The Location of Economic Activity With Imperfect Labour Markets

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Abstract

The first generation New Economic Geography (NEG) models assume perfect labour markets and perfect labour mobility between countries/regions. It is however obvious that labour markets are far from perfect and that - especially in Europe - labour mobility remains limited. The question we want to answer in this paper is whether the forces determining the location of firms (agglomeration and dispersion forces) change when we assume labour to be immobile and labour markets to be imperfect. It turns out that the introduction of unions per se does not really change the location decision of firms. Location outcomes remain similar to the ones in existing NEG models: a symmetric outcome is stable at both high and low transport costs while agglomeration is stable for intermediate transport costs. Moreover, the fact that we allow the number of unions to vary in our model makes it possible to analyse the effect of the level of wage setting on location decisions. We show that the degree of 'centralisation' (at firm, sectoral or national level) and 'regionalisation' (at regional or supra-regional level) of the wage setting affects the wage level - and therefore firms' location decisions - in a significant way.

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1 Introduction

The first generation New Economic Geography (NEG) models assume perfect labour markets and perfect labour mobility between countries/regions. It is however obvious that labour markets are far from perfect and that - especially in Europe - labour mobility remains limited. The question we want to answer in this chapter is whether the forces determining the location of firms (agglomeration and dispersion forces) change when we assume labour to be immobile and labour markets to be imperfect.

The key feature of the model in this chapter is the introduction of *the degree of centralisation of wage setting, reflected by the number of unions present* (more unions imply wage setting at a more decentralised level). This is a very particular feature of the European labour market, where we are confronted with different degrees of centralisation in different countries. The Nordic countries, for instance, have a highly centralised wage setting while in the UK, wages are rather set at a decentralised level.

Introducing this degree of centralisation, we can *first of all* prove that when the number of unions increases, the wages will decrease. *Secondly*, the location outcomes in this model are similar to the ones in existing NEG models: a symmetric outcome (equal number of firms in each region) is stable at both high and low transport costs while agglomeration (all firms in one region) is stable for intermediate transport costs. Note that in the benchmark NEG models, agglomeration is stable for low transport costs too. However, introducing extra dispersion forces in these models renders agglomeration unstable again for low transport costs. In our model we also have an extra dispersion force that stems from imperfections in the labour market.

Thirdly, we show that the degree of 'centralisation' (at firm, sectoral or national level) and 'regionalisation' (at regional or supra-regional level) of the wage setting affects the wage level in a significant way. Location decisions of firms will therefore be influenced by both the fact whether wages are set at a more or less centralised level ánd whether they are set in each of the regions separately or rather at a coordinating level. Based on these outcomes, *finally*, our model allows us to make some policy recommendations as far as the attraction of new firms is concerned. We prove that wage setting at the firm level is the best policy to attract firms when transport costs are low while wage setting at the national level is most effective to attract firms when transport costs are high. Moreover, wage setting at the supra-regional level will be beneficial for the more agglomerated region and hurt the peripheral region.

In order to situate our model in the literature, in the next section we give a short overview of the existing models and focus on their differences and similarities. In section 3, we present our own model, derive the conditions for the symmetry and agglomeration outcomes to be stable equilibria ánd have a look at the role unions play in these location outcomes. We show both that wages are a decreasing function of the number of unions ánd that agglomeration and symmetry are both stable but for different levels of transport costs - as is the case in other NEG models too. In section 4 we dig deeper into the role of unions. We show that the degree of 'centralisation' and 'regionalisation' of wage setting affects the wage level in a significant way ánd we make policy recommendations as far as the attraction of new firms is concerned. The last section states the conclusions and offers some ideas for future research.

2 Literature overview

This chapter combines two branches of the literature. Since we introduce union wage setting in a NEG model, we want to focus not only on traditional *union-models* but also on the existing *NEG models*.

2.1 Unionised wage setting

The essential point in this chapter is the link between the degree of (de)centralisation of wage setting on the one hand and the level of the wage on the other hand. In our model there is a monotonically increasing relationship between the level of centralisation and the wage: the more centralised the wage setting, the higher the wage. Note that the model is partial equilibrium and that it is only the price index effect that affects the union's maximisation problem. Calmfors and Driffil (1988) however find in their model a hump-shaped relationship between centralisation and real wages. Wage setting at a fully (de)centralised level will lead to lower wages while wages set at industry level will turn out highest. The reason for the difference in outcome is the fact that Calmfors and Driffil assume a general equilibrium model where apart from the price index effect, terms of trade effects and income effects play a role too. We however restricted ourselves to a partial equilibrium model because we wanted to be able to analytically investigate the location decisions of firms - something which would turn out to be impossible in a general equilibrium model.

2.2 New Economic Geography models

In order to clearly situate the new aspects of the model developed in this chapter, it is necessary to give a short overview of (some of) the NEG models we know thus far. We do not claim to give an exhaustive overview of all of them (we refer to e.g. Fujita, Krugman and Venables (1999), Fujita and Thisse (2001) and the first chapter of this dissertation for more extended overviews) but we want to tackle those models that allow us to situate our own. The three models we will mention are to be found in the pioneering articles from Krugman (1991), Venables (1996) and the more recent contribution of Picard and Toulemonde (2002). We contrast these three models with the model developed in this chapter. Table 1 gives an overview of the main characteristics of the four models. We will only highlight the main differences in set-up here. The different implications for the location of economic activity will be discussed and compared after we have developed our own model.

Note that in all four models the goods markets are modelled in the same way but the labour market modelling differs. The first two models assume labour markets to be perfect (no unions) while the one from Picard and Toulemonde, like the model discussed in this chapter, introduces unions in the analysis. The second basic difference between the models is that the Krugman (1991) one is the only one where (manufacturing) labour is assumed to be mobile between countries/regions. The other models - like ours - assume labour to be mobile only intersectorally.

All models help us gain insight in the location decision of firms by defining centripetal (pulling towards the centre) and centrifugal (pushing away from the centre) forces. In all models, the relative strength of these forces - and therefore the location outcome - will depend on the level of the transport costs. At high transport costs, centripetal forces will outweigh the centrifugal ones. When transport costs decline, centrifugal forces will start to dominate. At very low transport costs it depends on the model which of the forces dominates. We refer to a discussion of the relationship between transport costs and location in the different models to later in this chapter. If the centripetal forces outweigh the centrifugal forces, we will find firms agglomerating in one single country/region. If however the centrifugal forces dominate, agglomeration can never be an equilibrium.

Table 1: **Comparison of NEG models**

	Krugman (1991)	Venables (1996)	P&T (2002)	this chapter
unions	No	No	Yes	Yes
L mobile	Yes	No	No	No
centripetal force(s)	forward linkage: price index effect backward linkage: home market effect	forward linkage: lower costs of input backward linkage: larger demand	home market effect	home market effect
centrifugal force(s)	imports are costly	cost effect (wage) location of final demand	product market competition	cost effect (wage) product market competition

The Krugman (1991) model is the only one where agglomeration is determined by *mobile workers*. Workers are assumed to be mobile between regions/countries and move to the region where they get the highest real wage - they keep on moving until *real wages* are equalised. Workers thus prefer to be in agglomerated regions where both nominal wages are higher and prices are lower (price index effect). Firms on the other hand prefer to locate close to their demand (by the workers) so that agglomeration will be reinforced (home market effect). The force that possibly pulls firms away from the centre is the fact that the immobile agricultural labour also demands goods. This immobile labour force stays put and if all firms are located in one single region they will have to ship back their goods to the other market - which is costly. Some firms might therefore relocate to the peripheral region and supply the market there.

The other three models assume (manufacturing) labour to be immobile between countries but mobile intersectorally. In this case, the agglomeration effects follow from *firms* *that move* to the region where they can get a *higher profit*. Venables (1996) assumes vertical linkages between firms: one firm provides the input for the other. It is obvious that these linkages pull firms together: the input-producing firm wants to locate close to the firm(s) demanding its products while the firm(s) that use the input in their turn prefer to be close to the input-producing firm in order to minimise the cost of their input. Again however there are forces pulling firms away from the centre, more specifically the cost of higher wages in the centre and again the location of final demand (as in the Krugman model).

In the last two models labour markets are assumed to be imperfect. Wages are the outcome of either a bargaining process (Picard and Toulemonde (2002)) or are unilaterally set by unions (this chapter). Workers that do not get a job in the manufacturing sector flow to the agricultural sector. On the one hand firms are drawn to regions with a large share of their labour in manufacturing (higher total income and therefore higher demand - home market effect); on the other hand firms prefer to locate not too close to each other in order to avoid too fierce a competition (product market competition). Moreover, in the model developed in this chapter we assume wages to be set unilaterally by unions. Wages will therefore be higher than in the Picard and Toulemonde (2002) model with bargaining thus reinforcing the home market effect but also creating a (stronger) dispersion force because of the high wage (production) costs for the firms.

The second difference between Picard and Toulemonde (2002) and our model is that they assume bargaining at the firm level while we allow for the number of unions - and therefore the level at which wages are set - to differ. This interesting feature allows us to relate location decisions of firms to the level of wage setting in the countries considered (national, sectoral or firm level) - something which was not possible in the model of Picard and Toulemonde (2002). In the next section, we will go through the full model and draw some first conclusions.

3 Theoretical model

We assume two regions H (Home) and F (Foreign) and two kinds of goods: agriculture, denoted by A, and manufacturing, denoted by M. The agricultural sector is competitive while the manufacturing sector is characterized by monopolistic competition. As already mentioned before, there is no interregional labour mobility, but there is perfect intersectoral labour mobility within each region. Finally, we assume wages in the manufacturing sector to be set by unions. Workers who do not get a job in the unionized manufacturing sector spill-over to the non-unionized agricultural sector where wages are market-clearing. In setting up the model, we first focus on the demand side. In the second section, we analyse the production decision in both sectors and the wage setting by the unions. After deriving these basic tools, we can analyse the location decisions of firms in the third section. The purpose is to assume either agglomeration or symmetry as a point of departure and to analyse for which levels of transport costs these are (un)stable equilibria.

3.1 Consumption and labour supply

Every region has I agents, indexed by $i = 1, ..., I$, each with the following utility function:

$$U_i = C_{Ai}^{1-\alpha} C_{Mi}^{\alpha} \qquad 0 < \alpha < 1$$

where α $(1-\alpha)$ is the share of manufacturing (agriculture) in consumption, C_{Ai} is consumption by agent i of the agricultural good and C_{Mi} is his manufacturing consumption index, defined as

$$C_{Mi} = \left(\int_{j=0}^{n+n^*} C_{ji}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \qquad \sigma > 1$$

where C_{ji} is agent i's consumption of variety j of the manufacturing good, n (n^*) is the number of varieties (and the mass of firms) in the home-region (foreign-region) and σ is the constant substitution elasticity between any two varieties. The total number of firms equals N $(n + n^*)$. Each agent i has one unit of agent-specific labour which is supplied either to the agricultural sector or to the manufacturing sector.

