

# A Time-Consistent Agreement in an Interregional Differential Game on Pollution and Trade\*

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## Abstract

This paper studies a transboundary pollution problem between two neighbour regions as a dynamic game. These two regions do not only share an environmental problem but there are also engaged by interregional trade. A good produced in one region is traded to the other which uses it as an input. This intermediate good is supplied by the former and demanded by the latter. The supply-demand scheme determines the price and production of the intermediate good. Thus total production is fixed in both regions, and the emissions of pollutants are also determined as a by-product. Cooperation cuts down production and trade, and in consequence the emissions of pollutants. Therefore, the environmental gain from cooperation overcomes the shrink in the interregional trade. An allocation mechanism to share the surplus of cooperation is defined, which guarantees a time-consistent agreement between both regions.

Keywords: Cooperative and non-cooperative games, differential games, dynamic individual rationality, time-consistent agreement, trade and environment, transboundary pollution.

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## 1 Introduction

International trade and many environmental problems cannot be separately treated, but on the contrary they are necessarily interconnected. International trade has an unques-

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tionable effect on production activities in all trading countries where these activities may cause environmental problems. So that, decisions on trade will unavoidably influence the global environment and vice versa. Likewise, it is worth saying that production patterns in one country will affect not only the environment within its borders but also abroad. This is particularly clear for transboundary pollution problems, as it is the case in the acid rain problems in Europe and the US. Although these problems appear independently of the existing trade relationship among countries, international trade policies can be useful in order to better achieve an international agreement on a specific environmental issue. Countries with higher benefits from the conservation of the environment (which is a common good unevenly valued), would sign the agreement even if it means a better trade position for those countries with lower environmental gains.

This reasoning is in concordance with recent literature that points out how the success in international environmental agreement is more likely when linked to other international agreements. Along this line, (Carraro and Siniscalco 1995, 1997; Katsoulacos 1997) suggest to relate technological cooperation with environmental bargaining. This notion already appears in the climate change convention. Other idea is suggested by Barrett (1995) who proposes to link environmental negotiations to trade liberalization. The environmental clause in GATT is a good example of that.

Linking international trade and environmental agreements could make a global agreement more profitable in the sense that all countries signing the agreement would be better-off. Those countries with lower gains from the environmental agreement could obtain compensation from the trade agreement which reduces their incentive to free ride.

A dynamic framework may be more appropriate to deal with environmental issues in which the stock of pollutants rather than the flow of emissions is the crucial variable to monitor. To analyze the interrelationship between regions when the dynamic effect of their production processes on the environment is considered, we use a differential game. One of the seminal works that uses this methodology to treat this kind problems is (Ploeg and Zeeuw 1992) whose interest is on transboundary pollution problems. Among others authors that use this methodology focusing on pollution, (Dockner and Long 1993; Kaitala and Pohjola 1995; Jørgensen and Zaccour 2001a, 2001b; Petrosjan and Zaccour, 2003; Jørgensen et al. 2003) can be cited. The recurrent conclusion in all these papers states that no only total welfare increases when players decide to cooperate, but also emissions decrease and consequently the pollution stock is lower than in the non-cooperative case. If cooperation is in the interest of all signatory countries, why is so difficult to maintain environmental agreements in practice? Asymmetries between countries and incentives to free ride can be an answer to this question (see, e.g. Carraro 1999). The question of how to achieve international environmental agreements which are sustained over time seems more difficult, giving rise to numerous different answers. One of these answers, which is

the mechanism used in this paper, implies the study of individual rationality over time or dynamic individual rationality. In a differential game framework the study of dynamic individual rationality has been done using different concepts such as time-consistency and agreeability (for a survey on intertemporal individual rationality in differential games see Jørgensen and Zaccour, 2002. Other recent papers that address the question of individual rationality over time are Jørgensen and Zaccour 2001a, 2001b; Petrosjan and Zaccour, 2003; Jørgensen et al. 2003). The dynamic individual rationality property implies that each player's individual cooperative payoff to go is not lower than his non cooperative payoff to go. The comparison between the payoffs can be done at any intermediate instant of time. Therefore, the issue of dynamic individual rationality is important to ensure the sustainability over time of an agreement between the players.

In previously cited papers on differential games and transboundary pollution, players (regions or countries) do not negotiate in any other economic aspect. Moreover all these papers settle models in term of symmetric games. Conversely, we incorporate the trade matter and so that, the agreement will take into account trade as well as the environmental issue. At the same time, this gives rise to asymmetries between countries. They have to take decisions on different aspects and simultaneously, the impact of one country's decision on another country is uneven. Likewise, the effect of each player's decision on the pollution stock is also asymmetric.

The paper considers two regions that trade a single good in an international market. They are heterogeneous in the sense that one region,  $A$ , is the only producer and supplier of this good, while the other region,  $B$ , demands and uses it as an intermediate good in its production function. A differential game is settled between these two regions. The producer fixes the international traded good supply, while the other region chooses the demand for this good. The market determines the price and the total amount of the intermediate good, as well as the final production in region  $B$ , which is assumed proportional to the traded good. Emissions of pollutants are a by-product of production processes in both regions. However welfare losses in each region are caused by the pollution stock rather than by emissions. Within this context, countries do not only negotiate on environmental issues but also on international trade issues. Therefore a transboundary pollution problem is analyzed within an international trade framework. Both regions maximize their stream of discounted welfare taking into account the benefits from production and trade as well as the damage caused by the pollution stock. We will prove that total pollution stock will be reduced if they coordinate their emissions strategies.

The paper is organized as follows. Section 2 defines a differential game between two regions that trade each other and bear a disutility from the pollution stock accumulated in the atmosphere. Sections 3, 4 and 5 seek to characterize the optimal paths for the non-cooperative as well as the cooperative modes of play. In the non-cooperative case,

depending on the information structure, we distinguish between open-loop and feedback strategies. In Section 3 the non-cooperative open-loop game is solved and the steady-state and time paths are found for price, production and pollution stock. Same analysis is carried out in the feedback case in Section 4. In Section 5 the cooperative equilibria are found and we show that production and pollution decrease under cooperation, while total welfare increases. Section 6 studies two different concepts of dynamic individual rationality: global and instantaneous. In this section a time-consistent agreement between both regions is analyzed. Conclusions are presented in Section 7.

## 2 A model of pollution and trade

A transboundary pollution problem is analyzed within an interregional trade framework. We consider two different regions interconnected by a double relationship. On the one hand, pollution is a by-product of the productive process in each region. Emissions accumulated in the atmosphere provoke welfare losses although not necessarily identical in both regions. Thus the production in each region and indirectly the emissions of pollutants must be determined. On the other hand, a good produced in region *A* is traded to region *B*. Consequently, the price and the amount of the traded good must also be determined. Welfare depends on production and trade, but at the same time is reduced by pollution. Following (Bovenberg and Smulders 1995, 1996; Musu 1996) we assume that these losses are caused by the accumulated pollution stock rather than by the flow of emissions.

Region *A* produces the amount  $y(t)$  of the intermediate good at time  $t$ . This good is demanded by region *B* which uses it as a productive input. Welfare in region *A* is given by the difference between the economic benefits and the environmental damage of the production process.

To produce the intermediate good,  $y(t)$ , region *A* has to buy an input,  $q(t)$ , from the rest of the world in the international market. A linear technology is assumed for this region production process:  $y(t) = aq(t)$ .

