Optimal monetary policy in the area of the Euro: arbitrage in inflation - output- interest rate adjustment

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Abstract

The objective of our work has been here to calculate an optimal reaction function for the European Central Bank and this from the inflation and the output-gap formation model in the euro area.

Our method has consisted, in the first part, by presenting the model inspired by Peersman and Smets (1998) of prices and of output-gap formation in the euro area during the period 1990:1-2000:4. Thus, the output-gap is produced by an IS curve and depends on the past interest rate, the past output-gap and a demand shock. Inflation, itself, is formed according to a Phillip's curve. It is a function of past inflation, of past output-gap and a supply shock. So, as Ball (1997) specifies, all movement of interest rate by the central bank will have an effect on the economic variables of inflation and production, which in their turn will engender a reaction by the central bank. In the specific case of the European Central Bank, such as we are studying here, a movement of interest rate will first modify the evolution of the output-gap which depends on it directly, and modify only after the inflation, that is itself, a function of the output-gap and not directly of interest rates. In order to define its monetary policy, the European Central Bank must then take account of the before mentioned's channels of transmission. By defining a loss function for the European Central Bank, we define then what will have to be its optimal monetary policy taking into consideration the preceding interactions. Generally, the loss function of central banks is a function of three known arguments, the difference of inflation regarding its target, the output-gap and the interest rate difference regarding its level during the preceding period.

The second part of our work has first necessitate an estimation of the model previously defined. The obtained results correspond to the results generally presented in the economic literature, even if surprisingly, the IS curve estimated for the euro area seems closer than those estimated by other authors for the United States than those estimated for Germany. One also remarks that inflation, in the euro area, is particularly sensitive to the level of inflation attained in the trimester preceding the current period. By thus using an estimation method of constrained minimization, we are able, thanks to this model and the definition of the loss function associated with the monetary policy led by the European Central Bank, to define the rule of optimal monetary policy. We have calculated two rules of optimal monetary policy for the European Central Bank. These two rules correspond to two canonical cases. The first refers to the works of Rudebusch and Svensson (1998) and consider that the European Central Bank grant equal value in its strategy of stabilization of inflation and to that of output-gap. The second, on the contrary, supposes that the European Central Bank is not concern with inflation stabilization and ignores the evolution of output-gap. In any case, these two rules imply that the European Central Bank consider the effects of its policy on the evolution of interest rates. The optimal evolution of obtained rates through these two types of rules, during the 1990:1-2000:4, have thus been compared with the evolution of effective rates and with those of rates obtained throught the rule of Taylor, estimated for the same period in the euro area. From this confrontation, it thus appears that monetary policy in the euro area reveals itself to be close to these two types of rules of optimal monetary policy after 1996:2. After this date, the rule of optimal monetary policy taking into consideration output-gap seems to better describe the monetary policy directed in the euro area that the one we had qualified as a pure inflation target. It is moreover the same if one studies the links existing between the optimal rules and

the rule of Taylor that describes the behavior of effective rates in the euro area for the same period. One also notices that even the rule of optimal monetary policy of target inflation takes into account the output-gap as argument. In fact, this result emerges because of the mechanisms of transmission of the monetary policy previously described and the nature of the advanced indicator of inflation potentially attributed to the output-gap. Moreover, it is noteworthy that the two rules of optimal monetary policy incorporate past interest rates.

Finally, the third part of this work consists of calculating all over again what would be the rules of optimal monetary policy, for the euro are, by calibrating the Ball (1997) model, for the same period. This alternative scenario thus permits giving priority to the role of interest rate adjustment in monetary policy that is not able to be summarized, due to the volatility that would result on the rates, from a simple arbitrage between inflation variability and the output-gap volatility.

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Introduction

Since the beginning of the 90's, economic research regarding the rules of monetary policy have become more developped, especially by the work of Taylor (1993). By estimating a rule for monetary policy in the United States, between 1987 and 1992, he showed that monetary authorities were following a simple rule based on the conjugation of targets of inflation and output.

Yet since January 1st, 1999, the Eurpean Central Bank has become responsable for the monetary policies of the 11, and subsequently 12, countries who adopted the Euro. The national central banks are in charge of the application of this monetary policy that is in itself, decided at the heart of the European Central Bank, who fixes the intervention rates for the entire area. It is therefore possible to understand the monetary policy led by the European Central Bank with the help of the Taylor methodology.

However, only one in-depth study of the formation of the output-gap of gross domestic product from potential gross domestic product, on the one hand, and of the formation of inflation on the other hand, in the framework of the estimation of a theoretical model, allows for deciding on the supposed optimality of the reaction function of the central bank. In fact, with the help of such a model and a constrained optimization process, it is possible to determine endogenously, what are the coefficients that affect in an optimal manner, the difference of inflation regarding its target and the output-gap between the gross domestic product and the gross domestic potential product or the interest rate.

The objective of our work here has therefore been to calculate an optimal reaction function for the European Central Bank and this from the inflation and the output-gap calculation model in the euro area.

Our process consisted first of all, by presenting the model, inspired by Peersman and Smets (1998), of formation of prices and output-gap, applicable to the Euro's area, as the method from which one can deduce the rule of optimal monetary policy for the European Central Bank, taking into consideration the criteria of retained optimality, described in the loss function of the central bank.

In the second part, after having estimated it, with the method of Jondeau and Le Bihan (2000), we were able to calculate two rules of monetary policy for the European Central Bank. These two rules correspond to two canonical cases. The first refers to the works of Rudebusch and Svensson (1998) and suggests that European Central Bank grant equal importance in its strategy to inflation stabilization and output-gap stabilisation. The second, on the contrary, supposes that the European Central Bank does only concern itself with inflation stabilization and ignores the evolution of output-gap. In any case, these two rules suggest that the European Central Bank is conscious of the effects of its policy on the evolution of interest rates. The optimal evolutions of obtained rules through these two types of rules, in the period 1990:1-2000:4 are thus compared to the evolution of the effective rates and to those obtained by the rule of Taylor, for the same period, in the euro area.

Finally, the third part consists of calculating all over again what would be the rule of optimal monetary policy for the area of the Euro, by calibrating Ball's (1997) model for the same period. This alternative scenario permits, in fact,to present the role of interest rates smoothing in monetary policy which can not be summarized in a simple arbitration between variability of inflation and variability of output-gap.

I - Model of prices and output-gap formation, Taylor's optimal rule and central bank loss function in the euro area

I.1 - Model of prices and output-gap formation in the euro area

The « European » economy, composed of the different countries who have adopted the Euro, will be described here by a model inspired[1] by the one presented by Peersman and Smets (1998).

$$\begin{cases} y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \lambda\left(i_{t-1} - \bar{\pi}_{t-1}\right) + \varepsilon_t^y & (1) \\ \pi_t = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \alpha_3 \pi_{t-3} + \alpha_4 \pi_{t-4} + \beta y_{t-1} + \varepsilon_t^\pi & (2) \end{cases}$$

with : $0 < \sum \varphi_i < 1,\ \ \beta > 0,\ \ \lambda < 0,\ \ 0 < \sum \alpha_i < 1.$

π_t = inflation for the 11, and subsequently 12 countries forming the area of the Euro, in trimestrial frequence, expressed by annualized percentage.

