Should risk averse countries be "trade averse"?

I

Abstract

Taking seriously the stylized fact that some countries are more risk averse than others regarding business activities, we examine the consequences on international trade and welfare. When risk is idiosyncratic to each manager's project, international differences in managers' risk-aversion distribution become the determinant of trade pattern. Welfare analysis shows that the less risk averse country specialized in the risky commodity is always better off with free trade. But the more risk averse country specialized in unrisky commodity can be worse off. Consequently, risk averse countries can be "trade averse". Moreover, the world's welfare is sometimes decreasing.

- *Keywords:* International Trade, Risk-aversion, idiosyncratic risk, Inferior Trade.
- *J.E.L Classification*: F10 F13

1 Introduction

As reported by the Global Entrepreneurship Monitor [4], 10.51% of the adult population is involved in the creation and growth of start up businesses in the United States. The rate of entrepreneurial activity is lower in the main trade partners of the United States: 1.81% in Japan, 3.2% in France, 5.16% in Germany and 5.37% in the United Kingdom. According to Erkii Liikanen, member of the European Commission, responsible for Enterprise and the Information Society, "Europe suffers from an entrepreneurship deficit in comparison to the US". This stylized fact suggests that the United States are relatively well endowed in a production factor called "entrepreneur". But countries' endowments can be explained upward by the attitude towards risk of the citizens of each country. This assumption is supported by the European Commission's Eurobarometer survey [3] since 48% of European but 37% of American individuals surveyed agreed with the statement "a business should not be set up if there is a risk it might fail".

Consequently, national specialization could be grounded on the "entrepreneurship deficit" of European countries in comparison to the United States. Should European countries avoid risks and let the United States specialize in risky activities? What is the impact of free trade on welfare? In this paper we focus on these issues by developing a model which enlightens the choice of managers between risky (innovative) and certain (routine) activities.

The proportion of managers involved in innovative projects in an economy is often explained by the financial system's ability to provide funds for innovative projects[1]. But other considerations are fruitfull to understand the share of innovative activity in the economy. Following Manove and Padilla [5] and Allen and Gale [1], we stress the key role of the diversity of opinions rather than the financial system design. As already quoted by Schumpeter [8], some managers are more "entrepreneur" than others when they face the uncertainty inherent to innovative projects. Keeping this idea of diversity in mind, we consider the diversity of managers regarding their own degree of risk-aversion and also the diversity of countries regarding their managers' risk-aversion repartition functions. These functions synthetize psychological, cultural and sociological differences as well as financial systems specificities. This framework is consistent with the results of the Eurobarometer 107 poll [3]: some countries are more risk averse than others.

Our model is close in spirit to the seminal papers of Newbery and Stiglitz [6] and Shy [9] since we assume that risk averse managers freely choose between a risky project and a safe one and markets for risk sharing are incomplete. Nevertheless, we move away from these frameworks by introducing (i) a diversity of managers instead of a representative manager and (ii) an idiosyncratic risk rather than a global risk. According to us, this idiosyncratic perspective is more adapted to innovative activities: in each country some risky projects succeed while others fail.

[1] See Thakor [10] for a survey.

Under these assumptions, our model focuses on international differences in managers' risk-aversion distribution as the determinant of the trade pattern and examines each country's gain from trade and the global effect as well. Gains from trade are based on ex post utility comparisons resulting from effective consumptions.

We show that the globally less risk averse country allocates more managers in the risky activity than the globally more risk averse country. Then, the former country specializes in the production of the risky commodity while the latter specializes in the certain commodity resulting from the routine process. From the normative point of view, the impact of free trade on countries' welfare is contrasted: (i) the less risk averse country is always better off, (ii) the more risk averse country can be either better or worse off. Moreover (iii) lump-sums transfers are sometimes impossible since the world's welfare may decline.

The paper is organized as follows. Section 2 describes the framework of the model. Section 3 analyses the autarky equilibrium. Section 4 investigates the free trade equilibrium. Section 5 is the welfare analysis. Section 6 concludes. Proofs are provided in the appendix.

2 The framework

In this section, we describe the structure of an economy with a continuum of risk averse managers bound to choose between the production of a uncertain commodity and a certain commodity.

□ *Managers*

We consider a continuum of managers with a population normalized to 1. It is possible to identify each manager i with his specific Constant Relative risk-aversion (CRRA) α_i, strictly different from his peers's one. CRRA exhibits a decreasing absolute risk-aversion with revenue, which is consistent with empirical revealed preferences. This specification has also the advantage to conserve the relative position of managers for each level of revenue.

Moreover, we assume a twice differentiable and monotonous distribution of manager's risk-aversion. Risk-aversion is assumed to be strictly inferior to unity[2].

$\alpha_i = \Theta_A(i) : [0,1] \to [\underline{\alpha}, \overline{\alpha}]$ with $\alpha_i \in]0,1[$.

The less risk averse manager has the lowest CRRA : $\Theta_A(0) = \underline{\alpha}$. Similarly, the more risk averse manager has the highest CRRA : $\Theta_A(1) = \overline{\alpha}$.

Since population is normalized to 1, the population share having a CRRA lower or equal to the i^{th} manager's one (i.e α_i) is then equal to $i\%$.

[2] If we don't restrict the risk-aversion to be less than one, the risk-averse utility can be negative. Then, a lower probability would give a lower negative expected utility and then a superior expected utility! As in Shy [9], it would be necessary to have a non-zero paiement in case of project failure. This would unnecessarily complexify our model, without representative agent.