Hence all agents (total number of I) are fully employed and we have that

$$I = L_A + L_M$$

where L_A is total labour input in agriculture, and L_M is total labour input in manufacturing.

The optimization problem of agent i is now

$$\left| \begin{array}{l} \text{maximize } \; U_i = C_{Ai}^{1-\alpha} \left(\int_{j=0}^{n+n^*} C_{ji}^{\frac{\sigma-1}{\sigma}} dj \right)^{\alpha \frac{\sigma}{\sigma-1}}, \\ \text{under the constraint } p_A C_{Ai} + \int_{j=0}^{n+n^*} p_j C_{ji} dj = Y_i \end{array} \right.$$

where p_A is the price of the agricultural good, p_j is the price of the j-th variety of the manufactured good and Y_i is agent i's nominal income,

$$Y_i = \begin{array}{ll} W_A + D_i & \text{if employed in the agricultural sector} \\ W_M + D_i & \text{if employed in the manufacturing sector} \end{array}$$

where D_i denotes the dividends received by agent i[1]. We use the two-stage budgeting procedure to solve agent i's optimization problem and obtain the following demand equations:

$$C_{Mi} = \alpha \frac{Y_i}{p_M} \qquad \text{and} \qquad C_{Ai} = (1-\alpha) \frac{Y_i}{p_A} \tag{1}$$

[1] In order to have a closed model, we assume that profits are distributed among the workers.

$$C_{ji} = \left(\frac{p_j}{p_M} \right)^{-\sigma} C_{Mi} \qquad \text{where} \qquad j = 1, ..., n + n^* \tag{2}$$

A share α $(1-\alpha)$ of real income is devoted to manufacturing (agricultural) consumption while the demand for each variety j clearly depends on its relative price.

Finally note that the manufacturing price index equals:

$$p_M = \left(\int_{j=0}^{n+n^*} p_j^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \tag{3}$$

and that the consumption-based price index is the following:

$$p = \frac{1}{\gamma} p_A^{1-\alpha} p_M^{\alpha} \qquad \text{where} \qquad \gamma = \alpha^{\alpha} (1-\alpha)^{1-\alpha} \tag{4}$$

3.2 Production, price setting and labour demand

In this section we will first tackle the agricultural sector, then the manufacturing sector and finally the wage setting process.

3.2.1 Agricultural sector

Firms in the agricultural sector operate under constant returns to scale. In order to produce one unit of output, one unit of labour is required. If we denote by z_A the production of this sector, the production function is as follows: $L_A = z_A$. Since profits are zero: $p_A z_A - W_A z_A = 0$, we have that $W_A = p_A$. We take the agricultural good as the numeraire, such that we can set $W_A = p_A = 1$. Note that this implies that

$$p = \frac{1}{\gamma} p_A^{1-\alpha} p_M^{\alpha} = \frac{1}{\gamma} p_M^{\alpha} \tag{5}$$

3.2.2 Manufacturing sector

We consider a continuum of manufacturing firms $j \in (0, n)$. Each firm produces a quantity z_j of a differentiated good j and in order to produce one unit of the good, the firm needs one unit of labour.

The profit of firm j in region H is

$$\Pi_{Hj} = q_{Hj} z_{Hj} - W_{Hj} z_{Hj} - E \tag{6}$$

where q_{Hj} is the mill price and E are the fixed costs.

We first consider the total demand for variety j. The variable z_{Hj} refers to the demand for variety j produced in region H. We have that $z_{Hj} = C_{Hj} + \frac{1}{\tau} C_{Fj}$, where C_{Hj} (C_{Fj}) refers to total consumption of good j of agents residing in region H (F). The parameter τ captures the iceberg transport cost; if a unit is shipped to another region, only $\tau < 1$ units actually arrive at that region. For the agents of region H the consumption price equals the mill price, i.e. $p_{Hj} = q_{Hj}$. For agents of the other region, the consumer price equals $p_{Fj} = q_{Hj}/\tau$. Using the demand functions (1) and (2), we get the following expression for sales:

$$z_{Hj} = \left(\frac{p_{Hj}}{p_{HM}} \right)^{-\sigma} \frac{\alpha Y_H}{p_{HM}} + \frac{1}{\tau} \left(\frac{p_{Fj}}{p_{FM}} \right)^{-\sigma} \frac{\alpha Y_F}{p_{FM}} \tag{7}$$

Now defining the demand in terms of the mill price, we get

$$z_{Hj} = \left(\frac{q_{Hj}}{p_{HM}} \right)^{-\sigma} \frac{\alpha Y_H}{p_{HM}} + \frac{1}{\tau} \left(\frac{q_{Hj}}{\tau p_{FM}} \right)^{-\sigma} \frac{\alpha Y_F}{p_{FM}} = \left(\frac{q_{Hj}}{p_{HM}} \right)^{-\sigma} G_H \tag{8}$$

where

$$G_H = \alpha \left[\frac{Y_H}{p_{HM}} + \tau^{\sigma-1} \left(\frac{p_{HM}}{p_{FM}} \right)^{-\sigma} \frac{Y_F}{p_{FM}} \right] \tag{9}$$

can be interpreted as the "total" world demand for the manufacturing goods produced in region H.

The optimization problem of firm j in region H can then be written as

$$\left| \begin{array}{l} \text{maximize } \Pi_{Hj} = q_{Hj} z_{Hj} - W_{Hj} z_{Hj} - E, \\ \text{such that } z_{Hj} = \left(\frac{q_{Hj}}{p_{HM}} \right)^{-\sigma} G_H \end{array} \right.$$

from which we derive the profit maximising mill price:

$$q_{Hj} = \frac{\sigma}{\sigma - 1} W_{Hj}$$

Note that the mill prices do not differ between varieties. It follows that

$$q_{Hj} = q_H = \frac{\sigma}{\sigma - 1} W_{Hj} \tag{10}$$

Prices are thus set as a markup over the wage.

Substituting (10) in (8), it follows that

$$z_{Hj} = \left(\frac{q_{Hj}}{p_{HM}} \right)^{-\sigma} G_H = \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} W_{Hj}^{-\sigma} p_{HM}^{\sigma} G_H \tag{11}$$

and profits become:

$$\begin{aligned}\Pi_{Hj} &= (q_{Hj} - W_{Hj})\, z_{Hj} - E = \left(\frac{\sigma}{\sigma-1} W_{Hj} - W_{Hj}\right) z_{Hj} - E \\ &= \frac{1}{\sigma-1} W_{Hj} z_{Hj} - E = \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} W_{Hj}^{1-\sigma} p_{HM}^{\sigma} G_H - E\end{aligned} \tag{12}$$

Now that we have determined prices, sales and profits in both sectors, we focus on the wage setting in the model.

3.2.3 Wage setting

The purpose of this section is threefold. We first of all want to analyse a representative union's maximisation problem. After having determined the general wage expression, we want to focus on the wages in two extreme location outcomes - a situation of full agglomeration and one of full symmetry. We end the section with some comparative statics results that will help us to gain a better insight in the wage setting process.

The union's objective We assume K (K^*) unions in region H (F). Their number is "intermediate" in the following sense: each union is small in that it takes aggregate income or aggregate demand as given[2], but the union is large enough to take into account the effect of its wage on the price of manufactures. Note that if K equals one, wages are set at the 'national' level by one single union. When K equals n, wages are set at the firm level since the number of unions equals the number of firms. For intermediate values of K (smaller than n but larger than one) we might say that wages are set at a (sub)sectoral level. We might therefore argue that given the number of firms n, a higher number of unions K implies a lower union strength for each union independently since there are more unions to 'compete' with.

The sequence of decision making is as follows. In stage 1, unions simultaneously set the wage by maximising their objective function taking into account the firms' labour demand function. In stage 2, each firm chooses its output (and hence employment) level taking as given the wage set by the unions.

We assume that each union k maximizes the following expression[3]:

$$\frac{W_{Hk} - 1}{p_{HM}^{\alpha}} L_{Hk},$$

where W_{Hk} is the union's wage applying to all of its members and L_{Hk} is the demand for union k's labour. Each union therefore maximises the real wage differential (manufacturing

[2]The reason for this is that I do not want income effects to play a role in wage determination. I did try to incorporate this before but wage determination became intractable.

[3]Note that this boils down to rent maximisation for the union - a specific case of the Stone-Geary function where the union maximises the difference between the real wage bill in the unionised sector and the real wage bill in the perfectly competitive sector.

wage minus agricultural wage of one deflated by the price index[4]) multiplied by the demand for union k's labour.

Note that

$$\begin{aligned} p_{HM} &= \left[nq_H^{1-\sigma} + n^*\left(\frac{q_F}{\tau}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{\sigma-1}\left[\sum_{l=1}^{K}\frac{n}{K}W_{Hl}^{1-\sigma} + n^*\left(\frac{W_F}{\tau}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ L_{Hk} &= \frac{n}{K}z_{Hj} = \frac{n}{K}\left(\frac{q_{Hj}}{p_{HM}}\right)^{-\sigma}G_H = \frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}W_{Hk}^{-\sigma}p_{HM}^{\sigma}G_H \end{aligned}$$

Since union k is small in the sense of neglecting the effect of his wage on G_H, we can rewrite the unions' objective function as follows:

$$\frac{W_{Hk}-1}{p_{HM}^{\alpha}}W_{Hk}^{-\sigma}p_{HM}^{\sigma}$$

The first order condition is

$$\left[1-(W_{Hk}-1)\,\alpha p_{HM}^{-1}\frac{\partial p_{HM}}{\partial W_{Hk}}\right]\frac{W_{Hk}}{p_{HM}} - \sigma\frac{p_{HM}-W_{Hk}\frac{\partial P_{HM}}{\partial W_{Hk}}}{p_{HM}^2}(W_{Hk}-1) = 0 \tag{13}$$

Now note that

$$\frac{\partial P_{HM}}{\partial W_{Hk}} = \frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{W_{Hk}}{p_{HM}}\right)^{-\sigma} \tag{14}$$

so that the first order condition can be written as

$$W_{Hk} - (W_{Hk}-1)\frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{W_{Hk}}{p_{HM}}\right)^{1-\sigma}(\alpha-\sigma) - \sigma(W_{Hk}-1) = 0$$

from which we can derive

$$\frac{W_{Hk}}{W_{Hk}-1} = \sigma + \frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{W_{Hk}}{p_{HM}}\right)^{1-\sigma}(\alpha-\sigma)$$

Note that all manufacturing wages in a region will be equal. Hence

$$\frac{W_H}{W_H-1} = \sigma + \frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \tag{15}$$

$$\left(\frac{W_H^{1-\sigma}}{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left[nW_H^{1-\sigma}+n^*\left(\frac{W_F}{\tau}\right)^{1-\sigma}\right]}\right)(\alpha-\sigma) \tag{16}$$

$$= \sigma + \frac{1}{K}\left(\frac{nW_H^{1-\sigma}}{nW_H^{1-\sigma}+n^*W_F^{1-\sigma}\tau^{\sigma-1}}\right)(\alpha-\sigma) \tag{17}$$

[4]The price index equals actually $\frac{1}{\gamma}p_M^{\alpha}p_A^{1-\alpha}$ but since $\frac{1}{\gamma}$ is a constant and p_A equals 1 these terms do not influence the maximisation problem.