Economic benefits in region *A* are given by the income derived from the intermediate good traded to region *B* minus the production costs, which are assumed quadratic in the produced amount:  $p(t)y(t) - y^2(t)/2$ , where  $p(t)$  is the price of the intermediate good. From the hypothesis of linear production technology the economic benefits can be written as:

$$ap(t)q(t) - a^2q^2(t)/2.$$

On the other hand, pollution stock causes a negative impact on welfare due to two different effects. First, pollution reduces non-extractive services of the environment worsening productivity (e.g. pollution in the atmosphere reduces productivity in agriculture, worsens labour productivity and provokes the physical depreciation of the equipment; see,

Bovenberg and Smulders 1995; Smulders 1995). Second, as it is standard in this kind of models, it is the cause of disutility for the representative individual. The marginal damage caused by additional units of pollution is continuously growing with this stock. Thus, assuming a quadratic pollution damage function, welfare function in region  $A$  reads:

$$w^A(t) = ap(t)q(t) - a^2q^2(t)/2 - \beta_A s^2(t)/2, \quad (1)$$

where  $s(t)$  and  $\beta_A$  are the pollution stock at time  $t$ , and region  $A$  constant environmental damage coefficient, respectively.

Region  $B$  demands the intermediate good and uses it as the unique input in his production process. For the sake of simplicity, a linear production technology is also assumed in this region. Therefore total production in both regions is proportional to the amount,  $y(t)$ , of the intermediate good traded from  $A$  to  $B$ . The whole production in region  $B$  is consumed within his borders and the produced good is neither exported to region  $A$ . Furthermore, there are no investment activities in the former. Therefore, total economic benefits in region  $B$  equates production minus the amount paid to region  $A$  for the intermediate good and minus the cost of the production process. Assuming quadratic production costs and damage functions, the welfare in this region, that is, the difference between the economic benefits and the environmental damage, can be written as:

$$w^B(t) = \mu y(t) - p(t)y(t) - y^2(t)/2 - \beta_B s^2(t)/2, \quad (2)$$

where  $\mu$  is the marginal productivity in  $B$  (or equivalently  $B/A$  output ratio); and  $\beta_B$  is the constant damage coefficient for region  $B$ . Although pollution is generated by emissions in both regions, the damages caused by pollution do not necessarily match.

The dynamics of the pollution stock is given by the following ordinary differential equation:

$$\dot{s}(t) = \gamma y(t) - \delta s(t), \quad s(0) = s_0 > 0. \quad (3)$$

The right-hand side of this equation has two terms. The first term is the flow of emissions at time  $t$ . The emission of pollutants in each region is proportional to the final output. Moreover, since production is proportional in both regions, total emissions are proportional to the traded good production, being  $\gamma$  the emissions-traded good ratio. The second term in equation (3) is the natural degradation or depreciation of the pollution stock, where  $\delta \geq 0$  represents the constant depreciation rate.

Notice that when region  $A$  decides the amount of input,  $q(t)$ , to use in her productive process, is equally fixing the supply of the intermediate good traded to region  $B$ . Region  $A$  is well aware of how a higher amount of input increases her final output and consequently her income and production costs. However, this region does not internalize the effect of an increment of the input on pollution through a higher production. Therefore, we are assuming a myopic region  $A$  that does not consider the effect of her supply decisions on the

pollution matter. Conversely, region  $B$  realizes that a higher demand will finally increase output and as a consequence the emissions of pollutant.

As far as we know, most authors that study transboundary pollution problems using the methodology of differential games make the assumption of homogeneous countries, which leads to symmetric games (see, e.g. Ploeg and Zeeuw 1992; Jørgensen and Zaccour 2001a, 2001b). These symmetries make easier the analysis of the problem. Conversely two heterogeneous regions are presented in this paper, giving rise to an asymmetric behavior. In our setting two regions play a differential game in which region  $B$  decides demand for the traded good, and region  $A$  its supply. As a result of that, in equilibrium the price and production of this intermediate good are determined and correspondingly, global emissions to the atmosphere are also fixed. Thus decisions about supply or demand of the intermediate good not only determine the interregional trade, but also the emissions of pollutants. Under these hypotheses negotiations about transboundary pollution problems are necessarily linked to interregional trade bargaining. Unfortunately, asymmetries in the game formulation lead to a more difficult analysis.

Each region maximizes its welfare function. The objective functions in both regions are given by their stream of welfare discounted at a constant rate,  $r$ , which is assumed to be the same. The specified income and cost functions guarantee each player's welfare function to be concave in its control variable. Thus the maximization problem for each region can be written as<sup>1</sup>:

$$\begin{aligned}\max_q W^A &\equiv \max_q \int_0^\infty \exp(-rt) \left[ apq - (aq)^2/2 - \beta_A s^2/2 \right] dt, \\ \max_y W^B &\equiv \max_y \int_0^\infty \exp(-rt) \left[ (\mu - p)y - y^2/2 - \beta_B s^2/2 \right] dt,\end{aligned}$$

subject to equation (3) governing the accumulation of pollutant.

Given this specification and leaving the production costs apart, assuming an initial situation with a too high pollution stock, (position 1 in Figure 1) each region perceives differently the effect of her decisions, when the aim is to reduce production in order to slow down pollution.

On the one hand, region  $A$  is not aware of the impact of her decisions upon the emissions of pollutants. Therefore, in a too polluted scenario, myopic region  $A$  would take no action, although she actually bears welfare losses associated with higher levels of pollution.

On the other hand, region  $B$  is facing a dilemma. If he reduces demand ( $D \rightarrow D'$ ) in order to cut down output and consequently emissions, how would the other region react? Region  $A$  could share the burden of a better environment accepting a lower price for the traded good (staying at point 2); or conversely,  $A$  could reduce supply ( $S \rightarrow S'$ ) increasing

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<sup>1</sup>Henceforth time arguments are omitted when no confusion is caused by doing so.

the price of the intermediate good (moving from 2 to 3). In the latter situation pollution is better off although the price remains unchanged. This price could even augment if the supply reduction is stronger than the one needed to offset the drop in demand. Therefore, if region *A* depresses supply strongly enough, the main burden of the pollution abatement would be borne by region *B* that buys the intermediate good.

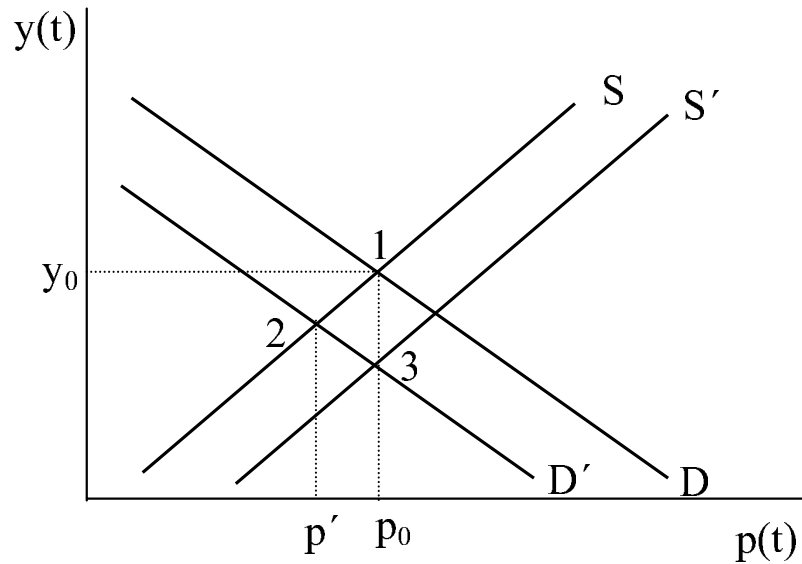


Figure 1: Supply-demand scheme

In what follows, the non-cooperative and the cooperative solutions are analyzed and compared. This paper first objective is to study whether cooperation may overcome this dilemma leading to lower global emissions. When players do not cooperate, we distinguish the open-loop Nash equilibrium that is not subgame-perfect, and the feedback Nash equilibrium that overcomes this problem. The main goal of the analysis is to attain a time-consistent agreement between both regions, which guarantees that cooperation is sustained over time.