Remark 1 *With* $\Theta_A(i) = (\overline{\alpha} \square \underline{\alpha})i + \underline{\alpha}$ *and* $\Theta_B(i) = (\overline{\alpha} \square \underline{\alpha})i^2 + \underline{\alpha}$ $\forall i, \Theta_A(i) > \Theta_B(i)$, *we obtain the following graphical illustration:*

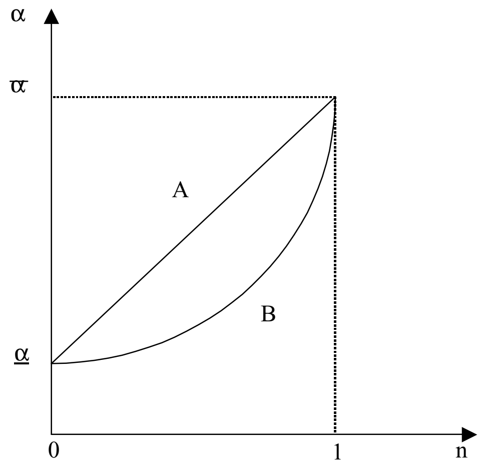

Figure 1.

□ *Uncertainty*

We assume that there is only two commodity C and U. In period 0, managers must choose exclusively between production project C or production project U. The former comes from a well-known and already experienced routine technology producing traditional commodity. Thus project C is riskless and products a certain quantity of a commodity after one period. On the contrary, commodity U requires a new technological process which can fail because of technological perfecting difficulties or skills learning problems. Hence, project U provides a quantity of an innovative commodity U with probability θ and fails (no production) with probability $1 \square \theta$.

We assume that the probability of success θ is correctly anticipated by managers. This assumption means that the difficulties of new technology adoption are close to similar cases in the past. Hence, projects U are incremental innovations rather than radical innovations in the sense of Schumpeter.

□ *Market structure*

There is a only one production factor, the manager's own labor: a_c and a_u denote the manager's productivity in each project. Because of constant returns, markets are perfectly competitive. Hence, the profit is exhausted in each industry and manager's remuneration is simply the competitive market wage: $w_c = p_c a_c$ and $w_u = p_u a_u$. We assume commodity C is the numeraire. Then $p_c = 1$ and p refers to the relative price of commodity U in terms of commodity C.

Choice is exclusive: each production project requires the total worktime of a manager, preventing any diversification choice. Managers choose between C and U prior to the resolution of uncertainty. After those initial choices, labor is frozen in each sector until the results of production processes are revealed. Then managers receive the competitive market wage specific to the industry, w_c or w_u which is contingent to success. In other words, project C provides a certain wage (revenue) paiement w_c in period 1 while the uncertain project U delivers w_u with probability θ and zero with probability $1 \square \theta$. This assumption of industry-specific wage is fundamental or else an hold-up problem would arised : if wages were equal ex post, risk averse managers would not take the risk of producing U.

□ *Consumption*

The manager-consumer buys commodities after the resolution of uncertainty in period 1.Then we assume that all consumers are identical and agregate utility is $V_i = d_U^b d_C^{1 \square b}$. Hence, the demand functions in period 1 are $d_u = \frac{by}{p}$ and $d_c = (1 \square b)y$ where y is the national income.

□ *Producers' decision rule*

In order to reach a decision, each manager-worker-consumer maximizes a Von Neumann-Morgenstern expected utility of his consumption function.

In period 0, a manager i chooses the project which gives the highest expected utility of revenue regarding his degree of risk-aversion α_i. Although the ultimate goal of a manager is to maximize his consumption utility function, this goal can be achieve by the prior indirect maximization of expected utility of business revenue of production.

Define $\Omega_i(R_i)$ the utility of business revenue R_i :

$$\Omega_i(R_i) = \frac{R_i^{1 \square \alpha_i}}{1 \square \alpha_i}$$

Then project C is prefered iff $\Omega_i(w_c) \geq \theta\Omega_i(w_u)$. Substituting $\Omega(.)$ by its expression gives a manager's i the following decision rule:

Decision rule: A manager i chooses to product good C iff:

$$\frac{w_c^{1 \square \alpha_i}}{1 \square \alpha_i} \geq \theta \frac{w_u^{1 \square \alpha_i}}{1 \square \alpha_i} \tag{1}$$

However, the decision is not trivial as long as the revenues are not initially given. The salient fact is that the price of each commodity (and consequently each revenue) depends on its aggregate supply i.e on how many managers are engaged in the project U. So individual decision needs first to anticipate others managers' choice and second to compute the production-consumption equilibrium. We assume that the distribution of managers' CRRA is common knowledge in autarky and in free trade as well.

Then, managers must include the risk-aversion distribution function in order to anticipate rationally the relative price of commodity U. This general equilibrium model with rational expectation is the same for all managers. But, their price contingent decision still only depends on their own CRRA.

3 Equilibrium in autarky

3.1 Production with perfect insurance market (or perfect market for risk management) or perfect diversification

In such cases, the managers are protected from idiosyncratic risk and choose the production according to their expected rewards. Hence, the first-best solution hence only on consumers' preferences and is not affected by manager's risk preferences. The proportion n of managers choosing the risky project is then equal to b, the taste of consumers for the risky commodity. The equilibrium price reflects the differences in expected productivities $p^a = \frac{a_c}{\theta a_u}$. For $n = b$ we have the maximum first-best welfare for the economy.