The expression for the wage we obtain is very different from the one obtained by Picard and Toulemonde (2002). The major reason for this difference is not so much that we assume unilateral wage setting while Picard and Toulemonde (2002) allow wages to be bargained over. Rather, Picard and Toulemonde (2002) assume wages not to influence the price index. We, on the other hand, allow unions to take into account that an increase in the wage they set will also increase the general price index (cfr. equation(14)). Indeed, if we assume $\frac{\partial p_{HM}}{\partial W_{Hk}} = 0$, expression (13) boils down to the expression Picard and Toulemonde (2002) obtain assuming the bargaining power of unions ϕ to equal 1 (as we do in our model); namely $w = 1 + \frac{1}{\sigma - 1}$. Picard and Toulemonde (2002) therefore obtain that wages are the same in every region, independent of the firms' location decision. In our model, it is obvious that wages do depend on firms' location and on the number of unions such that they can differ between regions. The strength of our model is therefore that we can analyse location issues assuming different levels of centralisation in the wage setting process.

Two extreme location outcomes Since expression (15) is too complicated to immediately draw general conclusions, we will first have a look at two extreme cases - agglomeration and symmetry. These wage expressions are essential to analyse the location decisions of firms in the next section. Full agglomeration implies that all firms are located in one region. In a symmetric case we assume both the number of firms and the number of unions in the two regions to be equal.

If there is *full agglomeration* in region H ($n^* = 0$), then

$$\begin{aligned} \frac{W_H}{W_H - 1} &= \sigma + \frac{1}{K}\left(\frac{NW_H^{1-\sigma}}{NW_H^{1-\sigma}}\right)(\alpha - \sigma) \\ &= \sigma + \frac{1}{K}(\alpha - \sigma), \end{aligned}$$

from which

$$W_H = \frac{\sigma (K-1) + \alpha}{\sigma (K-1) + \alpha - K}, \tag{18}$$

which only makes sense if the following condition holds true

$$\sigma (K-1) + \alpha > K \Rightarrow K(\sigma - 1) > -\alpha + \sigma$$

or

$$K > \frac{\sigma - \alpha}{\sigma - 1} \tag{19}$$

Note that $(\sigma - \alpha)/(\sigma - 1)$ is larger than one such that there definitely has to be more than one union. A single national union is therefore not an option. Moreover, from expression (18) we can see that W_H is larger than one - implying that manufacturing wages are higher than agricultural wages.

We derive the wage for region F in the same way

$$\frac{W_F}{W_F - 1} = \sigma + \frac{1}{K^*}\left(\frac{n^* W_F^{1-\sigma}}{n^* W_F^{1-\sigma} + n W_H^{1-\sigma}\tau^{\sigma-1}}\right)(\alpha - \sigma),$$

so that in the case of full agglomeration in region H

$$\begin{aligned} \frac{W_F}{W_F - 1} &= \sigma \\ W_F &= (W_F - 1)\,\sigma = \frac{\sigma}{\sigma - 1}, \end{aligned} \tag{20}$$

which again is higher than the agricultural wage of one. Note that in both regions the wage is independent of the number of firms. As long as we assume full agglomeration, the number of firms does not influence the wage level.

Why is it in our model impossible to get a single union if there is full agglomeration? The reason is fairly simple: in the limit case of full agglomeration in combination with a single union, the union's objective function will break down. Indeed, it is easily shown that in the case of full agglomeration, the expressions for price index and employment become:

$$\begin{aligned} p_{HM} &= \frac{\sigma}{\sigma - 1} N^{\frac{1}{1-\sigma}} W_H \\ L_H &= N z_{Hj} = N \left(\frac{q_{Hj}}{p_{HM}}\right)^{-\sigma} G_H = N^{\frac{1}{1-\sigma}} G_H \end{aligned} \tag{21}$$

The manufacturing price index is proportional to the nominal wage, and employment only depends on aggregate income. This implies that the objective function of the union in fact becomes

$$\frac{W_H - 1}{W_H^{\alpha}}$$

which is monotonically increasing in W_H for $W_H > 1$. Wages would thus turn out to be infinite.

This limit case of the model, combined with our assumption governing the wage setting of the union, depicts of course a very unrealistic and artificial kind of behaviour on the part of the union. Or, to put it another way, if there is only one union – or a limited number of unions – the assumption that the union takes aggregate income as given when setting the nominal wage cannot be maintained.[5]

So in the sequel of this chapter we will maintain the assumption that the number of unions K is "intermediate" in the sense that each union takes aggregate income as given, but takes into account the effect of its wage on the manufacturing price index.

[5]Recall that we made this assumption in order to avoid income effects. Introducing these effects along with differerences in centralisation of wage setting renders the model unsolvable. In order to gain insight however in centralisation issues we have to make abstraction of income effects.

In the case of *full symmetry*, $K = K^*, n = n^*$, we get that $W_H = W_F$ and hence

$$\frac{W_H}{W_H - 1} = \sigma + \frac{1}{K}\left(\frac{1}{1+\tau^{\sigma-1}}\right)(\alpha - \sigma),$$

from which

$$\begin{aligned} W_H &= (W_H - 1)\left[\sigma + \frac{1}{K}\left(\frac{1}{1+\tau^{\sigma-1}}\right)(\alpha - \sigma)\right] \quad (22) \\ &= \frac{\sigma K\left(1+\tau^{\sigma-1}\right) + \alpha - \sigma}{(\sigma - 1) K\left(1+\tau^{\sigma-1}\right) + \alpha - \sigma}, \quad (23) \end{aligned}$$

which only makes sense if the following condition holds true

$$(\sigma - 1) K\left(1+\tau^{\sigma-1}\right) > \sigma - \alpha$$

$$K > \frac{\sigma - \alpha}{\sigma - 1}\left(1+\tau^{\sigma-1}\right) \quad (24)$$

From expression (22) we can see that the manufacturing wage is larger than one - implying that manufacturing wages are higher than agricultural wages. Again the wage level is independent of the number of firms. As long as we assume an equal number of firms in both regions, their absolute number is irrelevant for wage setting.

Both the wage at symmetry (22) and the wage at agglomeration (18) are influenced by the number of unions, K, and the degree of competition, σ. It is easily shown algebraically that as the product market competition increases, unions will lower their wage demands. Indeed, as the substitution elasticity σ between the different varieties increases, firms are confronted with more severe competition and therefore have to lower their prices. Unions take this into account and lower their wage demands.

Comparative statics Before moving on to the (in)stability of the two extreme location outcomes, we want to give some comparative statics results that help us to gain a better insight in the wage setting and will be referred to frequently throughout this chapter. It is possible to derive the effect of the number of unions K, the number of firms n and n^*, and transport costs τ on wages:

$$\begin{aligned} \frac{\partial W_H}{\partial K} &< 0; \frac{\partial W_F}{\partial K} < 0 \quad (25) \\ \frac{\partial W_H}{\partial n} &> 0; \frac{\partial W_F}{\partial n} < 0 \quad (26) \\ \frac{\partial W_H}{\partial n^*} &< 0; \frac{\partial W_F}{\partial n^*} > 0 \quad (27) \\ \frac{\partial W_H}{\partial \tau} &< 0; \frac{\partial W_F}{\partial \tau} < 0 \quad (28) \end{aligned}$$

When the number of unions - and therefore the competition among them - in one region increases, the wage in both regions will decrease. More firms in one region - and therefore a higher labour demand - will increase the wage in that region and decrease it in the other region. Finally, a decrease in transport costs will decrease the wage in both regions (since competition becomes fiercer). This result is in line with Driffil and van der Ploeg (1993) who state that decreasing trade costs may indeed bid wages downwards. They continue to argue that this may provide an incentive for unions to cooperate with unions in other regions. We will come back to this point in section 4.

Our main point of interest is however the influence of the degree of centralisation of wage setting (the number of unions) on the wage. We will therefore dig deeper in the intuition behind the following proposition:

Proposition 1 *In a NEG model with unionised wage setting, we can say that when the number of unions K increases, wages will decline.*

As we can see from the objective function of the union, the number of unions affects the wage setting in two ways - through the real wage differential and through the demand for labour. When wages increase, so will prices and therefore the real wage will decline (or at least not increase as much as the nominal wage). When there are fewer unions, each union realises that it has a larger effect on the price level and hence it will moderate its wage claims. This would imply that an increase in the number of unions will increase the nominal wage. There is however another force at work that drives the nominal wage in the opposite direction. When there are more unions, the elasticity of labour demand with respect to the individual union's wage will increase thus lowering the nominal wage demands. The intuition behind this is as follows. The demand for each good depends on its relative price. Therefore, when the general price index increases along with the individual price of the variety, the decline in the demand will be smaller. If there is one single union, it will influence the general price index stronger and firms therefore will be confronted with a lower decrease in demand for their good and therefore with a lower decrease in demand for labour. In sum, more unions imply a higher elasticity of labour demand with respect to the wage. Since the second effect outweighs the first one, an increase in the number of unions decreases the wages. For an algebraic derivation and interpretation of the effect of the number of unions on the wage, we refer to Appendix 3.A. There we show that an increase in the number of unions both increases the marginal benefit and the marginal cost of the nominal wage. The effect on the marginal cost of a wage increase however outweighs the effect on the marginal benefit thus leading to a decrease in nominal wage.

3.3 (In)stability of agglomerated and symmetric location outcomes

In this section we want to analyse what our model can say about the (in)stability of extreme location outcomes. As in other NEG models, the (in)stability of the location outcomes is directly related to the transport costs. Depending on the level of transport costs, one equilibrium may be stable while the other one is unstable. The reason for analysing the link between (in)stability and location outcomes is that we want to make policy recommendations. Governments are of course interested in attracting (all) firms - and the accompanying rents - to their country and an important issue is how they best do this in a world that integrates more and more. The point of this section is to analyse how the extreme location outcomes change as the world integrates further - i.e. as transport costs continue to decrease. We will prove that full agglomeration (the preferred outcome of a government) is only a stable outcome for intermediate trade costs. When trade costs are very high or very low, a symmetric outcome is the only stable equilibrium.