### 3 Non-cooperative open-loop equilibria

Assuming that regions act non-cooperatively, the focus is firstly on the open-loop information structure. Players make decisions taking into account the initial value of the pollution stock and they commit themselves from the initial time onwards.

The dynamic equilibria in a non-cooperative framework are characterized by solving the maximization problems for each region separately. Assuming the open-loop information structure, the maximization problem solved by region *A* has no dynamics. This region

presents a myopic behavior about how she influences the environment. She is unaware of the effect of her supply decisions on the accumulation of pollutants. This region chooses  $q$ , given  $y$ , and therefore given the pollution stock,  $s$ . Then, the optimization problem faced by region  $A$  can be simplified to:

$$\max_{q(t)} \{ap(t)q(t) - (aq(t))^2/2\}, \quad \forall t \in [0, \infty).$$

Given that the kinematic equation (3) which defines the pollution stock dynamics does not depend on the amount of input,  $q$ , it is unnecessary to introduce a shadow price of pollution for region  $A$ . First-order optimal condition for region  $A$  maximization problem yields,  $p = aq$ , and taking into account the linear technology assumed for this region production process, we obtain the optimal supply function of the intermediate good:

$$p = y.$$

On the other hand, from the first-order necessary conditions associated to region  $B$  we can derive this region's optimal demand function:

$$p = \mu - y + \gamma\lambda_B,$$

where  $\lambda_B$  denotes region  $B$  shadow price of the pollution stock. This variable measures the marginal value of an additional unit of pollutant and thus will obviously present a negative sign.

From the demand-supply scheme it follows that the production of the intermediate good is positive if and only if  $-\lambda_B < \mu/\gamma$ . The larger is the marginal productivity in region  $B$ ,  $\mu$ , (or the lower are the emissions per unit of production,  $\gamma$ ), the lower can be the marginal value of an additional unit of pollutant,  $\lambda_B$ , to maintain a positive production.

Equating supply and demand the optimal traded good production and price are given by,

$$y = p = \frac{\gamma\lambda_B + \mu}{2}. \quad (4)$$

These price and production only depend on how region  $B$  values pollution at any time, measured by this region shadow price  $\lambda_B$ .

Region  $A$  determines the optimal amount of input,  $q$ , the traded good, and consequently, she fixes the supply for the traded good. At the same time, the optimal demand is chosen in region  $B$ , who fixes the optimal amount of the traded good at each price. The interaction between demand and supply determines both regions' productions,  $y$  and  $\mu y$ , as well as total emissions of pollutants,  $\gamma y$  and the optimal price of the traded good,  $p$ .

Furthermore, the necessary conditions for optimality in region  $B$  also yield the dynamic equations for the shadow price in this region and the pollution stock:

$$\dot{\lambda}_B = (r + \delta)\lambda_B + \beta_B s, \quad (5)$$

$$\dot{s} = \gamma y - \delta s, \quad s(0) = s_0 > 0. \quad (6)$$

In steady-state emissions exactly offset the natural degradation of the pollution stock, that is, the absorption capacity of the environment. Therefore, the pollution stock remains motionless through time and proportional to its negative shadow price, as well as to the intermediate good production.

Making use of the expression of the optimal traded good production given in (4) and the dynamics of the state and costate variables, (5) and (6), the following second order differential equation for the pollution stock can be deduced:

$$\ddot{s} - r\dot{s} - [(r + \delta)\delta + \gamma^2\beta_B/2]s = -\gamma\mu(r + \delta)/2.$$

The unique stable solution of this equation is given by:

$$s_{OL}(t) = (s_0 - s_{OL}^*)\exp(\nu t) + s_{OL}^*, \quad (7)$$

where

$$\nu = \frac{r - \Psi_{OL}}{2}, \quad s_{OL}^* = \frac{2\gamma\mu(r + \delta)}{\Psi_{OL}^2 - r^2} \quad \text{and} \quad \Psi_{OL} = \sqrt{(r + 2\delta)^2 + 2\gamma^2\beta_B}. \quad (8)$$

The steady-state pollution stock is denoted by  $s_{OL}^*$  where the subscript  $OL$  indicates that an open-loop information structure is being considered. Since  $\nu < 0$  depending on whether the initial pollution stock,  $s_0$ , lays below or above its steady-state, the time path for this variable increases or decreases toward  $s_{OL}^*$ .

The value of the steady-state pollution stock  $s_{OL}^*$  always depends positively on the  $B/A$  output ratio, that is, the marginal productivity in region  $B$ ,  $\mu$ . The greater  $\mu$  the larger the global production, and hence the pollution stock. The relationship between the discount rate,  $r$ , and the steady-state pollution stock is also positive since a higher discount rate requires a greater stream of output to obtain the same welfare, and thus, a larger level of pollution is by-produced. Conversely, a more important environmental concern, that is a larger damage coefficient, in the region  $B$ ,  $\beta_B$ , logically implies a lower pollution stock.

The effect of the emissions-traded good ratio,  $\gamma$ , and the depreciation rate,  $\delta$ , on the steady-state pollution stock depends on the size of region  $B$  damage coefficient. Thus,  $s_{OL}^*$  increases with  $\gamma$  if and only if  $\beta_B$  is lower than  $2\delta(r + \delta)/\gamma^2$ , whereas it increases with  $\delta$  if and only if  $\beta_B$  is greater than  $2(r + \delta)^2/\gamma^2$ . In both cases we can distinguish between a direct and an indirect effect.

For the emissions-traded good ratio, an increment in  $\gamma$  directly leads to greater levels of emissions. However, as it is clear from (4),  $\gamma$  measures the sensitivity of output to changes in region  $B$  shadow price. Thus, a higher  $\gamma$  induces a lower production, and as a result a shorter pollution stock. The more pernicious the pollution stock for region  $B$ , depicted by a higher  $\beta_B$ , the stronger this indirect effect of  $\gamma$  upon pollution. The depreciation rate of pollution,  $\delta$ , shows a symmetric behavior as  $\gamma$ . Being the natural degradation of the

environment, it directly lowers the pollution stock. On the other hand, by (5) is clear that a higher  $\delta$  would make region  $B$  value the pollution stock less negatively in steady-state (that is a lower  $\lambda_B$ , in absolute terms). This would mean an indirect increment in output and emissions. Now, the indirect effect decreases with the damage caused by pollution in region  $B$ ,  $\beta_B$ .

Let us note that region  $A$ , contrary to region  $B$ , does not consider the effect of her decisions on emissions, which are proportional to the amount of intermediate good traded from  $A$  to  $B$ . Therefore, a higher environmental concern in region  $A$ ,  $\beta_A$ , has no effect on the optimal time paths for the production and the price of the intermediate good, neither on the emission of contaminants or the pollution stock. These optimal paths only depend on region  $B$  shadow price,  $\lambda_B$ , which dynamics is a function of this region's damage coefficient,  $\beta_B$ .

The absolute value of parameter  $\nu$  represents the speed of convergence of the pollution stock toward its steady-state  $s_{OL}^*$ . The speed of convergence during this adjustment period depends on the discount rate, the depreciation rate, the emissions-traded good ratio and region  $B$  damage coefficient. The adjustment period can be shortened with a decrease in  $r$ , or an increment in  $\delta$ ,  $\gamma$ , and  $\beta_B$ .