3.2 Production without insurance market nor portfolio diversification

In a context of innovative firms, working on new technologies, it seems reasonable to assume that markets are incomplete. In our model, there is neither markets for contingent goods nor insurance markets nor markets for securities. Moreover diversification of production is impossible. This comes from the necessarily fulltime involvement of managers in such ambitious technological projects. Then, in such a context of incomplete markets for risk management, managers' preferences over risk matter[3]. Without insurance market, the equilibrium price of the risky commodity must be higher to prompt risk averse managers to bear their idiosyncratic risk : the risk averse managers involved in the risky project receive a risk premium equal to $p\theta a_u \square a_c$. Then market equilibrium moves away from first-best to second-best equilibrium.We now describe the properties of this second-best autarky equilibrium with incomplete markets.

3.2.1 The endogenous remunerations

The problem amounts to identify the quotation n^a of the manager indifferent between the two project, $w_c^{1\square\alpha^a} = \theta w_u^{1\square\alpha^a}$. According to the decision rule, managers having a lower CRRA than α^a choose project U. Consequently, the share of population allocated in this industry is equal to n^a.

The remuneration of each project depends on the sectorial market wage. As wage equals marginal productivity in value in each industry, we have

$$w_c = a_c \text{ and } w_u = p^a a_u, \text{ hence } \frac{w_c}{w_u} = \frac{a_c}{p^a a_u}$$

Market prices equate demand to supply, that is $d_u = y_u$. Since only $n^a\theta$ lucky managers succeed, production of commodity U is $y_u = n^a\theta a_u$. On the

[3] More precisely firms must build a preference function resulting from, for example, a weighting of large shareholders preferences or manager's preferences as quoted by J. Dreze [2].

demand side, aggregate demand for commodity U is given by $d_u = \frac{by}{p^a}$ where aggregate consumer income is

$$y = (1 \square n^a)a_c + p^a \theta n^a a_u. \tag{2}$$

So, the market clearing relative price is

$$p^a = \frac{1 \square n^a}{n^a} \times \frac{b}{1 \square b} \times \frac{a_c}{\theta a_u}$$

These endogenous relationships help each manager to compute the autarky equilibrium and to choose between project U and C.

3.2.2 Equilibrium allocation of managers in autarky

The following equations identify the autarky equilibrium without insurance market,

$$\begin{aligned}
\alpha^a &= \Theta(n^a) && (3) \\
\theta &= \left[\frac{w_c}{w_u}\right]^{1 \square \alpha^a} && (4) \\
\frac{w_c}{w_u} &= \frac{a_c}{p^a a_u} && (5) \\
p^a &= \frac{1 \square n^a}{n^a} \times \frac{b}{1 \square b} \times \frac{a_c}{\theta a_u} && (6)
\end{aligned}$$

This problem comes down to compute the value of n^a, that is the position of the indifferent manager between the two projects and the proportion of managers choosing U as well. Substituting $\frac{w_c}{w_u}$ and p^a into the decision rule gives

$$\theta \left[\frac{b}{1 \square b} \frac{1 \square n^a}{\theta n^a}\right]^{1 \square \Theta(n^a)} = 1 \tag{7}$$

It can be seen from appendix A that equation (7) has an unique solution. So each manager can compute the production of both commodities, the rational expectation of relative price p^a of commodity U and the wages that will prevail in period 1. Then, each manager can choose in period 0 according to the decision rule (1).

One can verify that the proportion of managers in industry U is lower in the absence of a perfect insurance market $(n^a < b)$. A higher wage is required in industry U to prompt risk averse managers to engage in the uncertain activity. Hence, the relative autarky price of commodity U is higher in the absence of a perfect insurance market $p^a > \frac{a_c}{\theta a_u}$.

3.2.3 Diagrammatic exposition

We provide a diagrammatic representation in order to enlighten the role of demand and managers' CRRA distribution in the autarky equilibrium. Consider again equation (7). Applying a logarithmic transformation, we have :

$$\begin{aligned} \theta \left[\frac{b}{(1 \Box b)\theta} \frac{1 \Box n^a}{n^a} \right]^{1 \Box \Theta(n^a)} &= 1 \\ \ln\left(\frac{b}{1 \Box b}\right) + \ln\left(\frac{1}{n^a} \Box 1\right) &= \Box \ln\theta \times \frac{\Theta(n^a)}{1 \Box \Theta(n^a)} \\ \text{that is } \phi(n,b) &= \delta(\theta, n) \end{aligned}$$

Figure 2 shows how the functions $\phi(n)$ and $\delta(n)$ determine the equilibrium allocation of managers in industry U. The downward-sloping curve $\phi\phi$ embodies demand conditions and the upward-sloping curve $\delta\delta$ includes uncertainty features with $\underline{\delta} = \Box \ln\theta \times \frac{\underline{\alpha}}{1 \Box \underline{\alpha}}$ and $\overline{\delta} = \Box \ln\theta \times \frac{\overline{\alpha}}{1 \Box \overline{\alpha}}$. Equilibrium is at point E^a where $\phi\phi$ and $\delta\delta$ intersect.