$\bar{\pi}_t$ = average inflation of four semesters for the countries in the Euro's area.

i_t =trimestrial average of the European day to day interest rates, expressed in annualized percentage.

y_t=output-gap (quadratic)=difference of gross domestic product from potential gross domestic product, estimated as the byproduct of the regression of the gross domestic product in its quadratic trend, for the Euro's area, in trimestrial frequence, expressed in annual percentage.

ε_t^y is a zero-mean demand shock.

The innovation ε_t^π is a zero-mean i.i.d cost-push supply shock.

Equation (1) is easily understood. The output-gap is a function of interest rate, controlled by the monetary authorities, in the past period. This way, the more the interest rates rise in the earlier period, the more that reduced the output-gap in the present period. Moreover, the greater the output-gap was in t-1, and t-2, the more the output-gap will be amplified by the date t. Equation (1) thus presents the agregate demand as an IS curve, the output-gap depends on the past interest rate, the past output-gap and a demand shock.

Equation (2) itself describes the formation of inflation in the present period as resulting from inflation already incurred in preceding periods and also as the output-gap established in the last period. This equation considers thus a certain rigidity in the formation of prices at the

[1] Contrary to Ball (1997), we do not suppose $\sum \alpha_i = 1$.We authorize this coefficient to vary and it is the final estimation that will furnish us with the value of $\sum \alpha_i$. However, we expect that $\sum \alpha_i \leq 1$. In fact, according to Peersman and Smets (1998), this coefficient is 0.74 for the United States and 0.92 for the Europe-5 (Germany, France, Austria, Belgium, Netherlands) between 1975:1 and 1997:4.

heart of the studied economy. It is a Phillips curve.[2] The current inflation is a function of past inflation, past output-gap and a supply shock.

This type of model, as well as models with similar structures, are presented to desribe the process of prices and the output-gap formation, in the case of the United States, Germany, Europe-5, in Rudebusch and Svensson (1998), Jondeau and Le Bihan (2000) or even Ball (1997).

I.2 - Taylor's optimal rule

A rule of monetary policy, as Ball (1997) specifies, serves to establish the interest rates, controlled by the monetary authorities, functionning with simple, observed economic variables, such as inflation or output-gap. Yet in reverse, the interest rate fixed by the monetary authority will have an effect on the economic variables in question such as the output-gap or the inflation.

In the framework of the model presented here, the monetary authorities fix the intervention rate according to the rule described by equation (3).[3]

$$i_t = g_1\pi_{t-1} + g_2\pi_{t-2} + g_3\pi_{t-3} + g_4\pi_{t-4} + g_5 y_{t-1} + g_6 y_{t-2} + g_7 i_{t-1} \qquad (3)$$

The choice of the interest rate level by monetary authorities with this rule allows the central bank to first have an influence on economic variables of output-gap and then through this biais, subsequently, on inflation. In fact, as equations (1) and (2) point out, the interest rate influences the level of the output-gap that in itself influences inflation. At the end of this process, the evolutions of output-gap and of the inflation incurred by the movement of interest rates will generate in turn, a new modification of interest rates. The choice of interest rate, and thus of coefficient g_i of the rule of monetary policy, in order that it be optimal, thus takes account of these interactions- interest rate/output-gap, inflation/interest rate.

I. 3- Criteria for determining the optimal rule

The traditional criteria of determination of the rule of optimal monetary policy is to define a loss function associated with this rule.

This loss function, in the economic literature consacrated to rules of optimal monetary policy, take most often the following form.

$$L_t = \gamma V(\pi_t) + (1-\gamma)V(y_t) + \nu V(i_t - i_{t-1}) \qquad (4)$$

where:

γ et ν=weight accorded in the function of loss to the variability of inflation relatively to that of the output-gap and weight accorded to that of interest rate from one period to another, γ and ν being fixed arbitrarily by the monetary authorities.

V=operator of variance=indicator of inflation variability, output-gap and interest rates variability.

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[2] As with Rudebusch and Svensson (1998), we don't reject a priori the hypothesis $\sum \alpha_i = 1$. In this case, equation (2) is a Phillips acceleration curve.

[3] Still in analogy with Peersman and Smets (1998).

$\bar{p}_t$ = average inflation for a year.
y_t=output-gap in the period t.
i_t - i_{t-1}=interest rate gap from the period t-1 to period t.

This signifies that the monetary authorities are mindful, since they fix their intervention rates to limit the variations that would prove to be excessive in inflation, and production as well as interest rate.[4] The value assigned to the variabilty of inflation relatively to the variability of output-gap and to the variability of interest rate diffence are traditionally fixed as γ=0.5 and ν=0.25 since the works of Rudebusch and Svensson (1998). In any case, nothing forbids thinking that, in the European framework, this value can reveal itself to be different from those chosen by the authors of the study of the American case. There emerges an infinity of rules of optimal monetary policy. Concerning comparability though, these are the traditionally defined values that are retained here. Anyhow, a second case will be equally studied to know the one corresponding to the situation where the central bank follows one strict inflation target, not granting any value in its strategy of stabilization of the output-gap. This case, which seems pertinent, takes into account the strategy declared by the European Central Bank, referring to the case where the values of γ and ν are 1 and 0.25, respectively.

We'll admit here that the central bank grants value in its loss function in the variability of interest rate difference. This means that the central bank is concerned with smoothing the fluctuations of this interest rate. In fact, according to Sack and Wieland (1999) or Jondeau and Le Bihan (2000), the central banks choose to adjust the evolution of interest rates notably within reason of possible errors in the measuring of economic variables or yet within reason of the necessity of limiting the overeactions associated with agents' anticipations. Indeed, Fuhrer and Moore (1995) note that a monetary policy rule, coming from a central bank loss function that is a simple arbitrage in inflation and output-gap, will only little reduce the inflation variability at price of a very importante increase in output-gap and interest rate variability. Fuhrer (1997) even shows that such a monetary policy rule promotes a very big increase in the inflation and output-gap variances when the monetary policy strictly targets one objective, that is when inflation or output-gap is not autorise to deviate from the target more than 2%. Also, taking into account the structure of the model of the economy, a loss function only considering the variability of inflation (γ=1 and ν=0) would bring the monetary authorities to define a simplistic monetary policy. Any variation of inflation, notably a rise, would entail a massive movement of interest rates involving some infinites values for the g (i=1...4) referring to inflation.[5] This case being hardly realistic, it will be thus left aside.

I. 4 - The method

The central bank, in this framework, will thus follow an optimal rule of monetary policy if it chooses its intervention interest rate in a way minimizing the function of loss (4) under constraint of the dynamic of the economy described by equations (1) and (2). It concerns then

[4] It's also within reason, given the strong variability on inflation variables, output-gap and on interest rates, that the Ball model (1997) which neither includes the interest rate nor in the reaction function nor in the function of loss, seems less realistic that the models of Peersman and Smets (1998), Rudebusch and Svensson (1998) or Jondeau and Le Bihan (2000).