Once we know the optimal time path for the pollution stock, we replace its expression in the dynamics of the second region's shadow price, getting a first order non homogeneous linear differential equation which solution is given by:

$$\lambda_B(t) = \beta_B \left[ \frac{(s_0 - s_{OL}^*) \exp(\nu t)}{\nu - (r + \delta)} - \frac{s_{OL}^*}{r + \delta} \right]. \quad (9)$$

Expressions (4) and (9) allow us to write the following optimal time paths for the traded good production and price:

$$y_{OL}(t) = p_{OL}(t) = \left[ \mu - \frac{\beta_B \gamma s_{OL}^*}{r + \delta} + \frac{\beta_B \gamma (s_0 - s_{OL}^*) \exp(\nu t)}{\nu - (r + \delta)} \right] / 2.$$

**Proposition 3.1** *Assuming open-loop strategies, traded good production,  $y_{OL}(t)$ , (and consequently its price,  $p_{OL}(t)$ ) remains positive for any  $t \geq 0$ , if and only if:*

$$s_0 < \frac{2\gamma\mu(r + \delta)}{[\Psi_{OL} - (r + 2\delta)][\Psi_{OL} + r]} \equiv \bar{s}_{OL}.$$

**Proof.** See Appendix A.

In steady-state the production is proportional to the pollution stock:

$$y_{OL}^* = p_{OL}^* = \delta s_{OL}^* / \gamma = \left[ \mu - \frac{\beta_B \gamma s_{OL}^*}{r + \delta} \right] / 2.$$

If the initial pollution stock is above (below) its steady-state, then the optimal production time path falls (grows) toward its equilibrium point,  $y_{OL}^*$ . Same result applies for the

optimal price. In steady-state, the effects of all parameters in the model (except  $\delta$  and  $\gamma$ ) are qualitatively the same on both the production, the price and the pollution stock. Regardless of region  $B$  damage coefficient,  $\beta_B$ , the steady-state production of the traded good is positively related with  $\delta$  and negatively with  $\gamma$ . That is, the faster the environment absorbs pollutants or the lower the emissions per unit of output, the higher the optimal level of output and its price, because the environmental costs are lower. Finally, it is worth saying that there exists a threshold for the pollution stock above which pollution is so damaging that no production (which by-products pollution) is encouraged. This production halt allows the natural degradation of pollution at the maximum rate.

## 4 Non-cooperative feedback equilibria

The usual criticism to open-loop strategies is that they rely on unrealistic information sets and require an infinite period of commitment. Players restrict themselves to take their decisions with the minimum information which corresponds to calendar time. Therefore, it is not a subgame-perfect equilibrium. This weakness can be overcome by the use of feedback strategies, for which player's current action depends on time as well as the state of the game at each moment, displayed by the state variables. In infinite horizon differential games, as it is the case in this paper, the interest is usually in stationary feedback Nash equilibria, which depend only on the value of the state variable and do not depend explicitly on time. In such a case, region  $A$  decides her supply of the traded good and region  $B$  his demand taking into account the current value of the pollution stock.

To obtain the stationary feedback solutions the value function  $V^i(s)$  is defined for region  $i \in \{A, B\}$ . This function represents the maximum value of the discounted welfare from a time  $t$  when the pollution stock is  $s$  onwards. The following Hamilton-Jacobi-Bellman equations have to be satisfied:

$$rV^A(s) = \max_q \left[ apq - (aq)^2/2 - \beta_A s^2/2 + (V^A)'(s) (\gamma y - \delta s) \right], \quad (10)$$

$$rV^B(s) = \max_y \left[ (\mu - p)y - y^2/2 - \beta_B s^2/2 + (V^B)'(s) (\gamma y - \delta s) \right]. \quad (11)$$

The maximization of the right hand side of each equation with respect to  $q$  and  $y$  respectively, gives the optimal feedback control for each region, and using again the hypothesis of a linear technology for the production process in region  $A$ , we get the optimal price and production:

$$y = p = \left( \mu + \gamma(V^B)'(s) \right) / 2. \quad (12)$$

These expressions are equivalent to the open-loop optimal controls but replacing the marginal value of an additional unit of pollution stock in region  $B$ ,  $\lambda_B$ , by the marginal variation of this region's value function with respect to the pollution stock,  $(V^B)'(s)$ .

For a linear quadratic game, a quadratic solution is guessed:

$$V^A(s) = \sigma_0^A - \sigma_1^A s - \sigma_2^A s^2/2, \quad (13)$$

$$V^B(s) = \sigma_0^B - \sigma_1^B s - \sigma_2^B s^2/2, \quad (14)$$

where  $\sigma_j^i$ ,  $i \in \{A, B\}$ ,  $j \in \{0, 1, 2\}$  are constants.

Now (12), (13) and (14) are substituted into the Hamilton-Jacobi-Bellman equations (10) and (11). Then equating coefficients a system of Riccati algebraic equations for  $\sigma_j^i$ ,  $i \in \{A, B\}$ ,  $j \in \{0, 1, 2\}$  can be obtained. The optimal price and optimal production of the traded good only depend on  $(V^B)'(s)$ , and so by (14), on  $\sigma_1^B$  and  $\sigma_2^B$ . Furthermore, the system of algebraic equations for these two coefficients,  $\sigma_1^B$  and  $\sigma_2^B$ , does only depend on themselves. Therefore to obtain the expression of  $(V^B)'(s)$  and the optimal price and production, the unique requirement is to solve the following system of two algebraic equations:

$$\begin{aligned} (r + \delta) \sigma_1^B + (\gamma \sigma_1^B - \mu) \gamma \sigma_2^B / 4 &= 0, \\ (r + 2\delta) \sigma_2^B + \gamma \sigma_2^B / 4 - \beta_B &= 0. \end{aligned}$$

Although the system has two pairs of solutions, only the positive pair leading to concave value functions and guaranteeing the fulfillment of the sufficient conditions for optimality is considered:

$$\begin{aligned} \sigma_1^B &= \mu [\Psi_F - (r + 2\delta)] / [\gamma(r + \Psi_F)] > 0, \\ \sigma_2^B &= 2 [\Psi_F - (r + 2\delta)] / \gamma^2 > 0, \end{aligned}$$

where  $\Psi_F = \sqrt{(r + 2\delta)^2 + \gamma^2 \beta_B}$ . The reminder values of parameters  $\sigma_j^i$ , needed to know the value functions of the players, are given in Appendix B.

The computed value for  $\sigma_1^B$  and  $\sigma_2^B$ , together with expression (12), give the price and the production of the traded good as functions of the pollution stock:

$$y = p = \frac{\mu(r + \delta)}{r + \Psi_F} - \frac{\Psi_F - (r + 2\delta)}{\gamma} s. \quad (15)$$

Replacing in the dynamics equation for the pollution stock (6), the expression of the optimal production given in (15) and solving the differential equation, the time path for the pollution stock can be written:

$$s_F(t) = (s_0 - s_F^*) \exp(\phi t) + s_F^*, \quad (16)$$

where the exponent  $\phi = r + \delta - \Psi_F$  is always negative and

$$s_F^* = \frac{\gamma \mu (r + \delta)}{(r + \Psi_F) (\Psi_F - (r + \delta))},$$

is the stable steady-state.

The sensitivity analysis of the steady-state pollution stock under feedback strategies carried out for the different parameters in the model, is qualitatively the same as in the open-loop non-cooperative game. The steady-state pollution stock does not depend on how pollution damages welfare in region  $A$ ,  $\beta_A$ . Conversely, region  $B$  damage coefficient,  $\beta_B$ , is negatively related with the steady-state pollution stock, whereas increments in the marginal productivity in region  $B$ ,  $\mu$ , or in the discount rate,  $r$ , always imply a higher pollution stock.

