

These results are intuitive since when the Home country is overwhelmingly big ($n \rightarrow 1$), the Home price index is not affected by any movement in the nominal exchange rate, whereas the Foreign price index goes down for a given nominal exchange rate depreciation ($\widehat{e}_t > 0$), depending on the fraction of firms that are not able to price to market ($1-s$).

The equivalent of (45) is given by

$$\widehat{C}_{t+1} - \widehat{C}_{t+1}^* = \widehat{C}_t - \widehat{C}_t^* - (s + \chi(1-s))\widehat{e}_t \quad (126)$$

The λ in (45) has effectively been replaced by χ .

The equivalent of (46) is given by

$$\widehat{e}_t = \frac{(\widehat{C}_t - \widehat{C}_t^*) + \frac{1}{(1-\frac{1}{2}(\kappa+\kappa^*))} \frac{\beta dF_t}{PC(1-n)} + \left(\frac{dG_t}{C} - \frac{dG_t^*}{C^*} \right)}{(1-s)(\rho - (1-\chi)\frac{(1-\rho\frac{1}{2}(\kappa+\kappa^*))}{(1-\frac{1}{2}(\kappa+\kappa^*))}) + s} \quad (127)$$

The equivalent of (47) is given by

$$\widehat{e}_t = \left\{ \begin{aligned} & \left(1 + \frac{\sigma}{r} - \frac{1}{2}(\kappa + \kappa^*) \left(1 + \frac{\sigma}{r\rho} \right) \right) (\widehat{C}_t - \widehat{C}_t^*) + (1 - \frac{1}{2}(\kappa + \kappa^*)) \left(\frac{dG_t}{C} - \frac{dG_t^*}{C^*} \right) \\ & + \left(\frac{1}{r} - \frac{\sigma}{r\rho} \frac{1}{2}(\kappa + \kappa^*) \right) \left(\frac{dG_{t+1}}{C} - \frac{dG_{t+1}^*}{C^*} \right) \end{aligned} \right\} / \left\{ \begin{aligned} & (1-s) \left[(\rho - 1) + \chi \left(1 + \frac{\sigma}{r} \right) - \chi \frac{1}{2}(\kappa + \kappa^*) \left(\rho + \frac{\sigma}{r\rho} \right) \right] \\ & + s \left[1 + \frac{\sigma}{r} - \frac{1}{2}(\kappa + \kappa^*) \left(1 + \frac{\sigma}{r\rho} \right) \right] \end{aligned} \right\} \quad (128)$$

The equivalent of (48) is given by

$$\widehat{e}_t = \left\{ \begin{aligned} & \left(1 + \frac{\sigma}{r} - \frac{1}{2}(\kappa + \kappa^*) \left(1 + \frac{\sigma}{r\rho} \right) \right) \epsilon (\widehat{M}_t - \widehat{M}_t^*) + (1 - \frac{1}{2}(\kappa + \kappa^*)) \left(\frac{dG_t}{C} - \frac{dG_t^*}{C^*} \right) \\ & + \frac{1}{r} \left(1 - \frac{\sigma}{\rho} \frac{1}{2}(\kappa + \kappa^*) \right) \left(\frac{dG_{t+1}}{C} - \frac{dG_{t+1}^*}{C^*} \right) \end{aligned} \right\} / \left\{ \begin{aligned} & (1-s) \left[\begin{aligned} & \epsilon \left(1 + \frac{\sigma}{r} \right) + (\rho - 1) - \epsilon \chi \left(1 + \frac{\sigma}{r} \right) \frac{(\epsilon-1)r}{(\epsilon r+1)} - \epsilon \frac{1}{2}(\kappa + \kappa^*) \left(1 + \frac{\sigma}{r\rho} \right) \\ & + \chi \frac{1}{2}(\kappa + \kappa^*) \left(\left(1 + \frac{\sigma}{r\rho} \right) \frac{(\epsilon^2 r+1)}{(\epsilon r+1)} - \left(\rho + \frac{\sigma}{r\rho} \right) \right) \end{aligned} \right] \\ & + s \left[\left(\left(1 + \frac{\sigma}{r} \right) - \frac{1}{2}(\kappa + \kappa^*) \left(1 + \frac{\sigma}{r\rho} \right) \right) \frac{\epsilon(1+r)}{(\epsilon r+1)} \right] \end{aligned} \right\} \quad (129)$$

10 Conclusion

Consistent with recent empirical evidence on imperfect international goods market integration, this paper introduces trade costs into a two-country general equilibrium model of pricing to market. Trade costs do not only incorporate transportation costs but basically any kind of border effect. In the flexible-price equilibrium trade costs reduce consumption, real wages and real profits but leave labour supply unaffected, hence making individuals worse off. The smaller a country is relative to the other, the more it suffers from trade costs.

As a general feature trade costs exhibit a ‘containment effect’, which means that they make imports relatively expensive and thus tend to contain spending in its country of origin. In other words, trade costs act like a buffer between countries and therefore obstruct the integration of international goods markets. Since imports are more expensive than domestically produced goods, consumption is biased towards domestically produced goods. Thus, trade costs give rise to a ‘Home Bias’ in consumption, which is consistent with the ‘Home Bias in Trade Puzzle’ pointed out by McCallum (1995) and others.