In order to analyse the location decision of a firm we look at the profit differential. Indeed, the firm's location decision hinges on the evaluation of this profit differential[6]:

$$\begin{aligned}\Delta\Pi_j &= \Pi_{Hj} - \Pi_{Fj} = W_H z_{Hj} - W_F z_{Fj} \\ &= W_H^{1-\sigma} p_{HM}^{\sigma} G_H - W_F^{1-\sigma} p_{FM}^{\sigma} G_F\end{aligned}$$

If the differential is positive, firms will prefer to locate in the home region while if the differential is negative, firms will settle in the foreign region. We therefore have to determine the sign of the profit differential for the two extreme location outcomes - full agglomeration and full symmetry. Recall that we want to analyse first of all the (in)stability of agglomerated and symmetric outcomes in function of *transport costs.* However, since we are also interested in the impact of the *level of wage setting* (number of unions) on location, we have special attention for the location outcome as a function of the number of unions.Agglomeration

In this section we will prove the following two propositions:

Proposition 2 *In a NEG model with unionised wage setting, agglomeration is unstable for both zero and infinite transport costs and (possibly) stable for intermediate values of transport costs.*

Proposition 3 *In a NEG model with unionised wage setting, when the number of unions increases, agglomeration is stable for a larger range of (intermediate) transport costs.*

Proof. We have to determine the conditions under which agglomeration in region H is an equilibrium. For this to be the case, the following condition should hold for $n = N$:

$$\begin{gathered}W_H^{1-\sigma} p_{HM}^{\sigma} G_H > W_F^{1-\sigma} p_{FM}^{\sigma} G_F \\ \left(\frac{W_H}{W_F}\right)^{1-\sigma} > \left(\frac{p_{HM}}{p_{FM}}\right)^{-\sigma} \frac{G_F}{G_H}\end{gathered}$$

Given that all firms are located in H, profits should be higher in H than in F so that no firm has an incentive to move to F. In that case agglomeration is a stable equilibrium.

[6]Note that the two terms that are subtracted from each other should actually be multiplied by $1/(1-\sigma)$. However, since this is a constant it will not influence the sign of the profit differential - and that is what we are interested in. So for simplicity, we drop the constant.

It follows from the definitions of G and p that in the case of agglomeration in region H, and hence $n = N, n^* = 0$, the condition for agglomeration becomes

$$\left(\frac{W_H}{W_F}\right)^{1-\sigma} > \tau^{1-\sigma}\frac{Y_F + \tau^{2(\sigma-1)}Y_H}{Y_F + Y_H}$$

Using the expressions for total income in both regions, we get - after tedious computations[7] - that the following inequality has to hold for agglomeration to be a stable equilibrium

$$\begin{aligned} F(\tau) &= \left(\frac{\sigma\left[\sigma\left(K-1\right)+\alpha\right]-\sigma K}{\sigma\left[\sigma\left(K-1\right)+\alpha\right]-\sigma K+\sigma-\alpha}\right)^{\sigma-1} \\ &\quad -\tau^{1-\sigma}\frac{\left[\sigma^2(K-1)+\alpha\sigma\right]\left[1+\tau^{2(\sigma-1)}\right] - \alpha K(\sigma-1)\left[1-\tau^{2(\sigma-1)}\right]}{2\left[\sigma^2(K-1)+\alpha\sigma\right]} > 0 \end{aligned} \tag{29}$$

It is obvious that it is very hard to determine the sign of this expression in general, so we first look at the two extreme cases - without transport costs and with infinite transport costs. If there are *no transport costs*, we have that

$$F(1) = \left(\frac{\sigma\left[\sigma\left(K-1\right)+\alpha\right]-\sigma K}{\sigma\left[\sigma\left(K-1\right)+\alpha\right]-\sigma K+\sigma-\alpha}\right)^{\sigma-1} - \frac{2\left[\sigma^2(K-1)+\alpha\sigma\right]}{2\left[\sigma^2(K-1)+\alpha\sigma\right]} < 0$$

This expression is negative because of (19)

If *transport costs are infinite*, we have that:

$$\lim_{\tau\to 0} F(\tau) = \lim_{\tau\to 0} -\frac{\left[\sigma^2\left(K-1\right)+\alpha\sigma\right]\left(1+\tau^{2(\sigma-1)}\right) - \alpha K\left(\sigma-1\right)\left(1-\tau^{2(\sigma-1)}\right)}{\tau^{\sigma-1}} = -\infty \tag{30}$$

Therefore, both for zero and for prohibitive transport costs, agglomeration turns out to be an unstable equilibrium. If all firms are located in H, profits in F are higher thus inducing firms to move and rendering the agglomeration in H unstable.

We however still do not know anything about the (in)stability of agglomeration for *intermediate values of trade costs*. Since an analytic solution cannot be derived, we resort to simulations in order to obtain results. It turns out that $F(\tau)$ is more negative the higher σ (more competition), the lower α (smaller demand effect) and the lower K (and therefore the higher W).

This result is as we would have expected. Agglomeration becomes more unstable the higher the competition (dispersion force), the smaller the demand effect (agglomerative force) and the higher the wages (dispersion force). However, since the contribution of this chapter is to analyse the importance of unions for firms' location decisions, we mainly focus on the impact of K. In order to get a better insight in the forces we therefore plot $F(\tau)$ as

[7] Cfr. Appendix

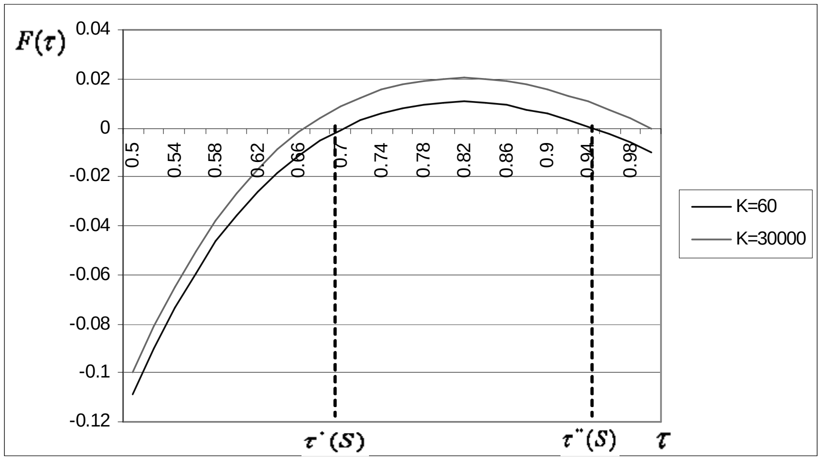

Figure 1: (In)stablity of agglomeration

a function of τ putting $\sigma = 2; \alpha = 0.8$ and $K = 30000$ or $K = 60$ in Figure 1. Since there are about 30000 firms in Belgium and about 60 sectors (NACE 2) we interpret the first case as wage setting at the firm level and the second one as wage setting at the sectoral level[8].

We observe that for high transport costs (low τ), agglomeration is definitely not an equilibrium. When transport costs decline, agglomeration becomes stable but when transport costs decline further, agglomeration becomes unstable again. Moreover, the range of transport costs for which agglomeration is stable decreases as the number of unions decreases. Indeed, if there are fewer unions in region H, wages are higher thus also increasing the cost of locating there. ■

As in Fujita, Krugman and Thisse (1999) we can also define the sustain point(s) for this model. The sustain point in general is defined as the point at which the agglomerated outcome, once established, can be sustained. We call $\tau^*(S)$ the sustain point for increasing transport costs (the point in Figure 1 where the curve first crosses the X-axis) and $\tau^{**}(S)$ the sustain point for decreasing transport costs (the point in Figure 1 where the curve crosses the X-axis for the second time).

Note that our proposition does not work for the traditional models assuming perfect labour markets and labour mobility. In these models agglomeration is an equilibrium for low values of transport costs. If transport costs are low, both firms and workers prefer to locate in the larger market. Firms have the highest demand there and workers are offered the highest real wage. Therefore, if for some reason there are more firms in one market, workers would move there taking demand with them thus inducing more firms to locate there. Moreover, if transport costs are low, firms can still cheaply ship their goods to the agricultural workers left behind in the peripheral region. In the model in

[8]Note that we do not include (almost) national wage setting since we already illustrated that a single union is not possible in combination with full agglomeration.

this chapter however, agglomeration can never be an equilibrium at zero transport costs. The reason is that in the model with immobile labour, wages are almost a pure dispersion force. Wages are higher in the agglomerated region but since there is no labour mobility, this will not attract more workers to the region and hence will not provide firms with an extra agglomeration force (demand effect). In view of the dispersion effect of higher wages, agglomeration is no longer an equilibrium.

After having analysed the (in)stability of agglomeration, we now want to have a look at the (in)stability of symmetric outcomes.

3.3.1 Symmetry

In this section we will provide proof for the following two propositions:

Proposition 4 *In a NEG model with unionised wage setting, symmetry is stable for both zero and infinite transport costs and (possibly) unstable for intermediate values of transport costs.*

Proposition 5 *In a NEG model with unionised wage setting, when the number of unions increases, symmetry is unstable for a larger range of (intermediate) transport costs.*

Proof. We start by imposing a symmetric equilibrium where $K = K^*$ and $n = n^*$. This also implies that $W_H = W_F = W$, $G_F = G_H = G$, $Y_F = Y_H = Y$, $p_{HM} = p_{FM} = p_M$, etc.

We define

$$\hat{n} = \frac{n}{n^*}, \hat{W} = \frac{W_H}{W_F}, \hat{Y} = \frac{Y_H}{Y_F}, \text{etc...}$$

The symmetric equilibrium will be unstable if, as a consequence of a reallocation of some firms to region H, i.e. $d\hat{n} > 0$, the profit differential widens, i.e. $d\Delta\Pi_j > 0$. We have that

$$\Delta\Pi_j = W_H^{1-\sigma} p_{HM}^{\sigma} G_H - W_F^{1-\sigma} p_{FM}^{\sigma} G_F$$

and hence

$$\begin{aligned} d\Delta\Pi_j &= (1-\sigma) W^{-\sigma} p_M^{\sigma} G \left(dW_H - dW_F\right) + \sigma p_{HM}^{\sigma-1} W^{1-\sigma} G \left(dp_{HM} - dp_{FM}\right) \\ &\quad + W^{1-\sigma} p_M^{\sigma} \left(dG_H - dG_F\right) \\ &= W^{1-\sigma} p_M^{\sigma} G \left[(1-\sigma) d\hat{W} + \sigma d\hat{p} + d\hat{G}\right] \end{aligned}$$

Now, since $W^{1-\sigma} p_M^{\sigma} G > 0$, the symmetric equilibrium will be unstable if

$$F = (1-\sigma) d\hat{W} + \sigma d\hat{p} + d\hat{G} > 0 \quad (31)$$

$$F = (1-\sigma) d\hat{W} + \varphi d\hat{Y} + \varphi (\sigma - 1) d\hat{p} > 0 \quad (32)$$

Note that we therefore have to find expressions for $d\hat{W}$, $d\hat{p}$ and $d\hat{G}$. For the derivation of these expressions we refer to Appendix 3.C. Assuming $d\hat{n}$ to be positive (i.e. firm(s) move from region F to region H), it is shown that:

$$\begin{aligned} d\hat{W} &> 0 \\ d\hat{p} &< 0 \\ d\hat{Y} &> 0 \end{aligned}$$

Recalling that $0 < \alpha < 1$ and $\sigma > 1$,we see that the first term of F $\left\{(1-\sigma)\, d\hat{W}\right\}$ is negative, the second term $\left\{\varphi d\hat{Y}\right\}$ is positive while the last term is negative again $\{\varphi(\sigma-1)\, d\hat{p}\}$. The second term is obviously the agglomeration force that draws firms to the H region. If more firms move to H, wages increase and so will income - attracting new firms to the region because of the *home market effect*. The first and last term are dispersion forces. The first term is what we call the *cost effect*. When more firms move into H, wages will rise thus inducing firms to move to the other region. The last term reflects the *competition effect* - when more firms move into H, the competition becomes fiercer such that firms have to lower their prices.