Likewise as in the open-loop case, the emissions-traded good ratio,  $\gamma$ , and the absorption rate,  $\delta$ , do not have a completely determined effect, but they depend on the size of region  $B$  damage coefficient,  $\beta_B$ . The sign of the derivative of  $s_F^*$  with respect to  $\gamma$  coincides with the sign of  $\Psi_F [\delta (3r + 4\delta) - \gamma^2 \beta_B] - \delta (r + 2\delta)^2$ . From this expression follows, as for the open-loop game, that low values of  $\beta_B$  ensure a positive effect of the emissions-output ratio on the pollution stock, and vice versa. On the other hand, the sign of the effect of  $\delta$  on  $s_F^*$  depends on expression  $\Psi_F [\gamma^2 \beta_B - (r + 2\delta) (3r + 2\delta)] + r\Psi_F^2 + 2\delta (r + \delta) (r + 2\delta)$ , which sign, contrary to previous expression, is positive for large values of  $\beta_B$ , and negative when the environmental losses in region  $B$  are low enough.

The speed of convergence of the pollution stock time path toward the steady-state value  $s_F^*$  depends negatively on the discount rate,  $r$ , and positively on region  $B$  damage coefficient,  $\beta_B$ , and the emissions-traded good ratio,  $\gamma$ . The relationship between the depreciation rate and the speed of convergence is positive if and only if the damage coefficient is short enough,  $\beta_B < 3(r + 2\delta)^2 / \gamma^2$ . Let us note that the higher the speed of convergence the shorter the period of adjustment to the equilibrium point, and vice versa. The effects of changes in the different parameters on the speed of convergence are the same for the open-loop and the feedback non-cooperative cases, except for the depreciation rate,  $\delta$ , that in the open-loop case was always positively related to the speed of convergence, while under the feedback hypothesis it is only true for low values of  $\beta_B$ .

The optimal time paths for the production,  $y_F(t)$ , and the price,  $p_F(t)$ , of the traded good can be obtained replacing in (15) the pollution stock optimal path given in (16).

**Proposition 4.1** *Assuming feedback strategies, traded good production,  $y_F(t)$ , and price,  $p_F(t)$ , remain positive for any  $t \geq 0$ , if and only if:*

$$s_0 < \frac{\gamma\mu(r + \delta)}{[\Psi_F - (r + 2\delta)][\Psi_F + r]} \equiv \bar{s}_F.$$

**Proof.** See Appendix C.

Relationship connecting the steady-states for traded good production and price,  $y_F^*$  and  $p_F^*$  with the pollution stock is the same as in the open-loop case. In consequence, the dependency of  $y_F^*$  and  $p_F^*$  on the different parameters of the model is qualitatively the

same than that of  $s_F^*$ , except for  $\gamma$  and  $\delta$ . Again as in the open-loop case, the production of the traded good and the price at the steady state is negatively related with  $\gamma$ . However, the effect of  $\delta$  on  $y_F^*$  and  $p_F^*$  is not fully determined.

#### 4.1 *Open-loop versus feedback non-cooperative strategies*

Depending on the information sets of the players, the outcomes of the game differ by assuming an open-loop or a feedback mode of play.

**Proposition 4.2** *In steady-state, if regions do not cooperate, the pollution stock under feedback strategies will not be smaller than under open-loop strategies. Likewise the price and demand for the traded good will be larger assuming subgame-perfect strategies.*

**Proof.** Inequality  $s_{OL}^* \leq s_F^*$  holds if and only if,

$$\gamma\mu(r + \delta)/[(r + \Psi_F)(\Psi_F - (r + \delta))] \geq \gamma\mu(r + \delta)/[2(r + \delta)\delta + \beta_B\gamma^2].$$

Previous inequality can be re-written as,

$$(r + \Psi_F)(\Psi_F - (r + \delta)) \leq 2(r + \delta)\delta + \beta_B\gamma^2,$$

or equivalently,

$$r + 2\delta - \Psi_F \leq 0,$$

which always holds. □

This is an unsurprising result. For a transboundary pollution problem (Ploeg and Zeeuw 1992; Mason 1997; Jørgensen and Zaccour 2001a) also show that the open-loop Nash equilibrium underestimates the damage of not coordinating policies and leads to lower emissions than the feedback strategies. Likewise (Ploeg 1987) obtains a similar result for a model of renewable resources extraction.

Next proposition extends the previous analysis, restricted to the steady-state values, to all the pollution stock time paths during the adjustment period to their long run values.

**Proposition 4.3** *When regions do not cooperate, the pollution stock time path under feedback strategies remains below this path under open-loop strategies:  $s_{OL}(t) \leq s_F(t)$  for any  $t \geq 0$ .*

**Proof.**

By the functional expressions for the pollution stock time paths under the open-loop and the feedback information structures in (7) and (16), inequality  $s_{OL}(t) \leq s_F(t)$  can be written as:

$$(s_{OL}^* - s_0) [1 - \exp(\nu t)] \leq (s_F^* - s_0) [1 - \exp(\phi t)].$$

It is straightforward to prove that  $\phi < \nu < 0$ , and therefore,

$$0 < 1 - \exp(\nu t) < 1 - \exp(\phi t) < 1. \quad (17)$$

Furthermore, by Proposition 4.2 it follows,

$$s_{OL}^* - s_0 \leq s_F^* - s_0. \quad (18)$$

Then, we distinguish three cases:

i  $s_0 \leq s_{OL}^* \leq s_F^*$ .

ii  $s_{OL}^* \leq s_0 \leq s_F^*$ .

For these two cases, proof immediately follows from inequalities (17) and (18).

iii  $s_{OL}^* \leq s_F^* \leq s_0$ .

In this last case, the pollution stock decreases towards its steady state value under both scenarios. Thus, a sufficient condition for  $s_{OL}(t) \leq s_F(t)$  is given by:

$$\dot{s}_{OL}(t) \leq \dot{s}_F(t), \quad (19)$$

or equivalently,

$$\nu(s_0 - s_{OL}^*) \exp(\nu t) \leq \phi(s_0 - s_F^*) \exp(\phi t). \quad (20)$$

A sufficient condition guaranteeing last inequality is:

$$(s_{OL}^* - s_0)\nu \geq (s_F^* - s_0)\phi,$$

or equivalently,

$$s_0 \leq \frac{2\gamma\mu(r + \delta)[\Psi_{OL} - \Psi_F]}{[2\Psi_F - \Psi_{OL} - (r + 2\delta)][\Psi_{OL} + r][\Psi_F + r]} \equiv \hat{s}.$$

As Propositions 3.1 and 4.1 state, a positive production under the open-loop and the feedback information structures establishes two upper bounds for the initial pollution stock,  $\bar{s}_{OL}$  and  $\bar{s}_F$ , respectively. It is not difficult although tedious to prove that  $\bar{s}_{OL} \leq \bar{s}_F \leq \hat{s}$ . Therefore, a positive production for the whole time horizon both under the open-loop and the feedback information structures, which is equivalent to  $s_0 \leq \bar{s}_{OL} \leq \bar{s}_F$ , guarantees  $s_0 \leq \hat{s}$ , which proves  $s_{OL}(t) \leq s_F(t)$  in case iii.  $\square$

Let us note that a null environmental concern in the second region, that is, a null damage coefficient  $\beta_B = 0$  leads to the equality between both steady-states,  $s_F^*$  and  $s_{OL}^*$ . Moreover, under this hypothesis the exponents of the pollution stock time paths for the open-loop and feedback cases are the same and in fact, both optimal trajectories coincide.

That is, when the second region is not concerned about the environmental consequences neither of its production process nor that of its neighbor the use of different kinds of strategies does not have effect either in the short or in the long run pollution stock.

The general specification of our game belongs to the well-known class of linear-quadratic differential games, which are characterized by the property that the system dynamics is a first order polynomial and the objective functions are second order polynomials with respect to the state and the control variables.

Another analytically tractable class of differential games is those games for which the system dynamics and the objective functions are polynomials of degree one with respect to the state variable and which satisfy a certain property concerning the interaction between control variables and state variables. In particular, if there is no multiplicative interaction at all between the state and the control variables in the game this property is fulfilled. This class of games are called linear state or state-separable games. This class of games has a very useful property. The linearity in the state variables together with the decoupled structure between the state variables and the control variables implies that the open-loop equilibrium is subgame perfect. Let us remark that the game we are studying in this paper belongs to the class of linear state differential games when the second region has a null environmental concern, that is,  $\beta_B = 0$ . Therefore, in this case, the open-loop and feedback strategies coincide.