[5] When inflation is very high, the central bank , in this case (γ=1 and ν=0), favor a sharp increase of its intervention rate since that causes the output-gap to diminish (2), then in turn, inflation (1) will also diminish. No matters what are the effects of its policy on the variables of production and of the interest rate since they no longer enter into the function of loss.

determining the coefficients g_i (i=1,...,7) by proceeding through scanning, thanks to a serie of included loops that gives the rule of monetary policy that minimize the loss function.[6]

II. Estimation of the rule of optimal monetary policy using euro zone data

II. 1 - The Data

The data used here to estimate an optimal monetary policy rule in the euro zone come from the Eurostat database.

Inflation is measured by the consumer price index, for all the countries within the euro zone, base 100 in 1990. It was, in the manner of Taylor (1993), taken into account on an annual basis so as not to be submitted to erratic variations.

The gross European domestic product is produced in volume, after a calculation of agregation of national European data considering the respective importance of each country in the gross European domestic product and taking into account the evolution of the exchange rate of each national currency before January 1999.

The European interest rate corresponds to the European interest rate of the European Central Bank from day to day, that is the "call for money" rate for the period after January 1999, and to a fictional interest rate constructed according to the same logic of that used for the gross domestic product for the preceding period, from January 1990 to December 1998. The national interest rates are agregated with reservation for the importance of a country in the gross European domestic product.

The data are trimestrial data for the period of 1990:1-2000:4. It is concerned with the observed data for the period 1990:1-1998:4. The choice to use "mixed" series was made in reference to the Mundell triangle of incomptability which indicates that the monetary policies of European countries, involved in a system of fixed exchange rate, associated by free circulation of capital (1990), were unable to be independant and thus totally divergent. In such an instance, the group of European countries were following a monetary policy derived from that of Germany. Moreover, we have equally justified our choice through the efforts of convergence that were undertaken by European countries after 1992 and the Treaty of Maastricht, in order to observe the criteria allowing them to converge for the passage to the Euro.

II.2 - Estimation of the model

The method used for estimating the model of the European economy was inspired by that of Jondeau and Le Bihan (2000) and put into action the method of maximum likelihood. For that, we estimate first of all the coefficients of equations (1) and (2) throught the ordinary least squared. These estimations are biased and they are thus used as a base value for beginning the estimation of maximum likelihood.

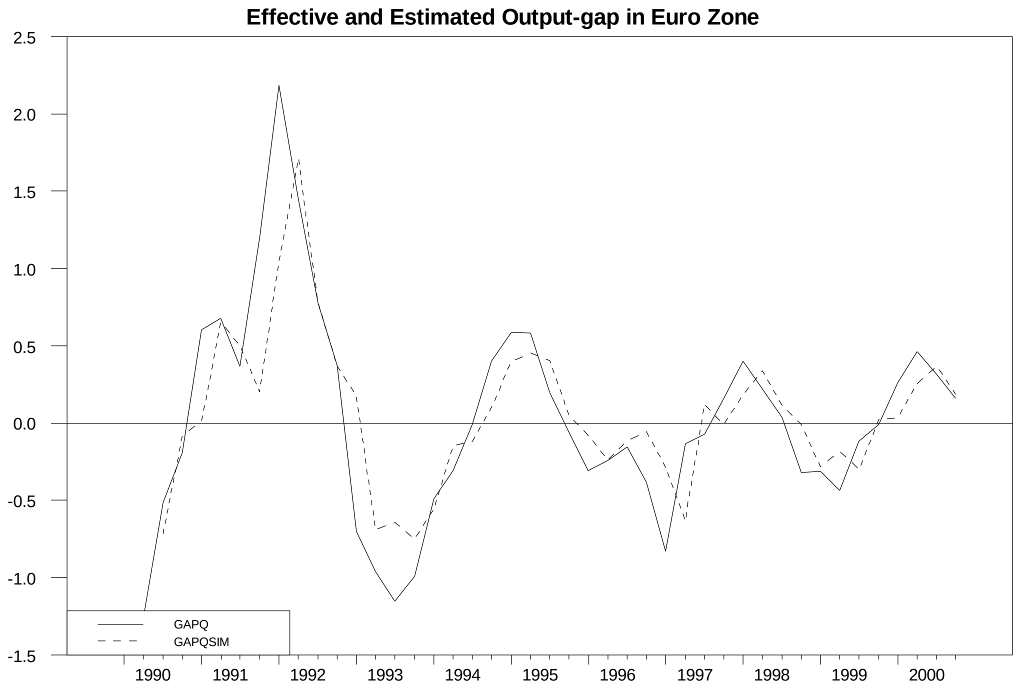
We obtain then the following results:

[6] It is also possible to resolve this problem in a matrix like fashion. See Peersman and Smets (1998) or Jondeau and Le Bihan (2000).

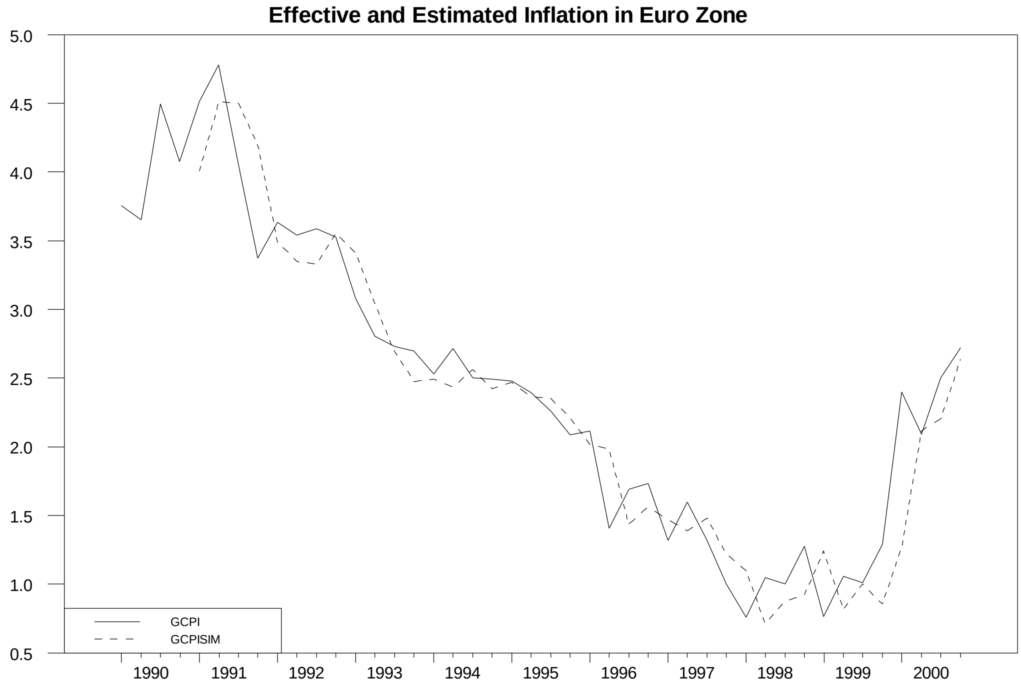
$$\begin{cases} y_t = 1{,}1423 y_{t-1} - 0{,}4588 y_{t-2} - 0{,}5218 i_{t-1} & (1) \\ \quad (20{,}7257) \quad (-4{,}8852) \quad (1{,}9755) & \\ p_t = 0{,}8154 p_{t-1} + 0{,}1303 p_{t-2} + 0{,}1154 p_{t-3} - 0{,}0989 p_{t-4} + 0{,}2151 y_{t-1} & (2) \\ \quad (97{,}7042) \quad (16{,}5138) \quad (13{,}8473) \quad (-11{,}6381) \quad (2{,}8422) & \end{cases}$$

The numbers in parentheses designate the statistics of Student associated with the coefficients to which they refer to.
With:
y_t=quadratic output-gap.
p_t=gap between inflation and its target (2% in the Euro's area).
i_t=difference between the real interest rate and the real natural rate (3.7% over the period studied).