Furthermore, with sticky prices and under full pricing to market there is an ‘income-switching effect’, namely in the case of a Home depreciation relative prices and thus relative demand do not change but Home profits are boosted, whereas Foreign profits fall. Due to the containment effect a greater share of the additional revenue is channelled towards Home goods, thus reinforcing the income-switching effect. With sticky prices and no pricing to market there is a ‘demand-switching effect’ in the sense that a Home depreciation makes Home goods relatively cheaper, boosting Home profits. Again, trade costs reinforce this demand-switching effect due to containment.

When an economy is hit by government spending shocks and there is full pricing to market, trade costs unambiguously reduce the variability of the nominal exchange rate. With no pricing to market the effects of government spending shocks are less clear-cut and depend on parameter values. In general, trade costs make overshooting more likely.

All these qualitative results are robust towards the extension of rebating trade costs to consumers. Thus, the results indicate that trade costs have wide-ranging implications for the behaviour of key economic variables. They may therefore help to gain insight into recent empirical findings on the imperfect integration of international goods markets.

A Households and firms

The derivations of the expressions in Section 2 are outlined here.

The demand function in (7) is derived by maximising C_t in (2) subject to the expenditure given by

$$K = \int_0^{\frac{1}{2}} p_{it} c_{it} \, di + \int_{\frac{1}{2}}^{\frac{1}{2} + \frac{1}{2}s} \left(\frac{1}{1-\tau} p_{it}^* \right) c_{it} \, di + \int_{\frac{1}{2} + \frac{1}{2}s}^1 \left(\frac{1}{1-\tau} e_t q_{it}^* \right) c_{it} \, di$$

Maximising utility (1) subject to a two-period intertemporal budget constraint

$$P_{t+1} C_{t+1} + M_{t+1} + d_{t+1} F_{t+1} = W_{t+1} h_{t+1} + \pi_{t+1} + \frac{d_t - 1}{d_t} M_t + Z_{t+1} + \frac{1}{d_t} [W_t h_t + \pi_t + M_{t-1} + Z_t + F_{t-1} - P_t C_t]$$

constructed from (5) yields (9), (10) and (11).

When $\tau = 0$ the results from the text simplify to the ones of Betts and Devereux (2000). Equations (1), (2) and (5)-(16) remain the same. The price indices are

$$P_t = \left[\int_0^{\frac{1}{2}} p_{it}^{1-\rho} \, di + \int_{\frac{1}{2}}^{\frac{1}{2} + \frac{1}{2}s} p_{it}^*{}^{1-\rho} \, di + \int_{\frac{1}{2} + \frac{1}{2}s}^1 (e_t q_{it}^*)^{1-\rho} \, di \right]^{\frac{1}{1-\rho}}$$

$$P_t^* = \left[\int_0^{\frac{1}{2}s} q_{it}^{1-\rho} \, di + \int_{\frac{1}{2}s}^{\frac{1}{2}} \left(\frac{1}{e_t} p_{it} \right)^{1-\rho} \, di + \int_{\frac{1}{2}}^1 q_{it}^*{}^{1-\rho} \, di \right]^{\frac{1}{1-\rho}}$$

Profits and markups are the same as in (17), (18) and (19).

B The initial equilibrium

The derivations of the expressions in Section 3 are outlined here.

(21) and (23) can be derived by inserting (14), (15), (18) and (??) into (17), also using (5) and (9) and noting that in the initial equilibrium $M_{t-1} = M_t$. Use the definition of the aggregate consumption index (2) to derive (22). For (28) use the fact that PPP holds in the initial equilibrium, rearrange (10) and the corresponding equation for the Foreign country and use the results in (21) and (22).

When $\tau = 0$ the results from the text simplify to the ones of Betts and Devereux (2000). Equations (21) and (23)-(29) remain the same. In the initial equilibrium consumption is

$$C = C^* = h = h^*$$

C Sticky prices

The derivations of the expressions in Section 4 are outlined here. The methodology follows Betts and Devereux (2000).