As in the agglomeration case, we would like to know for which values of transport costs the symmetric equilibrium is (un)stable. We again first investigate the extreme cases of infinite and no transport costs.

As *transport costs are infinite*, $F(0)$ becomes:

$$\begin{aligned} F(\tau=0) &= d\hat{Y} + (\sigma-1)\, d\hat{p} \\ &= \frac{\alpha(\sigma-1)\{K(\sigma-1)\, d\hat{p} + K d\hat{n}\}}{\sigma^2(K-1) - \alpha(\sigma-1)K} + (\sigma-1)\, d\hat{p} \end{aligned} \tag{33}$$

and since with $t = 0$

$$d\hat{p} = -\frac{1}{\sigma-1} d\hat{n}$$

we get

$$F(\tau=0) = -d\hat{n} < 0$$

As *transport costs are zero*, we get:

$$\begin{aligned} F(\tau=1) &= (1-\sigma)\, d\hat{W} \\ &= (1-\sigma) \frac{2K(\sigma-\alpha)\, d\hat{n}}{\begin{array}{c}[(\sigma-1)2K - (\sigma-\alpha)][2K\sigma - (\sigma-\alpha)] \\ +(\sigma-\alpha)K2(\sigma-1)\end{array}} < 0 \end{aligned} \tag{34}$$

because of (19). So both for zero and infinite transport costs the symmetric equilibrium is stable.

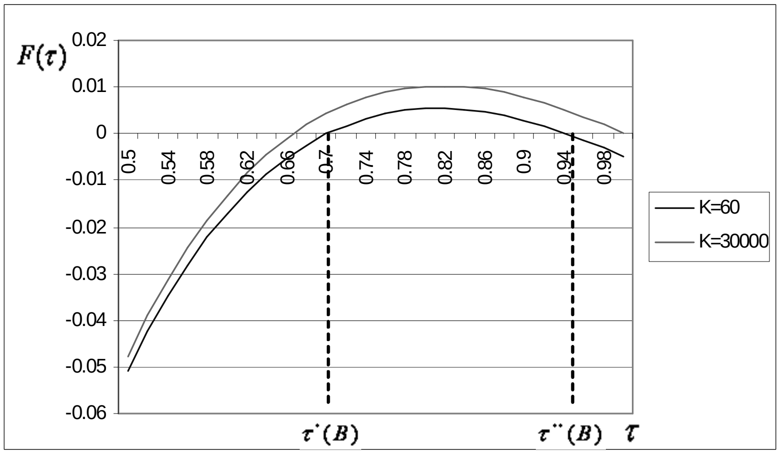

Figure 2: (In)stability of symmetric equilibrium

We however still do not know what happens for *intermediate values of* τ. Since an analytic solution cannot be derived, we simulate in order to arrive at interpretable results. We allowed the values for α, σ and K to vary. We investigate how $F(\tau)$ changes when $d\hat{n} = 1$, i.e. if there are (some) firms relocating to region H. Symmetry turns out to be unstable for intermediate values of trade costs if α is high (home market effect plays a big role), σ is small (competition effect does not really play a role), K is high (more unions, hence a lower wage in H, and therefore the cost effect is not that important).

These results are again as we would expect. If the home market effect plays an important role and there is an extra firm moving into region H, this will increase wages (and incomes) and therefore attract more firms to this region. Moreover, if the competition effect is less strong, firms will not 'mind' to be located close to other firms (in region H). Finally, when there are more unions in H, competition among the unions will decrease wage demands and therefore decrease the dispersion force originating from the cost side. For high values of α, low values of σ and a high number of unions K, the agglomerative force (home market effect) can dominate the dispersion forces (competition effect and cost) and thus render the symmetric equilibrium unstable.

However, since the contribution of this chapter is to analyse the importance of unions in firms' location decisions, we mainly focus on the impact of K. In order to get a better insight in the forces we therefore plot $F(\tau)$ - as in the agglomeration case - as a function of τ putting $\sigma = 2; \alpha = 0.8$ and $K = 60$ or $K = 30000$ in Figure 2.

We observe that for high transport costs (low τ), symmetry is a stable equilibrium. When transport costs decline, symmetry becomes unstable but when transport costs decline further, symmetry becomes stable again. Moreover, the range of transport costs for which symmetry is unstable decreases (or the possibility of a stable symmetric equilibrium increases) when the number of unions decreases. Indeed, if there are fewer unions in region H, wages are higher thus also increasing the cost of (re)locating there. ■

Table 2: (Un)stability of symmetry and agglomeration

$0 \leq \tau < \tau^*(B)$ high TC region	$\tau^*(B) \leq \tau \leq \tau^{**}(B)$ medium TC region	$\tau^{**}(B) < \tau \leq 1$ low TC region
symmetry stable agglomeration unstable	symmetry unstable agglomeration stable	symmetry stable agglomeration unstable

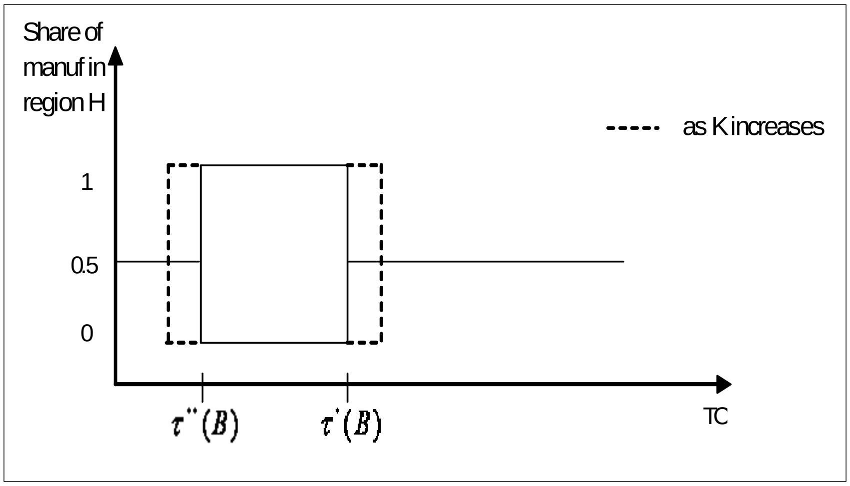

Figure 3: Bifurcation diagram

As in Fujita, Krugman and Thisse (1999) we can also define the break point(s) for this model. The break point in general is the point at which symmetry between the regions gets broken because the symmetric equilibrium becomes unstable. We call $\tau^*(B)$ the break point for decreasing transport costs (the point in Figure 2 where the curve first crosses the X-axis) and $\tau^{**}(B)$ the break point for increasing transport costs (the point in Figure 2 where the curve crosses the X-axis for the second time).

3.3.2 Agglomeration and symmetry combined

Now that we analysed the 'extreme' location decisions of firms separately, we combine the results both for symmetry and agglomeration in order to be able to say something about location in general and to compare the outcomes of this model with those of the other models discussed in the first section of this chapter. From simulations we derive that $\tau^*(B) = \tau^*(S)$; $\tau^{**}(B) = \tau^{**}(S)$[9] - such that the break and sustain points are the same. Table 2 summarises the possible location options for different levels of transport costs (TC).

The information in Table 2 is also visualised in Figure 3 which illustrates the bifurcation diagram as we know it.

[9] All simulation results are not included here but can be obtained from the author upon request.

If initially there is agglomeration (e.g. due to historical facts), economic activity will remain agglomerated as long as we are in the medium TC region but the agglomeration will no longer be sustainable when TC rise (into the high TC region) or decrease (into the low TC region).

So, assuming that transport costs continue to decrease in view of the mounting worldwide integration we would expect full agglomeration to become unstable. Note that we consider here only the full agglomeration cases and not the situations where there is more industry in one region and less (but still some) industry in the other region (what we will call henceforth *intermediate agglomeration*). Indeed, when full agglomeration becomes unstable, a symmetric outcome becomes a possibility. But note that as long as symmetry is not attained, there is no reason to assume that full symmetry is the only option - intermediate agglomeration remains a possibility. However, as soon as the economy reaches a perfect symmetrical division of the industry, this will remain an equilibrium. On the other hand, if we start from a symmetric equilibrium we will maintain it only in the high and (very) low TC region. Once entering the medium TC region, the symmetry gets broken and full agglomeration - if reached - is stable. Note again that intermediate agglomeration is also a possible outcome here.

We now want to compare our location outcome to the ones from the models discussed previously. Both the standard Krugman (1991) and the Venables (1996) model obtain an outcome where symmetry is stable only for high trade costs while agglomeration is stable for low trade costs. However, when these models introduce an extra dispersion force apart from transport costs for manufactures (i.e. TC in agriculture in the Krugman (1991) model and decreasing returns to scale in agriculture in the Venables (1996) model), symmetry turns out to be a stable equilibrium at (very) low TC too. The reason for this is that the centripetal forces are less strong at low transport costs and if then an extra dispersion force is added, agglomeration can no longer be stable. The relationship between transport costs and agglomeration indeed tends to be an inverted U-curve: no agglomeration at high transport costs, the emergence of a core-periphery at intermediate costs and finally a reversion to dispersed manufacturing to take advantage of low wages at low transport costs. Picard and Toulemonde (2002) find that symmetry is stable at high TC and agglomeration is stable at low(er) TC - there is no 'reversion' to a stable symmetry at very low transport costs. The reason for this outcome is that in their model wages are the same in both regions. The (wage) cost-dispersion force therefore does not play here and cannot render agglomeration unstable at very low transport costs.

We can therefore conclude that our results are similar to the ones of the existing NEG models. However, the interesting feature about this model is the introduction of the effect that the level at which wages are set (the number of unions) has on the location of economic activity. When there are more unions in a region - the wages are set at firm rather than at sectoral or more centralised level - agglomeration in this region is more likely. Indeed, agglomeration turns out to be a stable equilibrium for a larger range of transport costs when the number of unions increases. This is also illustrated in Figure 3.