The following graphs present the estimations that we have obtained and confront them with the evolutions of effective variables :



with :
GAPQ=effective quadratic output-gap in the euro zone.
GAPQCAL=estimated quadratic output-gap in the euro zone.



with :
GCPI=effective inflation in the euro zone.
GCPICAL=estimated inflation in the euro zone.

When one compares the results obtained here with those of Peersman and Smets (1998) for 5 European countries (Germany, France, Austria, Belgium, Netherlands) and for the United States, with those of Jondeau and Le Bihan (2000) for Germany and the United States, we have the following table:

Studies	Peersman and Smets (1975:1-1997:4)		Jondeau et Le Bihan (1968:1-1998:4)		0ur results (1990:1-2000:4)
Countries	United-States	Europe-5	United-States	Germany	Euro aera-12
y_t :					
φ_1	1.41	0.84	1.150	0.666	1.1423
φ_2	-0.52	0.10	-0.200	0.314	-0.4588
λ	-0.12	-0.10	-0.348	-0.508	-0.5218
π_t :					
α_1	0.48	0.45	0.597	0.287	0.8154
α_2	0.19	0.17	0.079	0.394	0.1303
α_3	0.13	0.06	0.208	0.319	0.1153
α_4	0.12	0.06	0.116	-	-0.0989
β	0.11	0.33	0.175	0.106	0.2151

The comparative study of these results is particularly interesting. If we stay with the study of the IS curve, we notice that the estimated coefficients for the euro zone, for that which is the effect of past production on the formation of the current output-gap (φ_1=1,1423 and φ_2=-0,4588) are closer to those estimated in the case of the United States (respectively φ_1=1,41 and φ_2=-0,52 or φ_1=1,150 and φ_2=-0,200) than of those who had already been estimated for Europe (φ_1=0,84 and φ_2=0,10 for Europe and φ_1=0,666 and φ_2=0,314 for Germany). Elsewhere, the coefficient λ of response of the current output-gap from the past interest rate, λ=-0.5218, is extremely close to the one estimated for Germany, λ=-0.508. This result yet again proceeds the estimation method that we are using. This one in fact agregates the national series of countries composing the euro aera in order to describe the behavior of the zone before its creation in order to have long series of statistics. Therefore, in the period 1990:1-1998:4, Germany was the leading country in matters of monetary policy in Europe in the framework of the system of fixe exchange rate and freedom of movement of capital that reigned there.

In that which concerns the Phillips curve and prices formation, the coefficient $\alpha=\sum a_i$, which corresponds in fact to past inflation in the formation of current inflation is 0.9621 for the euro zone. This result is very close to that found through the different authors named here. In any case, one will notice that the value of the coefficient associated with π_{t-1}, that is to say the value assigned to the inflation of the previous first trimester, is particularly important in the euro zone regarding the formation of current inflation. Elsewhere, the value assigned to the output-gap in the preceding period, in the formation of current inflation in the euro zone, β=0.2151, if it is greater to that which is forseen for the United States (0.11 and 0.175),it corresponds to that which was already observed for Germany or the Europe-5 (0.106 and 0.33, respectively).

II. 3- Optimal monetary policy rule(s) in the euro zone

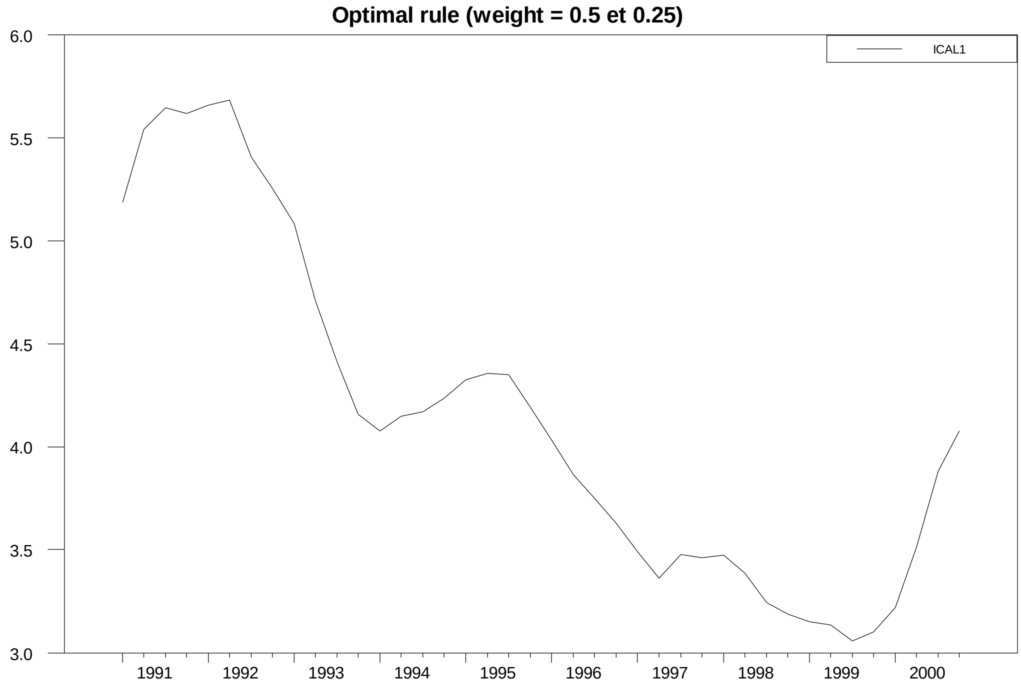
a - Optimal monetary policy rule when γ=0.5 and ν =0.25 (rule 1)

The rule of optimal monetary policy, obtained with the coefficients γ=0.5 and ν=0.25 in the loss function is a particular case if one considers the infinity of potentially existing rules of optimal monetary policy. However, these coefficients constitute a canonical case and are traditional in the literature since the works of Rudebusch and Svensson (1998).

With these coefficients, and the series previously mentioned, we obtain in the case of the euro area, the following optimal rule of monetary policy :

$$i_t = 0{,}12\pi_{t-1} + 0{,}24\pi_{t-2} + 0{,}21\pi_{t-3} + 0{,}03\pi_{t-4} + 0{,}27y_{t-1} + 0{,}03y_{t-2} + 0{,}03i_{t-1} \quad (5)$$

with Lmini=0,18891.



with ICAL1= optimal rates calculated with the optimal monetary policy rule n°1.