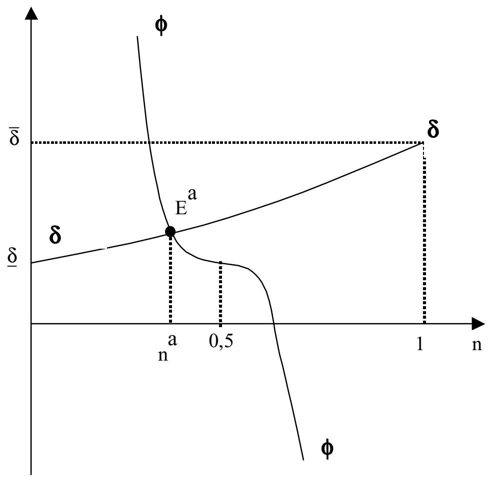

Figure 2. Autarky equilibrium

The curvature of $\delta\delta$ is indeterminate since it depends on the value of $\Theta''(n^a)$. If the country is risk averse enough ($\Theta''(n^a)$ is sufficiently negative), then $\delta\delta$ is concave. The reverse case is depicted in figure 2.

Now consider the effect of a change in demand conditions and in uncertainty features. Then, it is clear that the equilibrium allocation of managers in the industry U increases if :

□ demand for commodity U increases ($b \nearrow$) as $\phi(n)$ shifts to the north.

□ probability of success increases ($\theta \nearrow$) as $\delta(n)$ shifts to the north.

□ risk-aversion becomes globally lower in the country as the convexity (concavity) of $\delta\delta$ increases (decreases).

4 Free trade equilibrium

With perfect insurance markets $n = b$ in each country and no trade can be explained in this model. On the contrary, when a perfect insurance market (or complete markets) doesn't exist, the structure of international trade comes from international differences of managers' risk-aversion distributions.

4.1 The law of comparative advantage

Comparative advantage is specified with respect to autarky relative prices. In autarky, the relative price of commodity U depends on demand conditions, technology and on the behaviour of managers toward uncertainty. Recall that

$$p_J^a = \frac{1 \square n_J^a}{n_J^a} \times \frac{b}{1 \square b} \times \frac{a_c}{\theta a_u} \qquad J = A, B.$$

Assuming demand conditions and technology are the same in all countries, country B has a comparative advantage in commodity U if this country allocates more managers in the uncertain project:

$$p_B^a < p_A^a \Leftrightarrow n_B^a > n_A^a$$

Let us define the relative globally risk-aversion as follows: country A is globally more risk averse than a country B if

$$\forall \alpha \in \left]\underline{\alpha}, \overline{\alpha}\right[, \Theta_A^{\square 1}(\alpha) < \Theta_B^{\square 1}(\alpha).$$

Such a case is illustrated by figure 1. Then, we have the following result.

Proposition 1 *The globally more (less) risk-averse country has a comparative advantage in the production of the certain (resp. uncertain) commodity. A comparision of the autarky equilibria implies that the relative price of commodity U is higher in country A than in country B :* $p_A^a > p_B^a$.
Proof. See appendix B.

Figure 3 provides a diagrammatic illustration of this result. As explained before the curvature of the function $\delta_J\delta_J$ conveys the distribution of risk-aversion in country J : the more concave (or the less convex) δ_J δ_J is, the more risk averse country J is[4].

[4] However, notice that the curvature is not the only way to denote countries' risk-aversion. As $\underline{\alpha}_J$,and $\overline{a}_J$ can be country specific, the distribution functions can be simply illustrated by two parallel straight lines, the upper one denoting the more risk-averse country (see the welfare analysis, simulation 3).

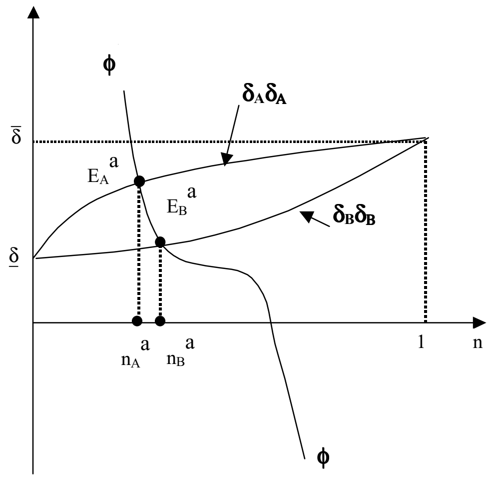

Figure 3.

Points E_A^a and E_B^a refer autarky equilibrium in A and B respectively. So country B allocates more managers in industry U than country A ($n^a{}_B > n_A^a$).

4.2 Equilibrium allocation of managers in free trade

Under free trade, each manager have now to anticipate the behavior of domestic and foreign managers. Managers must deal with the risk-aversion distribution of the two countries i.e the international risk-aversion distribution function. Such a calculus is possible because we assume that both countries' risk-aversion distribution are common knowledge. Under trade, less risk averse managers of both countries choose industry U.

Otherwise, the framework of the model is exactly the same as in autarky. The world supply and the world demand of commodity U are given by :

$$y_U^* = (n_A^* + n_B^*)\theta a_U \quad \text{and} \quad d_U^* = \frac{b\,[y_A^* + y_B^*]}{p^*}$$

where the star * refers to free trade value.