In the next section, we want to look into more specific policy questions that can be answered using the model developed in this chapter.

4 Wage setting and location

Based on our model, we can indeed shed some light on interesting policy questions. We would like to know whether the level at which wages are set has consequences for location. Note that we will interpret the level of wage setting in two different ways. The *first* one is the one that has been used throughout this chapter so far. Does it matter whether wages are set at national, sectoral or firm level? In other words, can a government attract firms by opting for the 'right' level of wage setting? The *second* level of wage setting we will analyse is different. We would like to know whether wages differ when they are set at the regional level or rather at the supra-regional level. In other words, would the wage setting alter if wages are e.g. set (by the same number of unions) at the Belgian level rather than at the Flanders and Walloon level separately? In order to make a clear distinction between the two cases we will refer to the first one as 'centralisation' of wage setting and to the second one as 'regionalisation' of wage setting. Our focus point is now whether the degree of centralisation/regionalisation of wage setting affects the level of the wages and therefore the location decisions of firms.

4.1 'Centralisation' of wage setting

Proposition 6 *In a NEG model with unionised wage setting, more unions - and therefore wage setting at a lower (firm) level - may be a good policy to attract firms in a world with decreasing transport costs. If transport costs between countries remain high however, it might be advisable to set wages at a higher level in order to attract more firms.*

Proof. In order to analyse and proof this proposition, we want to see what happens to profits in both regions as the degree of centralisation decreases in one of the regions - i.o.w. when there are more unions in this region. We start from two initially symmetric regions and analyse what happens if the symmetry gets broken due to an increase in the number of unions in region H - assuming the number of unions remains the same in F. Thus, $dK^* = 0$, $dK = 1$ and therefore $Kd\hat{K} = dK$. In order to know how the location decision of firms will change, we analyse the effect of a change in K on the profit differential between the two regions. We assume that initially no firm moves (i.e. $d\hat{n} = 0$) but that they will move as a reaction to a profit differential. If the profit differential does not change, symmetric equilibrium remains. If the change in the profit differential is positive, firms will relocate to the region with the largest number of unions (and therefore the lowest wage).

The way to proceed is as before, when we derived the condition for the (in)stability of the symmetric equilibrium. Recall that the sign of the following expression equals the sign of the change in the profit differential ($d\left(\Pi_H - \Pi_F\right)$):

$$F = (1-\sigma)\, d\hat{W} + \varphi d\hat{Y} + \varphi\left(\sigma - 1\right) d\hat{p} \tag{35}$$

When (35) is positive, firms will locate in H (the low wage region) and when (35) is negative, firms will locate in F (the high wage region). Finally, when (35) equals zero, the symmetric outcome remains.

We want to analyse how profits (and therefore location) change when there are more unions in H. As mentioned before, initially we therefore keep the number of firms constant.

Assuming *infinite transport costs,* F becomes:

$$\begin{aligned} F(\tau &= 0) = d\hat{Y} \\ &= \frac{\alpha(\sigma-1)\{K(\sigma-1)-\sigma(\alpha-1)\}}{\sigma^2[K-1]-\alpha(\sigma-1)K} \frac{-K(\sigma-\alpha)}{[\sigma K-(\sigma-\alpha)][\sigma K-(\sigma-\alpha)-K]} d\hat{K} < 0 \end{aligned}$$

This expression is always negative, implying that when there are more unions in H, firms will want to relocate to F.

If *transport costs are zero*, we get:

$$\begin{aligned} F(1) &= (1-\sigma)\, d\hat{W} \\ &= (1-\sigma) \frac{-2K(\sigma-\alpha)}{[2\sigma K-(\sigma-\alpha)][2\sigma K-(\sigma-\alpha)-2K]+2K(\sigma-\alpha)(\sigma-1)} d\hat{K} > 0 \end{aligned}$$

This expression is always positive, implying that when there are more unions in H, firms will want to relocate to H.

For intermediate transport costs we get the following expression after computation (cfr. appendix):

$$\begin{aligned} F(\tau) &= (1-\sigma)\, d\hat{W} + \varphi d\hat{Y} + \varphi(\sigma-1)\, d\hat{p} \\ &= -(\sigma-1) K(\sigma-\alpha)(1+t) \left\{ -1 + \varphi \frac{\alpha\{K(1-t)(\sigma-1)\varphi - \sigma(\alpha-1)\}}{\sigma^2[K(1+t)-1]-\alpha(\sigma-1)K(1-t)} + \varphi^2 \right\} \\ &\quad \frac{1}{\begin{array}{c}[\sigma K(1+t)-(\sigma-\alpha)][\sigma K(1+t)-(\sigma-\alpha)-K(1+t)] \\ -2K(\sigma-\alpha)\, t(1-\sigma)\end{array}} d\hat{K} \end{aligned}$$

Since this expression cannot be solved analytically, we simulate it in order to obtain some tentative results. We investigate how F changes when $d\hat{K} = 1$. For all values of α, σ and K, the change in profit differential turns out to be negative for high values of transport costs and positive for small and intermediate values of transport costs. This implies that when transport costs are high, firms prefer to relocate to the high wage region since they prefer to be close the their demand. When transport costs decline however, the location of final demand is less of an issue and firms prefer to relocate to the low-wage region to minimise their costs. There is only one value of transport costs for which the symmetry remains unbroken - which is therefore a highly unlikely scenario. ■

Note that the model does not allow us to predict whether once the symmetry is broken, it will be restored or we will rather end up in an agglomerative outcome. The only thing we can tell is that symmetry will be broken when the number of unions changes. Depending on the level of transport costs this will initially benefit one or the other region. Although our model does not allow us to analyse what happens after this initial movement of firms, we might predict some future developments. Once the relocation to one of the regions has started, this may lead to further concentration in this region due to other mechanisms that are not taken up in this model - like for instance vertical input-output linkages between firms.

4.2 'Regionalisation' of wage setting

Does it matter whether wages are set at the regional or at the supra-regional level? That is the question we want to tackle in this section. If we consider two symmetric regions, it is obvious that the wage - and therefore the location decisions - will be the same no matter at what level the wage is set[10]. Interesting insights are therefore only to be expected when we assume one region to have initially a different number of firms. Assume for now that we have more firms in region H than in region F $(n > n^*)$. We will prove that the following proposition holds:

Proposition 7 *In a NEG model with unionised wage setting, supra-regional instead of regional wage setting will increase the wage in both regions and may be a good policy to attract more firms to the more agglomerated region (and therefore a devastating policy for regions that lag behind).*

Proof. From derivation in section 3.2.3, we know that when wages are set at the *regional level* each union in region H and region F sets the wage W_H and W_F as

$$\begin{aligned} \frac{W_H}{W_H - 1} &= \sigma + \frac{1}{K}\left(\frac{nW_H^{1-\sigma}}{nW_H^{1-\sigma} + n^*W_F^{1-\sigma}\tau^{\sigma-1}}\right)(\alpha - \sigma) \\ \frac{W_F}{W_F - 1} &= \sigma + \frac{1}{K}\left(\frac{n^*W_F^{1-\sigma}}{n^*W_F^{1-\sigma} + nW_H^{1-\sigma}\tau^{\sigma-1}}\right)(\alpha - \sigma) \end{aligned}$$

We know from comparative static results that, when $n > n^*$, $W_H > W_F$[11].

Now assume each union k to operate at the supra-regional level and set a nominal wage W_k for all its members whether they are employed in region H or F. The number of unions K remains the same, and each union sets the wage for a number of firms equal to $(n + n^*)/K$.

As before, the union cares about the real rent of its employed members. Here, however, a complication arises since the true price index differs between the two regions. This implies that the objective function of union k now becomes

$$\frac{W_k - 1}{p_{HM}^{\alpha}} L_{Hk} + \frac{W_k - 1}{p_{FM}^{\alpha}} L_{Fk}$$

After some computation (cfr. Appendix 3.D), we obtain the first order condition for the union:

$$\{K[W - \sigma(W - 1)] + (W - 1)(\sigma - \alpha)\}\left(p_{HM}^{\sigma-\alpha} + p_{FM}^{\sigma-\alpha}\right) = 0 \tag{36}$$

[10]Indeed, the union's objective function would just be a monotonic transformation thus the wage (outcome of the maximisation problem) would be the same.

[11]As $n = n^*$, wages in both regions are equal (symmetric outcome). From comparative statics we know that W_H increases and W_F decreases as n increases - which establishes our result.

which can only be satisfied if

$$K\left[W-\sigma\left(W-1\right)\right]+\left(W-1\right)\left(\sigma-\alpha\right)=0 \tag{37}$$
$$\Longrightarrow W=\frac{\sigma\left(K-1\right)+\alpha}{\sigma\left(K-1\right)+\alpha-K}$$

which is - not surprisingly - the same result as under agglomeration. Indeed, when a union determines the wage supra-regionally, it is as if it sets the wage in one agglomerated region.

The question we are interested in now is whether the wage will be higher or lower when it is set at the supra-regional rather than at the regional level. Since it is difficult to compare the expression for W_H and W_F with the expression for W (37) - the 'agglomeration'-wage, we resort again to simulations. Simulations (cfr. Appendix 3.D) reveal that $W > W_H > W_F$. This finding can be demonstrated more generally as follows. First note that regional and supra-regional wage setting will yield the same result $W = W_H = W_F$ in the limit case of $\tau = 0$. Indeed, when transport costs are infinite, each region can be considered as a fully agglomerated region.[12] Furthermore, from the comparative static results we know that when transport costs decrease, wages will decrease too:

$$\frac{\partial W_H}{\partial \tau} < 0 \quad \text{and} \quad \frac{\partial W_F}{\partial \tau} < 0,$$

which establishes our result. The reasoning behind the proposition is simple. Initially the peripheral region has a lower wage than the more agglomerated region. This is to the benefit of the peripheral region because the lower production costs make it still profitable for firms to produce there and ship (part of) their goods to the more agglomerated region. However, when wages are set at the supra-regional level they will increase to the same level in both regions. This implies that the initial cost-advantage of the peripheral region disappears such that all firms will tend to move into the already more agglomerated region.
■

Finally note that this result is in line with Driffil and van der Ploeg (1993), who argue that decreasing trade costs will decrease wages and therefore provides an incentive for unions to cooperate with other unions. We indeed also obtain the result that supra-regional wage setting allows unions to set a higher wage.

5 Conclusion

The presence of unions and immobile labour are two key aspects of European labour markets. The model in this chapter tried to address the importance of these features for the location decisions of firms. The novelty of the model concerns the introduction of the degree of centralisation of (unilateral) union wage setting, reflected by the number of unions present (more unions imply wage setting at a more decentralised level).

[12] Recall that the wage under full agglomeration is independent of the number of firms in the region.

We first of all illustrate that when the number of unions increases, the wage will decrease. Leaving aside any income or terms of trade effects in our partial equilibrium model, an increase in the number of unions will have a larger impact on the marginal cost (employment effect) than on the marginal benefit (real wage effect) of a nominal wage increase - thus inducing wages to decrease.