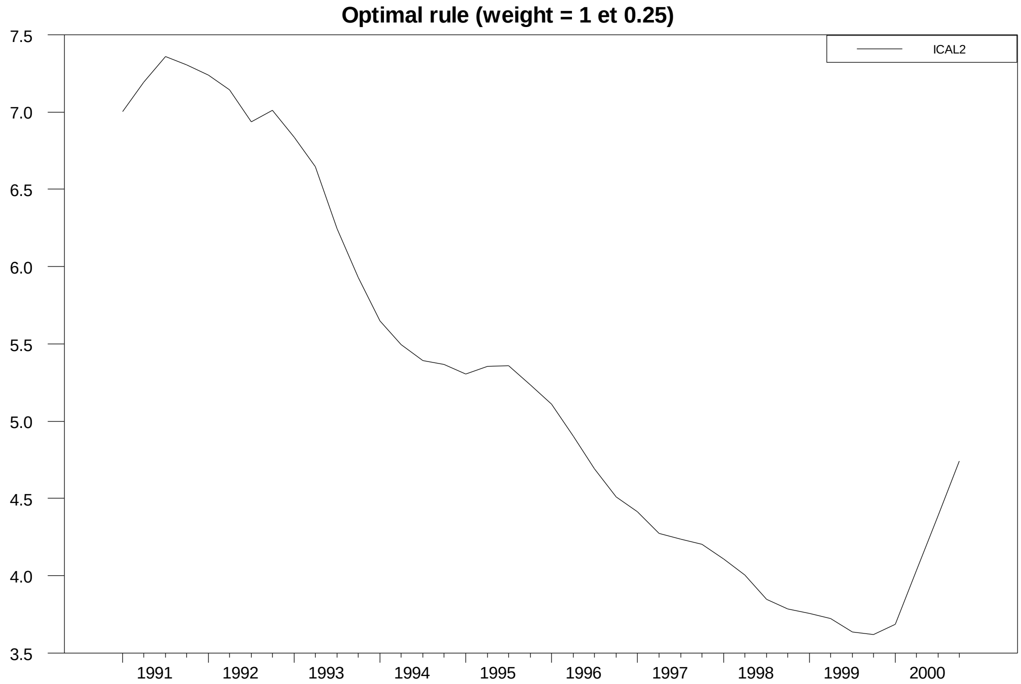
b - Optimal monetary policy rule when γ=1 and ν=0.25 (rule 2)

The rule of optimal monetary policy, obtained with the coefficients γ=1 and ν=0.25, in the function of loss is, there again, a particular case if one considers the infinity of potentially existing rules of monetary policy. However, the study of this case is interesting because it corresponds to a situation where monetary authorities are engaged in a strategy of pure inflation target in the sense where they don't grant any importance in their strategy to the variable of output-gap. In any case, they continue to take into account the effects of their actions on the variables of interest rate and smoothing their fluctuations.

With these coefficients, and the series that we use, we obtain in the case of the euro area, the following rule of optimal monetary policy :

$$i_t = 0{,}15\pi_{t-1} + 0{,}21\pi_{t-2} + 0{,}21\pi_{t-3} + 0{,}18\pi_{t-4} + 0{,}03y_{t-1} + 0{,}03y_{t-2} + 0{,}15i_{t-1} \quad (6)$$

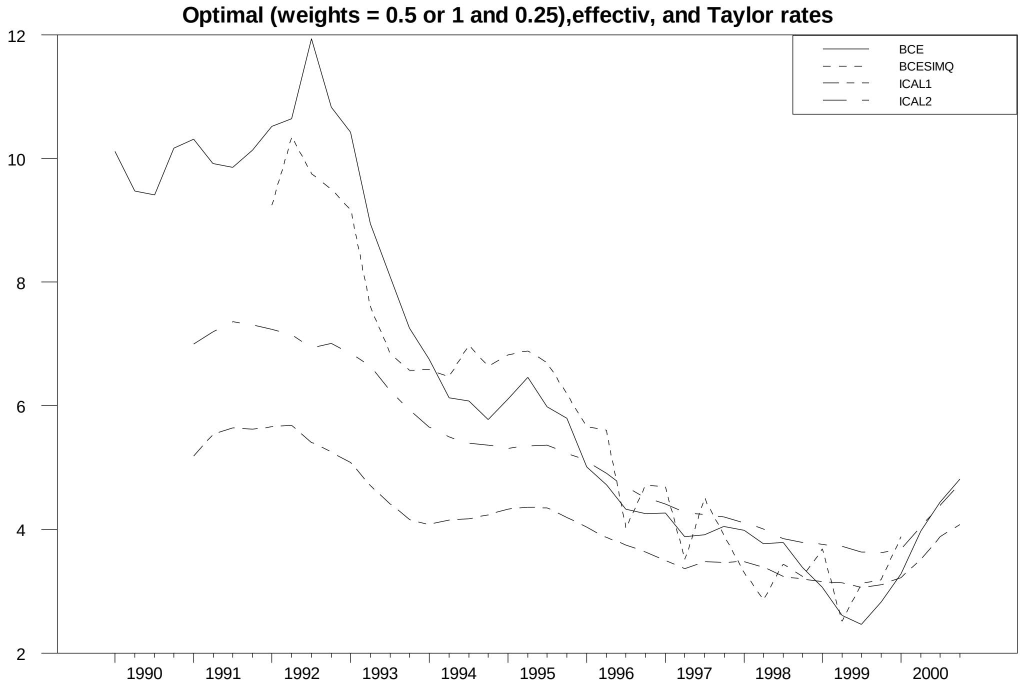
with Lmini=0,18004.



with ICAL2= optimal rates calculated with the optimal monetary policy rule n°2.

II.4 - Effective rates, Taylor's rule and optimal monetary policy in the euro area

The following graph and tables permit to compare the effective rates and the Taylor's rule in the euro zone with the two optimal rules of monetary policy defined here :



with :
BCE=effective rates in the euro zone
BCESIMQ=Taylor's rates calculated with a quadratic output-gap for the euro zone
ICAL1= optimal rates calculated with the optimal monetary policy rule n°1.

ICAL2= optimal rates calculated with the optimal monetary policy rule n°2.

Gap between the effective rates and the rates obtained with the optimal rules, in the euro zone

Gap between effective and optimal rates		Average gap	Standart-error	Minimum	Maximum
Rule 1	1990:1-2000:4	1,7632	1,4653	-0,6240	4,6645
	1996:2-2000:4	1,0647	1,3570	-0,6240	3,0327
	1999:1-2000:4	0,7357	1,0276	-0,6240	3,0327
Rule 2	1990:1-2000:4	0,7395	1,1786	-1,2085	3,2019
	1996:2-2000:4	0,5100	1,3534	-1,2085	3,2019
	1999:1-2000:4	0,0476	1,0294	-1,2085	2,5100

Gap between the rates obtained with the Taylor's rule and the rates obtained with the optimal rules, in the euro zone

Gap between Taylor's rates and optimal rates		Average gap	Standart-error	Minimum	Maximum
Rule 1	1990:1-2000:4	1,9754	1,9951	-0,5956	6,5349
	1996:2-2000:4	0,3318	0,4262	-0,5956	0,8579
	1999:1-2000:4	0,0402	0,5031	-0,5956	0,7329
Rule 2	1990:1-2000:4	0,8769	1,5790	-1,1720	5,0025
	1996:2-2000:4	-0,3563	0,3544	-1,1720	0,0702
	1999:1-2000:4	-0,5145	0,5040	-1,1720	0,0702

According to these graphs, we see that the Taylor's previously estimated rule by Sibi (2000) for the euro area, with a quadratic output-gap, summarized by equation (7):

$$r(t)=3{,}70\% + p(t) + \underset{(2{,}30641)}{0{,}5383}\,y(t) + \underset{(14{,}67167)}{1{,}3269}\,(p(t)-2\%) \qquad R^2=0{,}882264 \qquad (7)$$

describes relatively well the movement of effective interest rates in the euro aera.