Hence, the world income is:

$$y^* = y_A^* + y_B^* = \theta\,(n_A^* + n_B^*)\,p^* a_u + (2 \,\square\, n_A^* \,\square\, n_B^*)a_c$$

Then, the relative price of commodity U under free trade is:

$$p^* = \frac{b}{(1 \,\square\, b)} \times \frac{[2 \,\square\, (n_A^* + n_B^*)]}{n_A^* + n_B^*} \times \frac{a_c}{\theta a_u} \tag{8}$$

The international equilibrium implies $\alpha^* = \alpha_A^* = \alpha_B^*$, where $\alpha^*$is the free trade critical risk-aversion level splitting the world's population of managers between those choosing U or C. The parameter α^* denotes the level of risk-aversion of the world economy at equilibrium. The knowledge of α^* will resolve the problem since it gives n_A^* and n_B^*, and hence p^*. Substituting $\frac{w_c}{w_u}$ and p^* into the decision rule, we have the following equation:

$$\theta\left[\frac{b}{1-b}\times\frac{[2-(n_A^*+n_B^*)]}{\theta(n_A^*+n_B^*)}\right]^{1-\alpha_J^*}=1,\ J=A,B$$

Since $n_J^*=\Theta_J^{-1}(\alpha^*)$ by definition, the critical risk-aversion is the solution of the equation:

$$\theta\left[\frac{b}{1-b}\times\left(\frac{2}{\theta[\Theta_A^{-1}(\alpha^*)+\Theta_B^{-1}(\alpha^*)]}-\frac{1}{\theta}\right)\right]^{1-\alpha^*}-1=0 \qquad (9)$$

Proposition 2 *As stated in appendix C, α^* is unique and we have the following result. Under free trade, the relative price of commodity U and the critical level of CRRA lie somewhere between their two autarky values: $p_B^a<p^*<p_A^a$ and $\alpha_B^a<\alpha^*<\alpha_A^a$. Hence, country A (B) produces more certain (uncertain) commodity in free trade than in autarky: $n_A^*<n_A^a<n_B^a<n_B^*$.*

The trade pattern is consistent with the law of comparative advantage : the more (less) risk averse country exports the unrisky commodity and imports the risky commodity. In this framework, specialization is always incomplete since $\alpha^*\in]\underline{\alpha},\overline{\alpha}[$[5].

5 Welfare analysis

Though our framework includes competitive markets and well-informed managers, countries are not always better off under free trade. This result arises when welfare is based on effective consumptions, that is on ex post utility.

5.1 Ex ante versus ex post agregate utility

Welfare analysis can be drawn using ex ante utility or ex post utility. Ex ante utility function refers to the utility level prior to the resolution of uncertainty. Thus, it includes the risk-premium. On the contrary, ex post utility reflects the welfare level after the resolution of uncertainty. Thus, it depends on effective consumptions levels and no more on attitude towards risk.

Ex ante utility analysis would require social choice theory discussions. In fact, ex ante utility comparison between autarky and free trade is specific to each manager-consumer since it depends on its degree of risk-aversion; then, the global welfare assessment needs the building of a collective ex ante utility function. Hence, a ponderation of individual welfares is needed to aggregate

[5] Assuming $\underline{\alpha}_A>\underline{\alpha}_B$ and $\overline{\alpha}_A>\overline{\alpha}_B$ then complete specialization could arise.

ex ante utility functions. This would lead to equity, justice and social choice theory discussions which are outside the purpose of this paper[6]. In our model ex post utility is preferred to assess the impact of free trade on welfare. The citizens' perception of gains from trade seems to depend on the standard of living (i.e. on ex post utility) rather than on ex ante assessment. From the ex post consumer perspective, the ex ante decision rule based on risk-aversion has been forgotten for long time. Then politics should focuse on ex post agregate welfare to implement a trade policy.

5.2 A comparison of welfare under autarky and under free trade

For the less risk averse country, free trade has a negative price effect but a positive revenue effect on the (ex post) welfare. But the revenue effect always exceeds the price effect for country B.

Proposition 3 *When risk is idiosyncratic to each manager's project, the less risk-averse country is always better off with free trade.*

P roof. *Under free trade, assume country B consumes the same quantity of each commodity as in autarky. With* $p = p*$, *country B needs a revenue* $y\prime$ *equals to* $y\prime = p^*\theta a_u n_B^a + a_c(1 \square n_B^a)$. *Under free trade, the revenue of country B is in fact* $y^* = p^*\theta a_u n_B^* + a_c(1 \square n_B^*)$. *Since* $p^* > \frac{a_c}{\theta a_u}$ *and* $n_B^* > n_B^a$ *it is straightforward that* $y^* > y\prime$. *Hence, ex post utility of country is necessarily higher under free trade than in autarky.* ■

The same result doesn't hold for the more risk averse country. In fact, it is *a priori* analytically difficult to assess the impact of free trade on country's A welfare and the global world's welfare as well. This difficulty remains as long as the distribution functions are not specified.

Three particular cases are developped in order to evaluate the impact of free trade on welfare. Each simulation refers to particular distribution functions of CRRA (see figure 4). Then, we compute the equilibrium values of ex post utilities of country A (V_A), of country B (V_B) and the world (V_W) for different couples of b and θ with $a_c = a_u = 1$. The results in terms of welfare are summed up in table 1.

[6] It would be possible to base the comparison of aggregate utility welfare on the welfare of the average manager. This criterion implicitly assumes that managers are not aware of their type when the decision of openess is taken. We are then in a context of choice under a "veil of ignorance" following Rawls's theory of justice [7].