Secondly, like in other NEG models with an extra dispersion force, we find that the relationship between transport costs and agglomeration indeed tends to be an inverted U-curve: no agglomeration at high transport costs, the emergence of a core-periphery at intermediate costs and finally a reversion to dispersed manufacturing to take advantage of low wages at low transport costs. This finding strengthens again the relevance of the inverted U-curve. Even if labour markets are imperfect and labour is immobile, the standard results of the NEG remain valid.

In the last section, we show what the merits are of the model we developed - more importantly what role unions (can) play in location decisions. We focus on the impact of the degree of 'centralisation' (at firm, sectoral or national level) and 'regionalisation' (at regional or supra-regional level) of the wage setting on the wage level ánd the policy implications that follow from our analysis. Location decisions of firms will be influenced by both the fact whether wages are set at a more or less centralised level ánd whether they are set in each of the regions separately or rather at a coordinating level. We prove that wage setting at the firm level is the best policy to attract firms when transport costs are low while wage setting at the national level is most effective to attract firms when transport costs are high. Moreover, wage setting at the supra-regional level will be beneficial for the already more agglomerated region and hurt the peripheral region.

These are of course strong policy recommendations that depend on the assumptions we made in our model. Although it remains a partial equilibrium model, we do believe that it helps us to gain a better insight in what role the level of wage setting can play in location decisions of firms. We have taken the Dixit-Stiglitz framework to its boundaries in order to arrive at as many analytically solvable solutions as possible. We do however realise that computable general equilibrium models will make it possible to arrive at more all-embracing results and possibly policy recommendations. Future work could therefore focus on the impact of the level of wage setting in general equilibrium models.

Appendix A: Wage Setting and the Effect of the Number of Unions on the Wage

Wage setting

Union k maximizes

$$\frac{W_{Hk}-1}{p_{HM}^{\alpha}}L_{Hk}$$

where W_{Hk} is the union's wage applying to all of its members and L_{Hk} is the demand for union k's labour. Note that

$$\begin{aligned} p_{HM} &= \left[nq_H^{1-\sigma}+n^*\left(\frac{q_F}{\tau}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1}\left[\sum_{l=1}^{K}\frac{n}{K}W_{Hl}^{1-\sigma}+n^*\left(\frac{W_F}{\tau}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\ L_{Hk} &= \frac{n}{K}z_{Hj} = \frac{n}{K}\left(\frac{q_{Hj}}{p_{HM}}\right)^{-\sigma}G_H = \frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}W_{Hk}^{-\sigma}p_{HM}^{\sigma}G_H \end{aligned}$$

Union k is small in the sense of neglecting the effect of his wage on G_H. So the union maximizes

$$\frac{W_{Hk}-1}{p_{HM}^{\alpha}}W_{Hk}^{-\sigma}p_{HM}^{\sigma}$$

Using

$$\frac{\partial P_{HM}}{\partial W_{Hk}} = \frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{W_{Hk}}{p_{HM}}\right)^{-\sigma} \tag{38}$$

we get the following first order condition:

$$\frac{W_{Hk}}{W_{Hk}-1} = \sigma + \frac{n}{K}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{W_{Hk}}{p_{HM}}\right)^{1-\sigma}(\alpha-\sigma)$$

Note that all sectoral wages in a region will be equal. Hence

$$\frac{W_H}{W_H-1} = \sigma + \frac{1}{K}\left(\frac{nW_H^{1-\sigma}}{nW_H^{1-\sigma}+n^*W_F^{1-\sigma}\tau^{\sigma-1}}\right)(\alpha-\sigma)$$

The effect of the change in the number of unions on the wage: intuition

Recall that the FOC of the union can be written as:

$$L_{Hk}\frac{\partial \frac{W_{Hk}-1}{p_{HM}^{\alpha}}}{\partial W_{Hk}}+\frac{W_{Hk}-1}{p_{HM}^{\alpha}}\frac{\partial L_{Hk}}{\partial W_{Hk}}=0$$

The first term can be interpreted as the union's marginal benefit of the nominal wage, i.e. the additional real rent of its employed members resulting from a nominal wage increase, and the second term is the union's marginal cost of the nominal wage, i.e. the employment loss of the rise in the real wage resulting from the nominal wage increase.

We can rewrite this first order condition as follows:

$$p_{HM}-\alpha\left(W_{Hk}-1\right)\frac{\partial p_{HM}}{\partial W_{Hk}}=\sigma\frac{W_{Hk}-1}{W_{Hk}}\left(p_{HM}-W_{Hk}\frac{\partial p_{HM}}{\partial W_{Hk}}\right) \tag{39}$$

Note that the left hand side refers to the union's marginal benefit of the nominal wage and the right hand side to the union's marginal cost of the nominal wage.

Now suppose that the wage is such that the FOC holds. What will happen if the number of unions increases? The only channel through which the number of unions operates is the effect the wage will have on prices, i.e. K will only enter via $\frac{\partial p_{HM}}{\partial W_{Hk}}$.

From (38) we see that as the number of unions gets larger, a wage increase by an individual union will have a smaller impact on the general price level of manufactured goods. Hence, when the number of unions goes up, the union's marginal benefit of the nominal wage as well as the union's marginal cost of the nominal wage increases.

Write the FOC (39) as

$$p_{HM}-\alpha\left(W_{Hk}-1\right)\frac{\partial p_{HM}}{\partial W_{Hk}}=\sigma\frac{W_{Hk}-1}{W_{Hk}}p_{HM}-\sigma\left(W_{Hk}-1\right)\frac{\partial p_{HM}}{\partial W_{Hk}}$$

when $\sigma>\alpha$, an increase in the number of unions will have a larger impact on the marginal cost than on the marginal benefit of the nominal wage. As a consequence, the union's optimal wage decreases.

Appendix B: Agglomeration as a(n) (un)stable equilibrium

For agglomeration in region H to be an equilibrium, the following condition should hold:

$$\begin{aligned} W_H^{1-\sigma}p_{HM}^{\sigma}G_H &> W_F^{1-\sigma}p_{FM}^{\sigma}G_F \\ \left(\frac{W_H}{W_F}\right)^{1-\sigma} &> \left(\frac{p_{HM}}{p_{FM}}\right)^{-\sigma}\frac{G_F}{G_H} \end{aligned}$$

Using the definitions of G_H, G_F, p_{HM} and p_{FM}, the condition for agglomeration becomes:

$$\left(\frac{W_H}{W_F}\right)^{1-\sigma} > \tau^{1-\sigma}\frac{Y_F + \tau^{2(\sigma-1)}Y_H}{Y_F + Y_H}$$

After tedious computations and substitution of Y_F and Y_H by their respective expressions, we can rewrite the condition as follows:

$$\left(\frac{(\sigma-1)\left[\sigma\left(K-1\right)+\alpha\right]}{\sigma\left[\sigma\left(K-1\right)+\alpha-K\right]}\right)^{1-\sigma} > \tau^{1-\sigma}\frac{\left[\sigma^2(K-1)+\alpha\sigma\right]\left[1+\tau^{2(\sigma-1)}\right]-\alpha K(\sigma-1)\left[1-\tau^{2(\sigma-1)}\right]}{2\left[\sigma^2(K-1)+\alpha\sigma\right]}$$

Consider the function $F(\tau)$ defined by

$$F(\tau) = \left(\frac{(\sigma-1)\left[\sigma\left(K-1\right)+\alpha\right]}{\sigma\left[\sigma\left(K-1\right)+\alpha-K\right]}\right)^{1-\sigma} - \tau^{1-\sigma}\frac{\begin{array}{c}\left[\sigma^2(K-1)+\alpha\sigma\right]\left[1+\tau^{2(\sigma-1)}\right]\\ -\alpha K(\sigma-1)\left[1-\tau^{2(\sigma-1)}\right]\end{array}}{2\left[\sigma^2(K-1)+\alpha\sigma\right]}$$

Calculate now $F^{'}(\tau)$

$$F^{'}(\tau) = \frac{\begin{array}{c}\left\{-\left[\sigma^2(K-1)+\alpha\sigma\right]2\tau^{2(\sigma-1)}-\alpha K(\sigma-1)2\tau^{2(\sigma-1)}\right\}-\\ \left\{-\left[\sigma^2(K-1)+\alpha\sigma\right]\left[1+\tau^{2(\sigma-1)}\right]+\alpha K(\sigma-1)\left[1-\tau^{2(\sigma-1)}\right]\right\}\end{array}}{2\left[\sigma^2(K-1)+\alpha\sigma\right]\tau^{\sigma-1}\frac{1}{\sigma-1}\tau} \lesseqgtr 0$$

Wo now only look at the sign of the denominator since the nominator is positive:

$$-\alpha K(\sigma-1)(1+\tau^{2(\sigma-1)}) + \left[\sigma^2(K-1)+\alpha\sigma\right](1-\tau^{2(\sigma-1)}) \lesseqgtr 0$$

Appendix C: Symmetry as a(n) (un)stable equilibrium

The effect of a change in the number of firms on profits

Expressions for $d\hat{W}, d\hat{p}$ and $d\hat{Y}$

Effect on wages Starting from the wage expressions:

$$\frac{W_H}{W_H-1} = \sigma - \frac{\sigma-\alpha}{K}\left(\frac{nW_H^{1-\sigma}}{nW_H^{1-\sigma}+n^*W_F^{1-\sigma}\tau^{\sigma-1}}\right)$$

$$\frac{W_F}{W_F-1} = \sigma - \frac{\sigma-\alpha}{K}\left(\frac{n^*W_F^{1-\sigma}}{n^*W_F^{1-\sigma}+nW_H^{1-\sigma}\tau^{\sigma-1}}\right)$$

we obtain after tedious computations expressions for the change in the wage in both regions:

$$dW_F = -\frac{Kt(\sigma-\alpha)}{[\sigma K(1+t)-(\sigma-\alpha)][\sigma K(1+t)-(\sigma-\alpha)-K(1+t)] - 2K(\sigma-\alpha)t(1-\sigma)} W d\hat{n}$$

$$dW_H = \frac{Kt(\sigma-\alpha)}{[\sigma K(1+t)-(\sigma-\alpha)][\sigma K(1+t)-(\sigma-\alpha)-K(1+t)] - 2K(\sigma-\alpha)t(1-\sigma)} W d\hat{n}$$

and thus also the expression for $d\hat{W}$:

$$d\hat{W} = \frac{2Kt(\sigma-\alpha)}{[\sigma K(1+t)-(\sigma-\alpha)][\sigma K(1+t)-(\sigma-\alpha)-K(1+t)] - 2K(\sigma-\alpha)t(1-\sigma)} d\hat{n}$$