If one is concerned now with the two rules of optimal monetary policy obtenaited thanks the constrained minimization defined by our model, we see the one, like the other favoring a much less severe monetary policy in matters of interest rate than that led during the course of the period 1990:1-1996:2. This isn't suprising given the economic context of the aera. In fact, after the German reunification and the monetary reunification of the country, the Bundesbank led a severe monetary policy in order to avoid an eventual inflation explosion. The European countries bound by fixed exchange rates and free movement of capital have followed more or less the same type of policy as Germany, or eventualy more severe policies, given their own economic situations. This can evident be seen in the evolution of effective European rates such as we have calculated them.

In any case, after 1994, that is after the European Monetary System crises and its fluctuation marges widening at $\pm 15\%$, the effective monetary policy approach the rules of optimal monetary policy such as we have calculated them. It seems even to be relatively close to the rules of optimal monetary policy after 1996:2. From 1996:2 to 1999:1, these two optimal rules describe relaively well the movement of effective interest rates in the euro aera, allowing one to suppose that the monetary policy led therefore would have been a medium way between a pure inflation target and a monetary policy granting the same importance at the core of its objectives to inflation as well as to output-gap.

However in 1999, which corresponds to the first year of entry into effective function for the European Central Bank, seemed to have been marked by a particularly accomodating monetary policy. In fact, one sees that optimal rule n°1, like optimal rule n°2, favors a more severe monetary policy with relatively high interest rates.

On the contrary, the year 2000 leaves us to envision a certain conformity to the level of rates practiced by the Central European Bank with those proposed by optimal rule n°1, as well as optimal rule n°2.

If we look at the gap between the effective rates and the optimal rates, it seems that the optimal policy rule n°2, during the whole period 1990:1-2000:4, is the one which describes the better the monetary policy that has been followed in the euro zone. But, during the period 1996:2 or 1999:1 to 2000:4, optimal policy rule n°1, that take output-gap into account, is the one that describes the effective monetary policy the better in the euro zone.

We note the same think if we compare the European Taylor's rule to the optimal monetary policy rules n°1 and n°2.

One will notice that the monetary policy conducted in the euro area, which used to seem to be unsteady in the first years of the study, appear today as having a much smoother evolution. This would tend to suggest that the European Central Bank has effectively incorporated into its strategy the negative effects of too harsh variations of interest rates and that it smooth the evolution of this one. In this regard, one will notice that the Taylor's rule, which describes relatively well the evolution of European rates in the past 10 years and that does not take into account the variable of interest rates in his arguments, experiences a much rockier evolution than that of the effective rates.

Moreover, it is important to note that in these two rules of monetary policy defined as optimal, the relative values of inflation are 0.6 and 0.75 respectively in rule 1 and 2. The pure inflation target has a more important weight for inflation variability stabilisation in its arguments than rule n°2. The weights which refer to the output-gap are 0.3 for rule n°1 (weights : γ =0.5 and ν=0.25) or 0.06 for the said rule of inflation target (weights : γ=1 and ν=0.25). So even the suppposed optimal rule that is suppose to take only into account the inflation, leds the European Central Bank to consider the evolutions of the output-gap and this because of the transmission channels of monetary policy. In fact, the latter, by the biais of interest rate, acts in the first place on the output-gap and only then on inflation, through the intermediary of this one, which appears in the formation of inflation. Moreover, the presence of the output-gap term in the rules of monetary policy of the European Central Bank such as it is described here, is probably du to the fact there is only, in the rule, past variables and that it is not based on anticipations. So the output-gap appears to be an advanced indicator of inflation. Nevertheless, the weight attached to output-gap stabilisation in rules n°2 appears to be very week.

Finally, the two rules of optimal monetary policy described here take into account the interest rates of the last period. The values affecting those here are 0.03 for optimal rule n°1 (γ =0.5 and ν=0.25) and of 0.15 for optimal rule n°2 (γ=1 and ν=0.25). It appears that in the two optimals monetary policy rules the central bank smooths the interest rate evolution but this need is more important in rule n°2 because of the lack of the output-gap stabilisation in the central bank loss function.

II.5 - Reactivity of optimal monetary policy

When one compares the results that we obtain for the two rules corresponding to the two cases we have chosen within the framework of the euro area with those obtained through Peersman and Smets (1998) or through Jondeau and Le Bihan (2000) in the United States or for the Europe-5, we obtain the following table:

Optimal Rules	Jondeau and Le Bihan 1968:1-1998:4 United-States	Jondeau and Le Bihan 1968:1-1998:4 Germany	Peersman and Smets 1975:1-1997:4	Rule n°1 Euro Zone 1990:1-2001:1	Rule n°2 Euro Zone 1990:1-2001:1
Relativ Weights	$\mu\pi=\mu y=0{,}35$ $\mu i=0{,}30$	$\mu\pi=\mu y=0{,}35$ $\mu i=0{,}30$	Règle de référence $\gamma=0{,}5$ et $\nu=0{,}25$	$\gamma=0{,}5$ et $\nu=0{,}25$	$\gamma=1$ et $\nu=0{,}25$
on inflation gap (yearly)	1,89	1,93	0,65 (0,34+0,17+0,09 +0,05)	0,6	0,75
on output-gap (yearly)	1,87	1,94	1,29 (1,17+0,12)	0,3	0,06
on interest rate	0,71	0,78	0,56	0,03	0,15

Thanks to this table, we see that the euro zone has a propogating structure of monetary policy that implements an European optimal monetary policy much less reactiv than for the United States in particular in terme of output-gap reponse and of interest rate smoothing.

III - Alternative scenarios- Ball's model (1997) applied to the euro area

III.1- The Model

An alternative scenario can be envisioned by using the framework proposed by Ball (1997). Ball describes the functioning of the economy by the following simple model :

$$\begin{cases} y_t = -\beta i_{t-1} + \lambda y_{t-1} + \varepsilon & (8) \\ \pi_t = \gamma \pi_{t-1} + \alpha y_{t-1} + \eta & (9) \end{cases}$$

with $\beta>0$, $0 \leq \lambda \leq 1$, $\alpha>0$, $\gamma>0$,
and where :
y_t=output-gap (quadratic)=difference of the gross domestic product from the potential gross domestic product for the euro area, in trimestrial frequence, expressed in annualized percentage, estimated as the residual gross domestic product regressing on its quadratic trend.

i_t=difference between the real interest rate, calculated from the trimestrial average from the European interest rate from day to day and from inflation, in a trimestrial frequence, expressed in annualized percentage minus his equilibrium rate.
π_t=gap between inflation for the 11, and subsequently 12 countries forming the area of the Euro, in trimestrial frequence, expressed by annualized percentage and its target.[7]
α, β, γ et λ=parameters of the model.
ε and η are white noises.