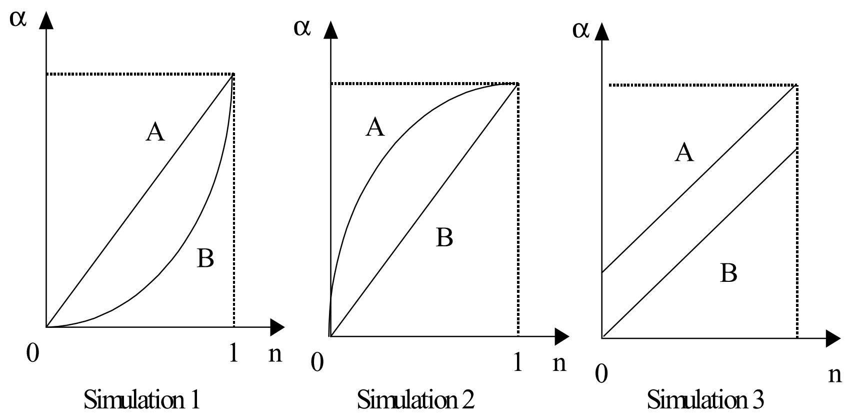

Figure 4.

5.2.1 Simulation 1

The manager's CRRA distributions in country A and in country B are respectively:

$$\alpha_A = \Theta_A(n) = n \quad \text{and} \quad \alpha_B = \Theta_B(n) = n^2.$$

The more risk averse country (country A) looses in trade while the less risk averse country (country B) always gain from trade, whatever the value for parameters b and θ. The world's welfare increases with the opening of trade. For example, with $b = 0.63$ and $\theta = 0.5$, ex post utilities are the following: $\Delta V_A = □0.01, \Delta V_B = 0.00124$ and $\Delta V_W = 0.0024$.

5.2.2 Simulation 2

The manager's CRRA distributions in country A and in country B are now respectively:

$$\alpha_A = \Theta_A(n) = n^{0.5} \quad \text{and} \quad \alpha_B = \Theta_B(n) = n.$$

Contrary to simulation 1 the world's welfare is now decreasing with the opening of trade whatever the values of b and θ; the gain from trade of country B never outweighs the loss experienced by country A. For example, with $b = 0.63$ and $\theta = 0.5$, ex post utilities are the following: $\Delta V_A = □0.06, \Delta V_B = 0.0202$ and $\Delta V_W = □0.00398$.

5.2.3 Simulation 3

The manager's CRRA distributions in country A and in country B are now respectively:

$$\alpha_A = \Theta_A(n) = 0.9n + 0.1 \quad \text{and} \quad \alpha_B = \Theta_B(n) = 0.9n.$$

In this simulation, we observe that the more risk averse country sometimes gains from trade. For example, with $b = 0.2$ and $\theta = 0.5$, ex post utilities are the following: $\Delta V_A = 0.0012, \Delta V_B = 0.005$ and $\Delta V_W = 0.0062$. When $b = 0.66$, we have $\Delta V_A = \square 0.0137, \Delta V_B = 0.0076$ and $\Delta V_W = \square 0.0061$.

5.2.4 Comments of simulations

Our results can be summed up in the following table.

	Country A	Country B	World
Simulation 1	worse off	better off	better off
Simulation 2	worse off	better off	worse off
Simulation 3	better off/worse off	better off	better off/worse off

Table 1: The impact of free trade on welfare

These simulations suggest that the more risk averse country can be worse off with the opening of trade. In country A some managers switch the production choice to the certain commodity and then give up the risk premium they earned in autarky to some country'B less risk averse managers. Country A pays to country B a risk premium which can exceed the price effect. From the world's point of view, the global effect of free trade can be positive or negative.

Proposition 4 *When risk is idiosyncratic to each manager's project, international trade doesn't always provide mutual gains from trade. The country the more risk-averse can be worse off under free trade while the country the less risk-averse is always better off. Moreover, from the world's point of view, we find that the global effect of free trade can be positive or negative.*

6 Concluding comments

We have shown that international differences in manager's CRRA distribution can explain the trade patterns in an idiosyncratic risky production context. Our analysis asserts that the concentration of risky and innovative technologies arises in the globally less risk averse country which is always better of under free trade. In this paper, international trade doesn't ensure a mutual gain from trade: with an idiosyncratic risk, there is no presumption that free trade will be Pareto optimal even if markets are competitive and producers well-informed. There exists a set of parameters values for which the more risk averse country's welfare decreases under free trade. Moreover, the world as a whole can be either better or worse off with the opening of trade. Trade barriers can be justified in the more risk averse country especially when lump-sums transfers are impossible because the world's welfare declines.

If the United States' comparative advantage in innovative products lies on financial systems or population's risk attitude rather than on factor's endowments, European countries could be worse off under free trade. More risk averse countries can consequently become "trade averse". Then it could be a good idea for European Union to subsidize risky activities and not let the US do innovative and risky business.

A Unicity of autarky equilibrium

We are investigating the equilibrium allocation of managers in industry U which is equivalent to compute the critical level of CRRA. Define

$$g_J(\alpha) = \theta \left[\frac{b}{(1 \Box b)\theta} \frac{1 \Box \Theta_J^{\Box 1}(\alpha)}{\Theta_J^{\Box 1}(\alpha)} \right]^{1 \Box \alpha} \Box 1.$$

We are looking for the number of roots of the function $g_J(\alpha)$ on the interval $]\underline{\alpha}, \overline{\alpha}[$ with $\underline{\alpha} > 0$, $\overline{\alpha} < 1$, $0 < \theta < 1$and $0 < b < 1$.