## 5 Cooperative equilibria

In this section both regions agree on cooperate and they jointly maximize the sum of their welfare functions. The instantaneous welfare function in region  $A$ , taking into account the linear technology assumed in this region production process, can be rewritten in terms of the amount of the intermediate good traded from  $A$  to  $B$ , as follows:

$$w^A(t) = p(t)y(t) - y^2(t)/2 - \beta_A s^2(t)/2.$$

The sum of both welfare functions is independent of the price of the traded good, and therefore, in the cooperative framework both regions jointly choose the amount of the intermediate good to trade from region to the other. Therefore, the solution of the cooperative game does not give an optimal price, but on the contrary this price is free. Later on we will see how this indeterminacy in the price under cooperation can play an important role as a sharing mechanism of the surplus of cooperation.

The cooperative maximization problem is defined as follows:

$$\max_y W_C \equiv \max_y [-y^2 + \mu y - (\beta_A + \beta_B)s^2/2],$$

subject to the dynamic evolution of the pollution stock given by (3). In the objective

functional the same bargaining power is assigned to both regions and it is easy to show that it is concave in the control variable. Solutions will be Pareto optimal.

To obtain the stationary feedback solutions the value function  $V_C(s)$  is defined. The following Hamilton-Jacobi-Bellman equation has to be satisfied:

$$rV_C(s) = \max_y [-y^2 + \mu y - (\beta_A + \beta_B)s^2/2 + V'_C(s)(\gamma y - \delta s)].$$

The optimal feedback control is given by

$$y = (\mu + \gamma V'_C(s))/2. \quad (21)$$

Following the same steps than in the non-cooperative feedback case, a quadratic solution is guessed:

$$V_C(s) = \sigma_{C0} - \sigma_{C1}s - \sigma_{C2}s^2/2,$$

where constants  $\sigma_{Ci}$  for  $i \in \{0, 1, 2\}$ , after some computations are determined:

$$\begin{aligned} \sigma_{C2} &= [\Psi_C - (r + 2\delta)] / (\gamma^2), \\ \sigma_{C1} &= [\Psi_C - (r + 2\delta)] \mu / [\gamma(r + \Psi_C)], \\ \sigma_{C0} &= (r + \delta)^2 \mu^2 / [r(r + \Psi_C)^2], \end{aligned}$$

where  $\Psi_C = \sqrt{(r + 2\delta)^2 + 2(\beta_A + \beta_B)\gamma^2}$  and all the coefficients are positive.

The demand for the traded good as function of the pollution stock, once the values for  $\sigma_{C1}$  and  $\sigma_{C2}$  have been computed, together with expression (21) can be written as:

$$y(t) = \mu(r + \delta)/(r + \Psi_C) - (\Psi_C - (r + 2\delta))/(2\gamma)s(t). \quad (22)$$

Solving the dynamic equation for the pollution stock (6) once the expression of the optimal production given in (22) is replaced, the time path for the pollution stock is:

$$s_C(t) = (s_0 - s_C^*) \exp(zt) + s_C^*, \quad (23)$$

where as in the previous cases, the exponent  $z = (r - \Psi_C)/2$  is always negative and

$$s_C^* = \frac{2\gamma\mu(r + \delta)}{\Psi_C^2 - r^2}, \quad (24)$$

denotes the stable steady-state.

Contrary to the non-cooperative case, under cooperation the first region's environmental concern,  $\beta_A$ , does affect the steady-state pollution stock as well as the speed of convergence during the adjustment period to this long-run equilibrium through  $\Psi_C$ . That is, in the cooperative case, the steady-state pollution stock not only depends on how pollution affects welfare in region  $B$ , but also in region  $A$ . An increment in  $\beta_A$  or in  $\beta_B$

induces a fall in the steady-state pollution stock,  $s_C^*$ . The same effect has a reduction in either the discount rate,  $r$ , or the  $B/A$  output ratio,  $\mu$ . The effect on  $s_C^*$  of changes in the emissions-traded good ratio,  $\gamma$ , or in the depreciation rate,  $\delta$ , depends on the global pollution damage that the two regions bear,  $\beta_A + \beta_B$ . Thus, the value of  $s_C^*$  is positively related with  $\delta$  and negatively related with  $\gamma$  if and only if  $\beta_A + \beta_B$  is large enough<sup>2</sup>. Let us note that the dependency of the steady-state pollution stock on the parameters of the model is the same as the dependency in the non-cooperative case. There is only one significant difference: now the effect of changes either in  $\delta$  or in  $\gamma$  does not depend only on the size of the second region damage coefficient, but on the global environmental concern,  $\beta_A + \beta_B$ .

The speed of convergence depends positively on both regions damage coefficients,  $\beta_A$  and  $\beta_B$ , as well as on  $\delta$  and  $\gamma$ , but depends negatively on the discount rate,  $r$ . That is, a greater environmental concern either in region  $A$  or  $B$  implies a shorter period of adjustment to the steady-state pollution stock, while the opposite happens when the regions discount their future welfare at greater discount rates. The main difference with the non-cooperative cases is that now the speed of convergence also depends on the first region damage coefficient,  $\beta_A$ .

**Proposition 5.1** *If regions cooperate, the production,  $y_C(t)$  remains positive for any  $t \geq 0$ , if and only if:*

$$s_0 < 2\gamma\mu(r + \delta)/[(\Psi_C - (r + 2\delta))(r + \Psi_C)] \equiv \bar{s}_C.$$

**Proof.**

The optimal time path for the demand of the traded good is determined replacing in (22) the pollution stock optimal path given in (23). It is easy to prove that this optimal path takes positive values if and only if the pollution stock path is below the upper bound  $\bar{s}_C$ , which is always greater than the steady-state pollution stock,  $s_C^*$ . Then, for any initial pollution stock below this bound,  $s_0 < \bar{s}_C$ , a positive time path for the demand can be guaranteed.  $\square$

As in the non-cooperative cases, the relationship connecting the demand for the traded good and the pollution stock at the steady-state is  $y_C^* = \delta s_C^*/\gamma$ . The effect of changes in the different parameters of the model on  $y_C^*$  is the same than that on  $s_C^*$ , except for  $\gamma$  and  $\delta$ . Now changes in these two parameters have a completely defined effect. Thus, any increment in  $\gamma$  implies a fall in  $y_C^*$ , while an increase in  $\delta$  leads to a greater  $y_C^*$ .

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<sup>2</sup> $\partial s_C^*/\partial \delta > 0 \Leftrightarrow \beta_A + \beta_B > 2(r + \delta)^2/\gamma^2$ .  
 $\partial s_C^*/\partial \gamma < 0 \Leftrightarrow \beta_A + \beta_B > 2\delta(r + \delta)/\gamma^2$ .

## 5.1 Cooperative versus non-cooperative strategies

The outcomes under the cooperative and the non-cooperative behavior are compared in this section. In the non-cooperative case, we distinguish between open-loop and feedback strategies. First, the effect on the steady-state pollution and production of the traded good is analyzed. Second, the interest is in the time paths for these variables toward their stationary positions.

**Proposition 5.2** *In steady-state, production, and consequently the pollution stock, are lower under cooperation,  $s_C^*$ , than under open-loop non-cooperative strategies,  $s_{OL}^*$ .*

**Proof.** From expressions (7) and (23), it is straightforward to show that the steady-state pollution stock is lower when players cooperate.  $\square$

**Corollary 5.1** *In steady-state, production, and consequently the pollution stock, are lower under cooperation,  $s_C^*$ , than under feedback non-cooperative strategies,  $s_F^*$ .*

**Proof.** This result is straightforward from the previous proposition, since proposition 4.2 establishes that  $s_{OL}^*$  is always lower than  $s_F^*$ .  $\square$

In what follows we search conditions which ensure that the pollution stock at any time reaches lower levels under cooperation, no matter whether in the non-cooperative case we assume an open-loop or a feedback information structure.