So, as before, equation (8) establishes that the output-gap is a function of interest rate, controlled by the monetary authorities, in the past period and on the level of output-gap in the preceding period. The more the interest rates raised in the preceding period, the more that reduces the output-gap in the current period. Moreover, the stronger output-gap was in t-1, the more the amplified the output-gap will be by the date of t. Equation (8) corresponds to an IS curve, the output-gap depends on past interest rate, on past output-gap and demand shock.

Equation (9) presents the formation of inflation in the present period as resulting from inflation already produced in the preceding period as well as the output-gap established in this same preceding period. This equation initially imposes then a certain rigidity on prices formation. It is concerned, once again, with the Phillip's curve. The current inflation is a function of past inflation, of past output-gap and supply shock.

III.2 - Derivation of Taylor's optimal rule

A rule of monetary policy, as Ball (1997) defines it, serves to establish interest rate, controlled by monetary authorities, in the function of simple observed economic variables such as inflation or output-gap. Conversely, the interest rate fixed by monetary authorities will have, as we previously reported, an effect on the economic variables in question, such as the output-gap or inflation.

We have, following Ball (1997), to minimise the central bank loss function under contrainst of inflation, output-gap, and interest rate formation equations. We have to find g_i $(i=\pi, y)$ coefficients of the optimal monetary policy rule that solves the following problem :

$$\begin{cases} Min\ L_t=\mu V(\pi_t)+(1-\mu)V(y_t) & (10) \\ s.c.\ \ y_t=-\beta i_{t-1}+\lambda y_{t-1}+\varepsilon & (8) \\ \pi_t=\gamma\pi_{t-1}+\alpha y_{t-1}+\eta & (9) \\ i_t=g_\pi \pi_{t-1}+g_y y_{t-1} & (11) \end{cases}$$

With :
V=variance operator=inflation and output-gap variability indicator.
y_t=output-gap (quadratic)=difference of gross domestic product from potential gross domestic product.
i_t=gap between the real interest rate and its equilibrium rate.
π_t =difference between inflation for the 11, and subsequently 12 countries forming the area of the Euro, in trimestrial frequence, expressed by annualized percentage and its target.
α, β, γ et λ=parameters of the model.

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[7] We return to the framework of Ball (1997), but we use the same data as before, in order to be able to perform comparisons. Elsewhere, in Ball's model he assumes γ= 1.

Equation (11) designs the optimal monetary policy rule and equation (10) is the central bank loss function. It only takes into account in its preferences of inflation and output-gap variability. We note $\mu \in [0,1]$ the weight, chosen by the monetary autorithy, accorded in its strategy to inflation stabilisation relatively to the output-gap one.

By using the same method than in precedent section, a simple programm constituted from a serie of included loops, gives us the $g_i(i=\pi, y)$ *coefficients*, and then the optimal interest rate, which minimise the central bank loss function.

III.3 - Calibrating Ball's model

If we calibrate Ball's (1997) model with the results obtained by estimating equations (1) and (2), equations (8) and (9) become:

$$\begin{cases} y_t = 0{,}6835 y_{t-1} - 0{,}5218 i_{t-1} \\ \pi_t = 0{,}9622 \pi_{t-1} + 0{,}2151 y_{t-1} \end{cases}$$

with :
α=0,2151
β=0,5218
γ=0,9622
λ=0,6835
$\mu \in$ [0,1]

These parameters will permit to obtain the coefficients of the optimal monetary policy rule associated with the economy behavior that is here discribe.

III.4 - Results

With an economy where the prices and the output-gap are calculated according to the system of equations (8) and (9), the rule of optimal monetary policy is thus identical to equation (11) and that is :

$$i_t^* = g_y y_{t-1} + g_\pi \pi_{t-1} \qquad (11)$$

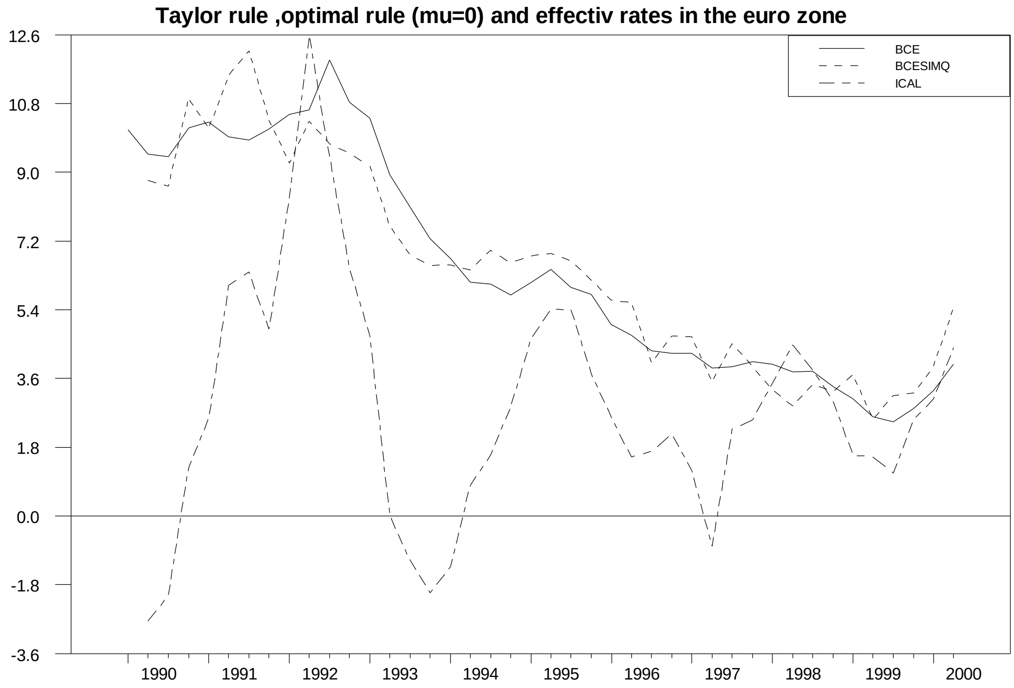
One obtains then the following results :

0ptimal monetary policy rule coefficients obtain with Ball methodology.