Recall that $\Theta_J^{\Box 1}(\alpha)$ is continuous and strictly increasing on $]\underline{\alpha}, \overline{\alpha}[$ with $\underset{\alpha \to \underline{\alpha}}{lim} \Theta_J^{\Box 1}(\alpha) = 0$ and $\underset{\alpha \to \overline{\alpha}}{lim} \Theta_J^{\Box 1}(\alpha) = 1$. Let

$$k = \frac{b}{(1 \Box b)\theta} > 0, \qquad = \ln \frac{1}{\theta} > 0, \quad r_J(\alpha) = \frac{1}{\Theta_J^{\Box 1}(\alpha)} \Box 1 \quad \text{and} \quad v(\alpha) = 1 \Box \alpha.$$

Then,

$$g_J(\alpha) = 0 \Leftrightarrow \left[k \cdot (\frac{1}{\Theta_J^{\Box 1}(\alpha)} \Box 1) \right]^{v(\alpha)} = \frac{1}{\theta}$$

$$g_J(\alpha) = 0 \Leftrightarrow \ln \left([k \cdot r_J(\alpha)]^{v(n^a)} \right) =$$

$$g_J(\alpha) = 0 \Leftrightarrow v(\alpha) \times (\ln k + \ln [r_J(\alpha)]) =$$

Let $h_J^a(\alpha) = \ln k + \ln [r_J(\alpha)]$.

$$g_J(\alpha) = 0 \Leftrightarrow v(\alpha) \times h_J^a(\alpha) =$$

$h_J(\alpha)$ is continuous and strictly decreasing on $]\underline{\alpha}, \overline{\alpha}[$. Moreover $\underset{\alpha \to \underline{\alpha}}{lim} r_J(\alpha) = +\infty$ and $\underset{\alpha \to \overline{\alpha}}{lim} r_J(\alpha) = 0$, so we have $\underset{\alpha \to \underline{\alpha}}{lim} h_J^a(n^a) = +\infty$ and $\underset{\alpha \to \overline{\alpha}}{lim} h_J^a(n^a) = \Box\infty$. Consequently, $\exists! \widehat{\alpha}_J \in]\underline{\alpha}, \overline{\alpha}[\mid h_J^a(\widehat{\alpha}_J) = 0$.

$\Box$ First let us reason on $]\underline{\alpha}, \widehat{\alpha}_J]$

On this interval $h_J^a(\alpha) \geq 0$. Notice that $v(\alpha)$ is continuous, strictly decreasing and positive on interval $]\underline{\alpha}, \overline{\alpha}[$. Hence $v(\alpha) \times h_J^a(\alpha)$ is strictly decreasing since the product of two positive decreasing functions is strictly decreasing. Then, since $v(\widehat{\alpha}_J) \times h_J^a(\widehat{\alpha}_J) = 0$ and $>$ 0,

$$\exists! \alpha_J^a \in]\underline{\alpha}, \widehat{\alpha}_J] \mid v(\alpha_J^a) \times h_J^a(\alpha_J^a) =$$

Hence, $g_J(\alpha)$ has only one root on interval $]\underline{\alpha}, \widehat{\alpha}_J]$.

$\square$ Second, let us reason on $]\widehat{\alpha}_J, \overline{\alpha}]$.

On this interval, $v(\alpha) > 0$ and $h_J^a(\alpha) < 0$. In this case, $v(\alpha) \times h_J^a(\alpha) < 0$. Then, the function $g_J(\alpha)$ has no root on $]\widehat{\alpha}_J, \overline{\alpha}]$.

$$\nexists \alpha \in]\widehat{\alpha}_J, \overline{\alpha}[| \; v(\alpha) \times h_J^a(\alpha) =$$

.

Hence the function $g_J(\alpha)$ has only one root on $]\underline{\alpha}, \overline{\alpha}[$: the autarky equilibrium is unique (QED):

$$\exists! \alpha_J^a \in]\underline{\alpha}, \overline{\alpha}[\quad \text{such as} \quad g(\alpha_J^a) = 0.$$

B Proof of proposition 1

Let us show first that the autarky critical level of CRRA is higher in country A than in country B: $\alpha_A^a > \alpha_B^a$.

Because country A is globally more risk averse than B, we have $\forall \alpha \in]\underline{\alpha}, \overline{\alpha}[, \Theta_A^{\square 1}(\alpha) < \Theta_B^{\square 1}(\alpha)$.

We also have $\Theta_A^{\square 1}(\alpha) < \Theta_B^{\square 1}(\alpha) \Leftrightarrow h_A^a(\alpha) < h_B^a(\alpha)$.

By definition $g_B(\alpha_B^a) = 0 \Leftrightarrow v(\alpha_B^a) \times h_B^a(\alpha_B^a) =$.

Then, $h_A^a(\alpha) < h_B^a(\alpha) \Rightarrow v(\alpha_B^a) \times h_A^a(\alpha_B^a) <$.

Because $v(\alpha_A^a) \times h_A^a(\alpha_A^a) =$, we have $\alpha_A^a > \alpha_B^a$.