In the non-cooperative case, Proposition 4.3 states that the pollution stock is greater assuming feedback strategies rather than open-loop strategies, not only at the steady state, but along the transition paths towards this point, that is,  $s_{OL}(t) \leq s_F(t)$  for any  $t > 0$ . Therefore, we only need to study under which conditions  $s_C(t) \leq s_{OL}(t)$  for any  $t \geq 0$ .

Next proposition compares the pollution stock time paths associated to the cooperative and non-cooperative behavior during the adjustment period to their steady-states, when in the non-cooperative case the players use open-loop strategies.

**Proposition 5.3** *The pollution stock time path under cooperation remains always below the pollution stock time path when players do not cooperate, irrespective of whether they follow an open-loop or a feedback information structure.*

**Proof.**

We have to prove that the pollution stock time path under cooperation is lower than the pollution stock assuming an open-loop information structure,  $s_C(t) < s_{OL}(t)$ . This is equivalent to

$$(s_C^* - s_0) [1 - \exp(zt)] \leq (s_{OL}^* - s_0) [1 - \exp(\nu t)]. \quad (25)$$

First, it is straightforward to see that the speed of convergence of the time paths during the adjustment period to its corresponding steady-state will be greater under the cooperative mode of play,  $z \leq \nu \leq 0$ , or equivalently,

$$0 \leq 1 - \exp(\nu t) \leq 1 - \exp(zt) < 1. \quad (26)$$

Furthermore, since  $s_C^* < s_{OL}^*$ , then

$$s_C^* - s_0 \leq s_{OL}^* - s_0. \quad (27)$$

Likewise as in the proof of Proposition 4.3, we distinguish three cases:

- i  $s_C^* \leq s_{OL}^* \leq s_0$ . Proof immediately follows from inequalities (26) and (27).
- ii  $s_C^* \leq s_0 \leq s_{OL}^*$ . The left hand side of (26) is negative while the right hand side is positive, thus, proof is obvious.
- iii  $s_0 \leq s_C^* \leq s_{OL}^*$ .

Since the initial pollution stock,  $s_0$ , is lower than  $s_C^*$  and  $s_{OL}^*$ , the pollution stock time path grows towards its stationary state either in the cooperative or the non-cooperative open-loop case. We will prove that the pollution stock grows faster under the non-cooperative open-loop information structure, that is,  $\dot{s}_C(t) \leq \dot{s}_{OL}(t) \forall t \geq 0$ . Or equivalently,

$$z(s_0 - s_C^*) \exp(zt) \leq \nu(s_0 - s_{OL}^*) \exp(\nu t), \quad \forall t \geq 0.$$

Since  $s_0 - s_C^*$  and  $s_0 - s_{OL}^*$  are both negative, and  $\exp(zt) \leq \exp(\nu t)$ , a sufficient condition which ensures this inequality is:

$$z(s_C^* - s_0) \geq \nu(s_{OL}^* - s_0).$$

And since  $z \leq \nu$ , the fulfillment of this condition for  $s_0 = 0$ , is a sufficient condition:

$$zs_C^* \geq \nu s_{OL}^*.$$

This condition always holds. □

## 6 A time-consistent agreement between both regions

In this section we focus on the definition of a sharing mechanism which allows to distribute the surplus of cooperation between both regions in such a way that no player has incentive to deviate from the agreement. For the cooperation to be sustainable, each region's welfare

has to be proved greater under cooperation than under the non-cooperative mode of play. An agreement signed by the players at an initial position is sustained over time only if cooperation is individually rational throughout the game. Thus an important component of an agreement between the players who wish to ensure its sustainability over time is the study of two individual rationality concepts. In a first stage, the global individual rationality implies that for the entire duration of the agreement, in the cooperative solution each region receives a payoff which is no less than its disagreement payoff. In a second stage, we analyze a more stringent concept called instantaneous individual rationality. This property requires the dominance of cooperative payoffs at each point in time. If the agreement satisfies this property it is called time-consistent (definitions of the different concepts of dynamic individual rationality used in the differential games literature can be found in Jørgensen and Zaccour 2002).

The propositions established in previous section show that less pollution requires a lower production in both regions. Besides the reduction in the pollution stock due to cooperation would make both regions better-off.

By virtue of joint maximization, total welfare is not lower when regions cooperate: the joint and global surplus of cooperation is never negative. This surplus is the difference between the optimal joint payoff and the sum of the disagreement payoffs. Although joint welfare is higher under cooperation, this does not assure a higher welfare for each single region. That is, although the cooperative surplus for the entire planning period is always non-negative, this fact does not guarantee the satisfaction of the global individual rationality property.

To analyze the first concept of global overall individual rationality, the first step is to compare the payoffs to go of each player under the cooperative and non-cooperative settings. The value functions given by (13) and (14) have been computed for region A and Region B under non-cooperative feedback strategies. Conversely, the value function under cooperation has been defined for the joint coalition. Thus, to compare cooperative and non-cooperative optimal welfares, a distribution mechanism of the joint cooperative value,  $V_C(s)$ , between both regions must be defined. We will assume that the payoff to go of each player under cooperation corresponds to the discounted value of his or her flow of welfare, given by (1) and (2) respectively, assuming that the cooperative strategy is played. Let us define  $w^A(t)$ ,  $w^B(t)$  as the instantaneous non-cooperative optimal welfare in regions A and B, obtained by replacing production, price and pollution stock by their optimal time paths in (15) and (16), in the feedback non-cooperative case. Likewise,  $w_C^A(t)$ ,  $w_C^B(t)$  are defined as the optimal cooperative instantaneous welfare, obtained replacing  $y$ , and  $s$  by their optimal paths under cooperation, given by (22) and (23). Notice that these instantaneous cooperative optimal welfare functions are not fully determined, but they depend on the price of the traded good.

Thus, we can write the global welfare in each region in the non-cooperative case:

$$V^A(s) = \int_0^\infty w^A(t) e^{-rt} dt, \quad V^B(s) = \int_0^\infty w^B(t) e^{-rt} dt,$$

as well as in the cooperative case:

$$V_C^A(s, p) = \int_0^\infty w_C^A(t) e^{-rt} dt, \quad V_C^B(s, p) = \int_0^\infty w_C^B(t) e^{-rt} dt,$$

where  $V_C^A(s, p)$ ,  $V_C^B(s, p)$  are the payoffs to go under cooperation for region  $A$  and  $B$ , respectively. Both functions are continuous in  $s$  and  $p$  and  $V_C(s) = V_C^A(s, p) + V_C^B(s, p)$ .

Although joint global cooperative welfare is independent of the price of the intermediate good,  $p$ , and it is never below this welfare under the disagreement, this is not true for each region payoff to go. Given the optimal time path for the production of the intermediate good, any non-negative price of this good is compatible with a maximum of the cooperative problem. However, at any point in time, as  $p$  grows, also do the revenues in the exporting region  $A$ , and the input costs in region  $B$  who buys the intermediate good. Therefore, in this model, the price of the traded good can be used as a tool to distribute the joint cooperative welfare between both regions.

When players cooperate, no region will cease cooperation if this brings him a lower discounted welfare. A non-negative joint surplus from cooperation always exists,  $V_C(s) - (V^A(s) + V^B(s))$ . However, the sharing mechanism in the cooperative case does not guarantee that each region obtains a higher welfare under cooperation. The sign of  $V_C^i(s, p) - V^i(s)$ ,  $i \in \{A, B\}$ , is not fully determined for both regions<sup>3</sup>.