An equilibrium in period t can be described by the following equations

$$\begin{aligned} \frac{M_t}{P_t} &= \left(\frac{\gamma C_t}{1 - d_t} \right)^{\frac{1}{\epsilon}} \\ \frac{M_t^*}{P_t^*} &= \left(\frac{\gamma C_t^*}{1 - d_t \frac{e_{t+1}}{e_t}} \right)^{\frac{1}{\epsilon}} \\ P_t C_t + P_t G_t + d_t F_t &= s(p_t x_t + e_t q_t (1 - \tau) z_t^{PTM}) + (1 - s)(p_t x_t + p_t z_t^{NPTM}) \\ P_t^* C_t^* + P_t^* G_t^* + \frac{d_t}{e_t} F_t^* &= s(q_t^* z_t^* + \frac{p_t^*}{e_t} (1 - \tau) x_t^{*PTM}) + (1 - s)(q_t^* z_t^* + q_t^* x_t^{*NPTM}) \\ x_t &= \frac{1}{2} \left(\frac{p_t}{P_t} \right)^{-\rho} (C_t + G_t) \\ (1 - \tau) z_t^{PTM} &= \frac{1}{2} \left(\frac{q_t}{P_t^*} \right)^{-\rho} (C_t^* + G_t^*) \\ (1 - \tau) z_t^{NPTM} &= \frac{1}{2} \left(\frac{\frac{1}{(1-\tau)} p_t}{e_t P_t^*} \right)^{-\rho} (C_t^* + G_t^*) \\ z_t^* &= \frac{1}{2} \left(\frac{q_t^*}{P_t^*} \right)^{-\rho} (C_t^* + G_t^*) \\ (1 - \tau) x_t^{*PTM} &= \frac{1}{2} \left(\frac{p_t^*}{P_t^*} \right)^{-\rho} (C_t + G_t) \\ (1 - \tau) x_t^{*NPTM} &= \frac{1}{2} \left(\frac{\frac{1}{(1-\tau)} e_t q_t^*}{P_t^*} \right)^{-\rho} (C_t + G_t) \\ d_t P_{t+1} C_{t+1} &= \beta P_t C_t \\ d_t \frac{e_{t+1}}{e_t} P_{t+1}^* C_{t+1}^* &= \beta P_t^* C_t^* \end{aligned}$$

Note that $Z_{t-1} = Z_{t-1}^* = F_{t-1} = F_{t-1}^* = G_{t-1} = G_{t-1}^* = 0$. Take log-linear approximations of these equations, using that in the initial equilibrium $z^{PTM} = z^{NPTM} = z$ and $x^{*PTM} = x^{*NPTM} = x^*$ and

$$\frac{x}{z} = \frac{z^*}{x^*} = (1 - \tau)^{1-\rho}$$

Combining the approximations and also using (41) and (42) yields (46) and subsequently (47) and (48) can be derived. A similar procedure for period $t + 1$ results in (38) and (39).

The following results are taken from Betts and Devereux (2000) and given for reference only. They can be directly derived from the more general expressions in Section 4 when $\tau = 0$, i.e. they represent the economy without trade costs.

The long-run response of the exchange rate

$$\widehat{e}_{t+1} = \widehat{M}_{t+1} - \widehat{M}_{t+1}^* - \frac{1}{\epsilon}(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)$$

is given by the money market equilibria. The long-run values of consumption

$$\widehat{C}_{t+1} - \widehat{C}_{t+1}^* = \frac{1}{\sigma} \frac{2(1-\beta) dF_t}{PC^w} - \frac{1}{\sigma} \frac{(dG_{t+1} - dG_{t+1}^*)}{C^w}$$

are determined by goods market and labour market clearing as well as by the countries' budget constraints.

The responses of the price indices to shocks are given by

$$\widehat{P}_t = \frac{1}{2}(1-s)\widehat{e}_t$$

$$\widehat{P}_t^* = -\frac{1}{2}(1-s)\widehat{e}_t$$

The exchange rate in period t moves according to

$$\widehat{e}_t = \frac{\epsilon^2 r + \epsilon}{\epsilon^2 r(1-s) + \epsilon - s} \left(\widehat{M}_t - \widehat{M}_t^* - \frac{1}{\epsilon}(\widehat{C}_t - \widehat{C}_t^*) \right)$$

The corresponding equations of (46)-(48) are

$$\begin{aligned} \widehat{e}_t &= \frac{\widehat{C}_t - \widehat{C}_t^* + \frac{2\beta dF_t}{PC^w} + \frac{(dG_t - dG_t^*)}{C^w}}{(1-s)(\rho-1) + s} \\ \widehat{e}_t &= \frac{\left(1 + \frac{\sigma}{r}\right) (\widehat{C}_t - \widehat{C}_t^*) + \frac{(dG_t - dG_t^*)}{C^w} + \frac{1}{r} \frac{(dG_{t+1} - dG_{t+1}^*)}{C^w}}{(1-s)(\rho-1) + s \left(1 + \frac{\sigma}{r}\right)} \\ \widehat{e}_t &= \frac{\left(1 + \frac{\sigma}{r}\right) \epsilon (\widehat{M}_t - \widehat{M}_t^*) + \frac{(dG_t - dG_t^*)}{C^w} + \frac{1}{r} \frac{(dG_{t+1} - dG_{t+1}^*)}{C^w}}{(1-s) \left(\epsilon \left(1 + \frac{\sigma}{r}\right) + \rho - 1 \right) + s \left(1 + \frac{\sigma}{r}\right) \left(\frac{1+1/r}{1+1/\epsilon r} \right)} \end{aligned}$$

D Welfare analysis

To see why under local-currency pricing a monetary expansion is a “beggar-thy-neighbour” instrument so that $dU_t^{*R} < 0$ in (84), plug in (55) and (61) and rearrange to yield

$$dU_t^{*R} = -\frac{1}{2\rho}(1-\lambda)(\rho-1) \left(\frac{\epsilon + 1/r}{1 + 1/r} \right) \widehat{M}_t$$

For $\tau, r, \epsilon > 0$ this expression is negative since $\rho > 1$.

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