Second, recall that the equilibrium under autarky implies

$$\left[\frac{w_c}{w_u}\right]_J^a = \theta^{1/(1 \square \alpha_J^a)} \quad \text{and} \quad p_J^a = \frac{a_c}{a_u} \times \left[\frac{w_u}{w_c}\right]_J^a.$$

Hence,

$$\alpha_A^a > \alpha_B^a \Leftrightarrow \left[\frac{w_c}{w_u}\right]_A^a < \left[\frac{w_c}{w_u}\right]_B^a \Leftrightarrow p_A^a > p_B^a.$$

Since p_J^a is a decreasing function of the proportion of managers devoted to the production of commodity U, we have

$$p_A^a > p_B^a \Leftrightarrow n_A^a < n_B^a \quad \text{(QED)}.$$

C Proof of Proposition 2

Let us demonstrate now that the free trade equilibrium is unique. Define

$$g^*(\alpha) = \theta \left[\frac{b}{(1 \square b)\theta} \times \frac{\left[2 \square (\Theta_A^{\square 1}(\alpha) + \Theta_B^{\square 1}(\alpha))\right]}{[\Theta_A^{\square 1}(\alpha) + \Theta_B^{\square 1}(\alpha)]} \right]^{1 \square \alpha} \square 1.$$

Like we did for the autarky equilibrium, we are looking for the number of roots of the function $g^*(\alpha)$ on the interval $]\underline{\alpha}, \overline{\alpha}[$. We have

$$g^*(\alpha) = 0 \Leftrightarrow v(\alpha) \times (\ln k + \ln [r^*(\alpha)]) =$$

with

$$r^*(\alpha) = \frac{2}{\Theta_A^{\square 1}(\alpha) + \Theta_B^{\square 1}(\alpha)} \square 1.$$

Note that $r^*(\alpha)$ has the same properties as in autarky since $\underset{\alpha \to \underline{\alpha}}{lim} r^*(\alpha) = +\infty$ and $\underset{\alpha \to \overline{\alpha}}{lim} r^*(\alpha) = 0$.

Hence the same method allows us to assert that $g(\alpha_i^*)$ has only one root on $]\underline{\alpha}, \overline{\alpha}[$.

$$\exists ! \alpha^* \in]\underline{\alpha}, \overline{\alpha}[\quad \text{such as} \quad g(\alpha^*) = 0.$$

Hence the free trade critical level of risk-aversion is unique under free trade and $n_A^* = \Theta_A^{\square 1}(\alpha^*)$ and $n_B^* = \Theta_B^{\square 1}(\alpha^*)$.(QED)

Moreover, let us show that, under free trade, the relative price of commodity U lies somewhere between the two internal relative prices: $p_B^a < p^* < p_A^a$.

As stated before, the equilibrium relative price of commodity U increases with the critical level of CRRA:

$$p_J^a = \frac{a_c}{a_u} \times \theta^{\square 1/(1 \square \alpha_J^a)} \quad and \quad p^* = \frac{a_c}{a_u} \times \theta^{\square 1/(1 \square \alpha^a)}$$

Hence,

$$p_B^a < p^* < p_A^a \Leftrightarrow \alpha_B^a < \alpha^* < \alpha_A^a$$

$$\forall \alpha \in]\underline{\alpha}, \overline{\alpha}[, \Theta_A^{\square 1}(\alpha) < \Theta_B^{\square 1}(\alpha)$$

$$\Leftrightarrow \quad \frac{1}{\Theta_B^{\square 1}(\alpha)} < \frac{2}{\Theta_A^{\square 1}(\alpha) + \Theta_B^{\square 1}(\alpha)} < \frac{1}{\Theta_A^{\square 1}(\alpha)}$$

$$\Leftrightarrow \quad v(\alpha) \times h_A^a(\alpha) < v(\alpha) \times h^*(\alpha) < v(\alpha) \times h_B^a(\alpha)$$

In particular, for $\alpha = \alpha^*$, we have

$$v(\alpha^*) \times h_A^a(\alpha^*) < \; < v \;\; (\alpha^*) \times h_B^a(\alpha^*)$$

Hence, $v(\alpha_A^a) \times h_A^a(\alpha_A^a) > v(\alpha^*) \times h_A^a(\alpha^*)$ and $v(\alpha^*) \times h_B^a(\alpha^*) > v(\alpha_B^a) \times h_B^a(\alpha_B^a)$.

Regarding the properties of these functions stated in appendix 2, we can conclude that

$$\alpha_B^a < \alpha^* < \alpha_A^a \qquad \text{QED}$$

References

[1] Allen F, Gale D [1999], "Diversity of Opinion and Financing of New Technologies", *Journal of Financial Intermediation,* 8, 68-89.

[2] Drèze J [1982], "Decision Criteria for business Firms", in *Essays on Economic Decision under Uncertainty,* Cambridge University Press.

[3] Eurobarometer Flash [2001], "Entrepreneurship", Report realised by EOS Gallup Europe upon request of the European Commission, N°107, september.

[4] Global Entrepreneurship Monitor [2002], *GEM Executive Report 2002*, downloadable at www.gemconsortium.org

[5] Manove M, Padilla A [1999], "Banking (Conservatively) with Optimists", *Rand Journal of Economics*, Summer.

[6] Newbery D. M.G, Stiglitz J. E [1984], "Pareto Inferior Trade", *Review of Economic Studies,* 51, 1-12.

[7] Rawls J. [1971], *A Theory of Justice*, Belknap Press of Harvard University Press.

[8] Schumpeter, J.A [1934], *The Theory of Economic Development,* Cambridge, Mass.: Harvard University Press.

[9] Shy Oz [1988], "A General Equilibrium Model of Pareto Inferior Trade", *Journal of International Economics,* 25, 143-154.

[10] Thakor A [1996], "The Design of Financial Systems: An Overview", *Journal of Banking and Finance,* 20, 917-948.