Next we define the gap between the gains from cooperation in region  $B$  and region  $A$ :

$$G_A^B(s, p) = V_C^B(s, p) - V^B(s) - [V_C^A(s, p) - V^A(s)].$$

For a given initial pollution stock, the gains from cooperation are the maximum in the importing region  $B$  (and the minimum in the exporting region  $A$ ) when the cooperative price of the traded good is fixed equal to zero. Region  $B$  gets his input at no cost and region  $A$  obtains no revenues from her production. Thus,  $G_A^B(s, p)$  is a continuous and decreasing function of  $p$ . We define  $\hat{p}$  as the constant price of the traded good for which  $G_A^B(s, p) = 0$ . This price guarantees that each region obtains an equal part of the surplus of cooperation. This price is in concordance with the egalitarian principle of cooperative game theory (see Moulin 1998 and Jørgensen and Zaccour 2001a, 2001b for two applications to pollution problems). Each region global cooperative welfare for this price is:

$$\bar{V}_C^i(s) = V^i(s) + [V_C(s) - (V^A(s) + V^B(s))] / 2, \quad i \in \{A, B\}.$$

For this price each region obtains the same welfare as when they played non-cooperative feedback strategies plus half the surplus of cooperation. Both regions are better-off

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<sup>3</sup>Due to joint maximization, this sign cannot be negative for both regions.

under cooperation. Therefore, this price ensures the satisfaction of the global individual rationality property.

In what follows we decompose the net gains from cooperation over time. Our aim is to achieve not only global but also instantaneous dynamic individual rationality. That is, we look for an instantaneous welfare under cooperation, which dominates the instantaneous welfare under non-cooperative feedback strategies.

When the price of the traded good is equal to  $\hat{p}$ , each region attains the same global gain from cooperation, defined as the difference between each region global cooperative welfare for  $\hat{p}$ , minus his non-cooperative welfare. These identical gains from cooperation can be written as:

$$\bar{V}_C^i(s) - V^i(s) = \int_0^\infty e^{-rt} [w_C^A(t) + w_C^B(t) - (w^A(t) + w^B(t))] / 2 dt, \quad i \in \{A, B\}.$$

Although a constant price equal to  $\hat{p}$ , for the whole time horizon, ensures a higher global cooperative welfare, the amount  $\Delta(t) = [w_C^A(t) + w_C^B(t) - (w^A(t) + w^B(t))]$  is not necessarily positive at every point in time. One way to overcome this shortcoming is to define the instantaneous gain from cooperation as the constant amount  $\Delta$  which satisfies:

$$\int_0^\infty e^{-rt} \Delta(t) dt = \int_0^\infty e^{-rt} \Delta dt.$$

That is,

$$\Delta/2 = (\bar{V}_C^i(s) - V^i(s)) / r \geq 0, \quad i \in \{A, B\}.$$

The instantaneous amount of welfare allocated to each region will be defined as the instantaneous non-cooperative optimal welfare plus constant  $\Delta/2$ :

$$\bar{w}_C^i(t) = \Delta/2 + w^i(t), \quad \forall i \in \{A, B\}.$$

From the specification of  $\Delta$ , it is obvious that,

$$\int_0^\infty e^{-rt} \bar{w}_C^i(t) dt = \bar{V}_C^i(s), \quad \forall i \in \{A, B\}.$$

Likewise, at the same time it is easy to observe that instantaneous dynamic individual rationality holds since  $\Delta \geq 0$ , and consequently,

$$\bar{w}_C^i(t) \geq w^i(t), \quad \forall t \in [0, \infty), \forall i \in \{A, B\}.$$

Note that constant  $\Delta$  can be understood as the positive constant payment that results from distributing the global gains from cooperation as an infinite horizon continuous annuity with an interest rate equal to  $r$ . Therefore, this mechanism allocates to each region and at any moment in time, his or her instantaneous non-cooperative optimal welfare,  $w^i(t)$ , plus the constant payment of an infinite continuous annuity which present value (being  $r$  the interest rate) is given by half the global gain from cooperation.

## 7 Conclusions

To analyze the interrelationship between trading regions taking into account the dynamic effect of their production processes on the environment, a differential game between these two heterogeneous regions has been studied. Region  $A$  determines the supply of an intermediate good and region  $B$  its demand. This good is traded from region  $A$  to region  $B$  which uses it as a productive input. Emissions of pollutants in each region are a by-product of production and are accumulated giving rise to a transboundary pollution problem. This pollution stock negatively affects welfare in each region. Cooperative and open-loop and feedback non-cooperative equilibria have been found.

In the non-cooperative case steady-state price, production and pollution stock are lower under the open-loop than under the feedback information structure. Moreover, the pollution stock is lower in the open-loop case also at any time along the optimal trajectory.

In contrast with non-cooperation, when players cooperate, a lower production in each region and therefore, a lower pollution stock have been established. These results apply not only in steady state but also along the overall time paths.

Given that total welfare is greater when players cooperate and fixing the price  $\hat{p}$  that leads to the egalitarian rule as the sharing mechanism, each one of the regions would be worse-off by playing non-cooperative feedback strategies. The sharing mechanism has been defined to guarantee in a first stage global individual rationality, and in a second stage instantaneous individual rationality.  $\hat{p}$  is the price that region  $B$  has to pay to region  $A$  for the traded good, so that both regions obtain an equal part of the surplus from cooperation, which give them the necessary incentive to cooperate. This sharing mechanism guarantees a time-consistent agreement between regions  $A$  and  $B$ .

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## Appendix A

By (4), a positive traded good production occurs if and only if the negative valuation of pollution is sufficiently low in absolute terms  $-\lambda_B < \mu/\gamma$ . By expression (9) this inequality can be rewritten in terms of the pollution stock:

$$s_{OL}(t) < \frac{2\gamma\mu(r+\delta)}{[\Psi_{OL} - (r+2\delta)][\Psi_{OL} + r]} \equiv \bar{s}_{OL}.$$

It is easy to prove that  $s_{OL}^* < \bar{s}_{OL}$  which guarantees,

$$s_0 < \bar{s}_{OL} \Leftrightarrow s_{OL}(t) < \bar{s}_{OL}, \quad \forall t \geq 0. \square$$

## Appendix B

$$\begin{aligned} \sigma_2^A &= (\beta_A\gamma^2 - (r+2\delta - \Psi_F)^2)/[(2\Psi_F - (r+2\delta))\gamma^2], \\ \sigma_1^A &= \mu(r+\delta)[\Psi_F^2 + \beta_A\gamma^2 - \Psi_F(r+2\delta)]/[\gamma(r+2\delta - 2\Psi_F)(\delta - \Psi_F)(r + \Psi_F)], \\ \sigma_0^A &= [(r+\delta)^2\mu[\mu(\delta(r+2\delta) - 2\beta_A\gamma^2) + \Psi_F r]/[2r(r+2\delta - 2\Psi_F)(\delta - \Psi_F)(r + \Psi_F)^2], \\ \sigma_0^B &= \mu^2(r+\delta)^2/[2r(r + \Psi_F)^2]. \end{aligned}$$

## Appendix C

From equation (15) follows that the optimal time path for the production, and therefore, for its price, takes positive values if and only if the pollution stock path is below an upper bound:

$$s_F(t) < \frac{\gamma\mu(r+\delta)}{[\Psi_F - (r+2\delta)][\Psi_F + r]} \equiv \bar{s}_F.$$

As for the open-loop information structure,  $s_F^* < \bar{s}_F$ , and therefore,

$$s_0 < \bar{s}_F \Leftrightarrow s_F(t) < \bar{s}_F, \quad \forall t \geq 0. \square$$