Effect on prices Evaluating $d\hat{p}$ at the symmetric equilibrium, we get the following expression:

$$d\hat{p} = \frac{1}{1-\sigma}\frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} \times \left\{(1-\sigma)\, d\left(\frac{W_H}{W_F}\right)_S + d\hat{n}\right\}$$

Effect on income Using the expressions for Y_H and Y_F and Y, z_j, G and p_M at the symmetric equilibrium, we obtain the following expression for $d\hat{Y}$

$$d\hat{Y} = \alpha\frac{\sigma-1}{\sigma}\left\{\frac{W-1}{W}\left(d\hat{G} + \sigma d\hat{p}_M - \sigma d\hat{W} + d\hat{n}\right) + d\hat{W}\right\}$$

Where $d\hat{G}$ can be shown to be:

$$d\hat{G} = \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}} d\hat{Y} + \frac{1-\tau^{\sigma-1}}{1+\tau^{\sigma-1}}(\sigma-1)\, d\hat{p}_M - \sigma d\hat{p}_M$$

Sign of $d\hat{W}, d\hat{p}$ and $d\hat{Y}$

Sign of $d\hat{W}$

$$d\hat{W} = \frac{2Kt(\sigma-\alpha)}{[\sigma K(1+t)-(\sigma-\alpha)][\sigma K(1+t)-(\sigma-\alpha)-K(1+t)] - 2K(\sigma-\alpha)t(1-\sigma)} d\hat{n}$$

The nominator is always positive but note that the denominator, denoted by $D(t)$ is also strictly positive since

$$D(0) = [\sigma K - (\sigma-\alpha)][\sigma K - (\sigma-\alpha) - K] > 0$$

since $(\sigma-1)K>\sigma$ because of (19). Furthermore

$$D(1)=[2\sigma K-(\sigma-\alpha)][2\sigma K-(\sigma-\alpha)-2K)]-2K(\sigma-\alpha)(1-\sigma)>0$$

Since the first two terms are positive (since $(\sigma-1)K>\sigma$) and the third term is negative.

Finally

$$\begin{aligned} D'(t) &= [\sigma K(1+t)-(\sigma-\alpha)][\sigma K-K]+[\sigma K(1+t)-(\sigma-\alpha)-K(1+t)]\sigma K \\ &\quad -2K(\sigma-\alpha)(1-\sigma) \\ &= K\left\{\begin{array}{c}[\sigma K(1+t)-(\sigma-\alpha)][\sigma-1]+[\sigma K(1+t)-(\sigma-\alpha)-K(1+t)]\sigma \\ -2(\sigma-\alpha)(1-\sigma)\end{array}\right\}>0\end{aligned}$$

This expression is positive for all values of t>0 since the first two terms are positive and the last one negative (see previous).

Sign of $d\hat{p}$

$$\begin{aligned} d\hat{p} &= \frac{1}{1-\sigma}\varphi\left[(1-\sigma)d\hat{W}+d\hat{n}\right] \\ &= \frac{1}{1-\sigma}\varphi\left[\frac{[(\sigma-1)K(1+t)-(\sigma-\alpha)][K\sigma(1+t)-(\sigma-\alpha)]}{\begin{array}{c}[(\sigma-1)K(1+t)-(\sigma-\alpha)][K\sigma(1+t)-(\sigma-\alpha)] \\ -(\sigma-\alpha)K2t(1-\sigma)\end{array}}d\hat{n}\right]\leq 0,\end{aligned}$$

because of (19)

Sign of $d\hat{Y}$ The sign of $d\hat{Y}$ equals the sign of

$$K(1+t)[\varphi(\sigma-1)d\hat{p}_M+d\hat{n}]-\sigma d\hat{W}$$

and

$$\begin{aligned}&\varphi(\sigma-1)d\hat{p}_M+d\hat{n} \\ =\ & \frac{(1-\varphi^2)[(\sigma-1)K(1+t)-(\sigma-\alpha)][K(1+t)-(\sigma-\alpha)]+(\sigma-\alpha)2Kt(\sigma-1)}{[(\sigma-1)K(1+t)-(\sigma-\alpha)][K\sigma(1+t)-(\sigma-\alpha)]-(\sigma-\alpha)K2t(1-\sigma)}d\hat{n}\end{aligned}$$

hence the sign of $d\hat{Y}$ equals the sign of

$$\frac{\begin{array}{c}K(1+t)(1-\varphi^2)[(\sigma-1)K(1+t)-(\sigma-\alpha)][\sigma K(1+t)-(\sigma-\alpha)] \\ +2Kt[K(1+t)(\sigma-1)(\sigma-\alpha)-(\sigma-\alpha)]\end{array}}{[(\sigma-1)K(1+t)-(\sigma-\alpha)][\sigma K(1+t)-(\sigma-\alpha)]-(\sigma-\alpha)2Kt(1-\sigma)}d\hat{n}\geq 0$$

The effect of a change in the number of unions on profits

Expressions for $d\hat{W}, d\hat{p}$ and $d\hat{Y}$

Note that we derive all expressions assuming $d\hat{n} = 0$.

Effect on wages Again starting from the wage expressions in both regions, assuming $d\hat{n} = 0$ and lots of computations, we arrive at the expression for $d\hat{W}$ and note that at

$$d\hat{W} = \frac{-K(\sigma - \alpha)(1+t)}{\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right)\right]\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right) - K(1+t)\right] - 2K\left(\sigma - \alpha\right)t(1-\sigma)} d\hat{K}$$

Effect on prices idem before

Effect on income idem before

Sign of $d\hat{W}, d\hat{p}$ and $d\hat{Y}$

Sign of $d\hat{W}$ The sign of $d\hat{W}$ is not clear in this case since it depends both on the number of firms and on the number of unions. We observe however that when the number of firms increases - given a number of unions - so does the wage. When the number of unions increases - given the number of firms - wages will decrease.

$$d\hat{W} = \frac{2Kt(\sigma - \alpha)}{\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right)\right]\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right) - K(1+t)\right] - 2K\left(\sigma - \alpha\right)t(1-\sigma)} d\hat{n} + \frac{-K(\sigma - \alpha)(1+t)}{\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right)\right]\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right) - K(1+t)\right] - 2K\left(\sigma - \alpha\right)t(1-\sigma)} d\hat{K}$$

Sign of $d\hat{p}$ idem before

Sign of $d\hat{Y}$ idem before

Sign of F for intermediate trade costs

$$\begin{aligned} F &= (1-\sigma)\, d\hat{W} + \varphi d\hat{Y} + \varphi\left(\sigma - 1\right) d\hat{p} > 0 \\ &= -\left(\sigma - 1\right) K(\sigma - \alpha)(1+t)\left\{-1 + \varphi\frac{\alpha\left\{K\left(1-t\right)\left(\sigma - 1\right)\varphi - \sigma\left(\alpha - 1\right)\right\}}{\sigma^2\left[K\left(1+t\right) - 1\right] - \alpha\left(\sigma - 1\right)K\left(1-t\right)} + \varphi^{2}\right\} \\ &\quad \frac{1}{\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right)\right]\left[\sigma K\left(1+t\right) - \left(\sigma - \alpha\right) - K(1+t)\right] - 2K\left(\sigma - \alpha\right)t(1-\sigma)} d\hat{K} \end{aligned}$$

Appendix D: 'Regionalisation' of wage setting

Union k maximizes

$$\frac{W_k - 1}{p_{HM}^{\alpha}} L_{Hk} + \frac{W_k - 1}{p_{FM}^{\alpha}} L_{Fk}$$

where W_k is the union's wage applying to all of its members and L_{Hk}/L_{Fk} is the demand for union k's labour in region H/F.

Note that

$$\begin{aligned}
p_{HM} &= \left[n q_H^{1-\sigma} + n^* \left(\frac{q_F}{\tau} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma - 1} \left[\sum_{l=1}^{K} \frac{n}{K} W_l^{1-\sigma} + \sum_{l=1}^{K} \frac{n^*}{K} \left(\frac{W_l}{\tau} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
p_{FM} &= \left[n^* q_F^{1-\sigma} + n \left(\frac{q_H}{\tau} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma - 1} \left[\sum_{l=1}^{K} \frac{n^*}{K} W_l^{1-\sigma} + \sum_{l=1}^{K} \frac{n}{K} \left(\frac{W_l}{\tau} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
L_{Hk} &= \frac{n}{K} z_{Hj} = \frac{n}{K} \left(\frac{q_{Hj}}{p_{HM}} \right)^{-\sigma} G_H = \frac{n}{K} \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} W_k^{-\sigma} p_{HM}^{\sigma} G_H \\
L_{Fk} &= \frac{n^*}{K} z_{Fj} = \frac{n^*}{K} \left(\frac{q_{Fj}}{p_{FM}} \right)^{-\sigma} G_F = \frac{n^*}{K} \left(\frac{\sigma}{\sigma - 1} \right)^{-\sigma} W_k^{-\sigma} p_{FM}^{\sigma} G_F
\end{aligned}$$

Union k is small in the sense of neglecting the effect of his wage on G_H. So the union maximizes

$$\frac{W_k - 1}{p_{HM}^{\alpha}} W_k^{-\sigma} p_{HM}^{\sigma} + \frac{W_k - 1}{p_{FM}^{\alpha}} W_k^{-\sigma} p_{FM}^{\sigma}$$

Using

$$\begin{aligned}
\frac{\partial p_{HM}}{\partial W_k} &= \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p_{HM}^{\sigma} W_k^{-\sigma} \left[\frac{n}{K} + \frac{n^*}{K} \tau^{\sigma-1} \right] \\
\frac{\partial p_{FM}}{\partial W_k} &= \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} p_{FM}^{\sigma} W_k^{-\sigma} \left[\frac{n^*}{K} + \frac{n}{K} \tau^{\sigma-1} \right]
\end{aligned}$$

we get the following first order condition:

$$\{ K \left[W - \sigma \left(W - 1 \right) \right] + \left(W - 1 \right) \left(\sigma - \alpha \right) \} \left(p_{HM}^{\sigma-\alpha} + p_{FM}^{\sigma-\alpha} \right) = 0$$

which can only be satisfied if

$$K\left[W-\sigma\left(W-1\right)\right]+\left(W-1\right)\left(\sigma-\alpha\right)=0$$
$$\Longrightarrow W=\frac{\sigma\left(K-1\right)+\alpha}{\sigma\left(K-1\right)+\alpha-K}$$

which is the same result as under agglomeration (not surprisingly).

We present some simulation results based on the following parameter configurations: $K=10,\tau=0.7,\alpha=0.9,n=100,n^{*}=50$.

	W_H	W_F	W
$\sigma=3$	1.5458	1.5283	1.5587
$\sigma=4$	1.3655	1.3553	1.3717
$\sigma=5$	1.2751	1.2687	1.2786
$\sigma=6$	1.2207	1.2166	1.2227
$\sigma=7$	1.1844	1.1817	1.1855

These simulations reveal that $W>W_H>W_F$.