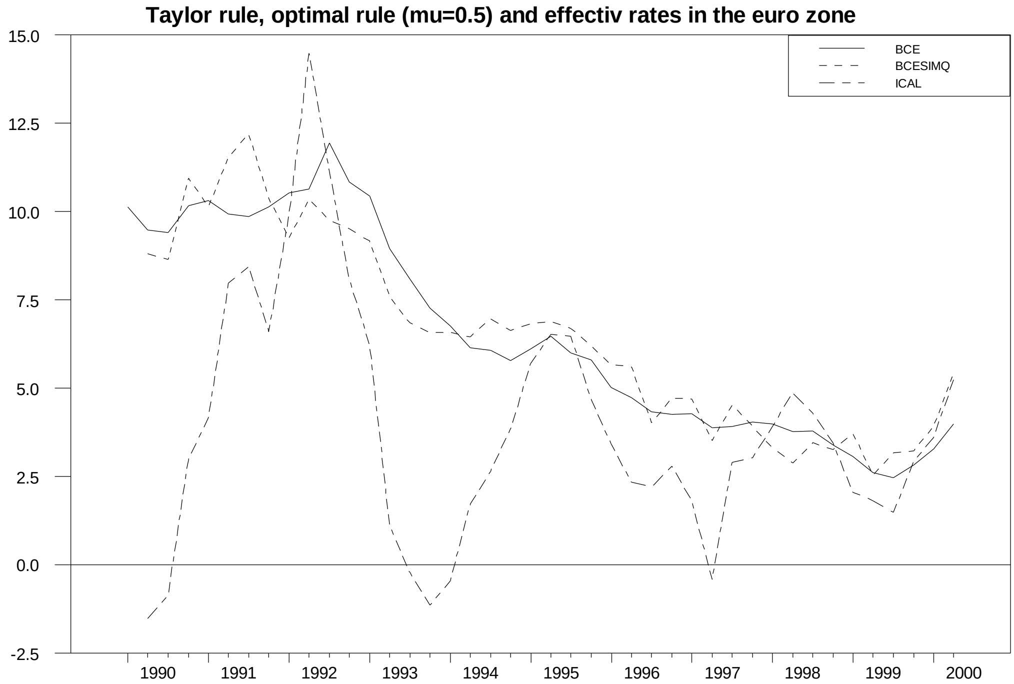
μ	g_π	g_y	Lmini	Vi=Interest rate variance
0	0,2	4,3	0,07191	9,38296[a]
0,1	0,2	4,3	0,09170	9,38296
0,2	0,3	4,3	0,11129	9,59879
0,3	0,4	4,4	0,13073	10,28409
0,4	0,5	4,4	0,14986	10,56525
0,5	0,6	4,5	0,16855	11,33210
0,6	0,8	4,6	0,18647	12,52700
0,7	1,2	4,8	0,20289	15,35745
0,8	1,8	5,2	0,21588	21,26643
0,9	3,4	6,0	0,21731	42,11850
1	19,5	7,4	0,07225	663,7249

a - The central bank doesn't care about inflation.
b - Inflation Target.

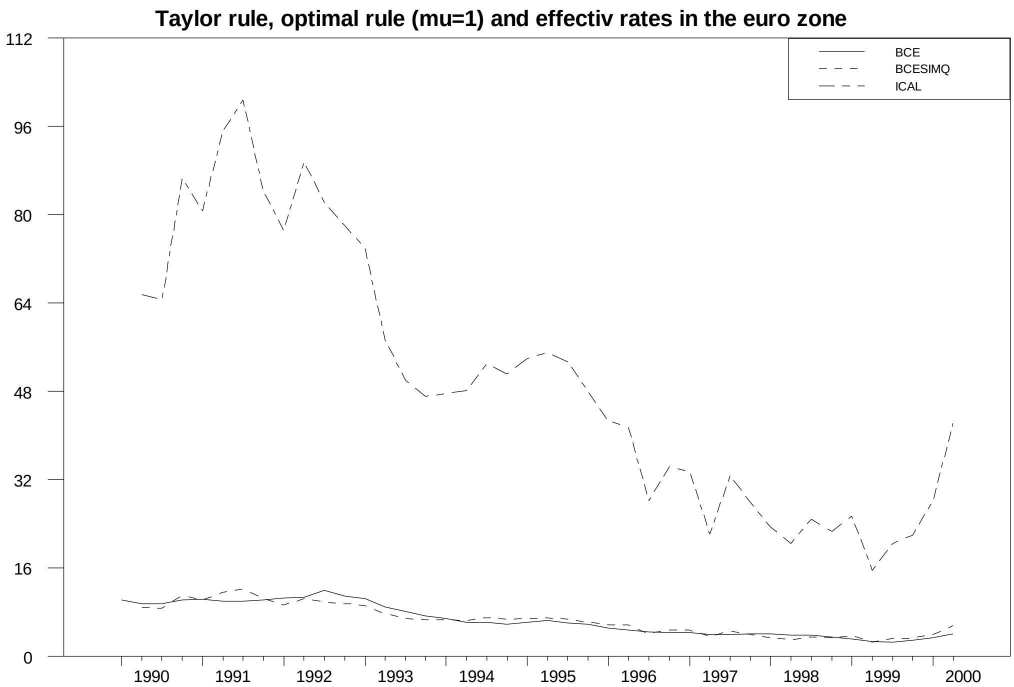
The three following graphs illustrate three situations presented by the preceding table. It is concerned with cases where the European Central Bank would target in an "optimal" fashion, the output-gap only *(m=0)*, or in equal parts inflation and output-gap *(m=0,5)*, or the inflation only *(m=1)*.



.
with :
BCE=effective rates in the euro zone.
BCESIMQ=Taylor's rates calculated with a quadratic output-gap.
ICAL=optimal rates obtained with a Ball's rule with m = *0*.



with :
BCE=effective rates in the euro zone.
BCESIMQ=Taylor's rates calculated with a quadratic output-gap.
ICAL=optimal rates obtained with a Ball's rule with $\mu = 0.5$.



with :
BCE=effective rates in the euro zone.
BCESIMQ=Taylor's rates calculated with a quadratic output-gap.
ICAL=optimal rates obtained with a Ball's rule with $\mu = 1$.

In these three cases, one will notice the choice of a said optimal policy based on a simple arbitrage between inflation and production, generates an extraordinary variability of interest rates. That comes naturally from the absence of interest rates as argument in the loss function previously described. It is therefore unlikely, as we discover it here, that the European Central Bank gives up taking it into consideration the evolution of interest rates, when it engages its monetary policy.

Moreover, one will notice, that this interest rate volatility is in reality particularly strong since the European Central Bank uniquely targets inflation.

Finally, one will notice also that the weight of the output-gap in the rules of "optimal" monetary policy are here particularly important regarding inflation and this regardless of the weight being assigned to these variables in the function of loss of the central bank. It is probable that the role of output-gap, as indicator projecting inflation, being here the cause of it.

III.5- Extension of the Ball model, a function of loss with past interest rates

The variabilty of interest rates engendered by the type of preceding rule, measured by the variance of interest rates causes one to think that the criteria of optimality used here have a very strong importance. This causes one then to re-examine the optimality of the rule of monetary policy being conducted, by using a loss function that takes into account the evolution of interest rates. We use therefore a loss function defined as:

$$L_t = \gamma V(\pi_t) + (1-\gamma)V(y_t) + \nu V(i_t) \qquad (12)$$

where γ and ν are, as previously, the values assigned by the monetary authorities, in their monetary policy, to inflation relatively to the output-gap and the evolution of interest rates.

As before, two canonical cases will be mentioned, where γ=0.5 and ν=0.25 and γ=1 and ν=0.25 corresponding to a pure inflation target.

The results[8] are then :

a - Case 1 : γ=0,5 et ν=0,25

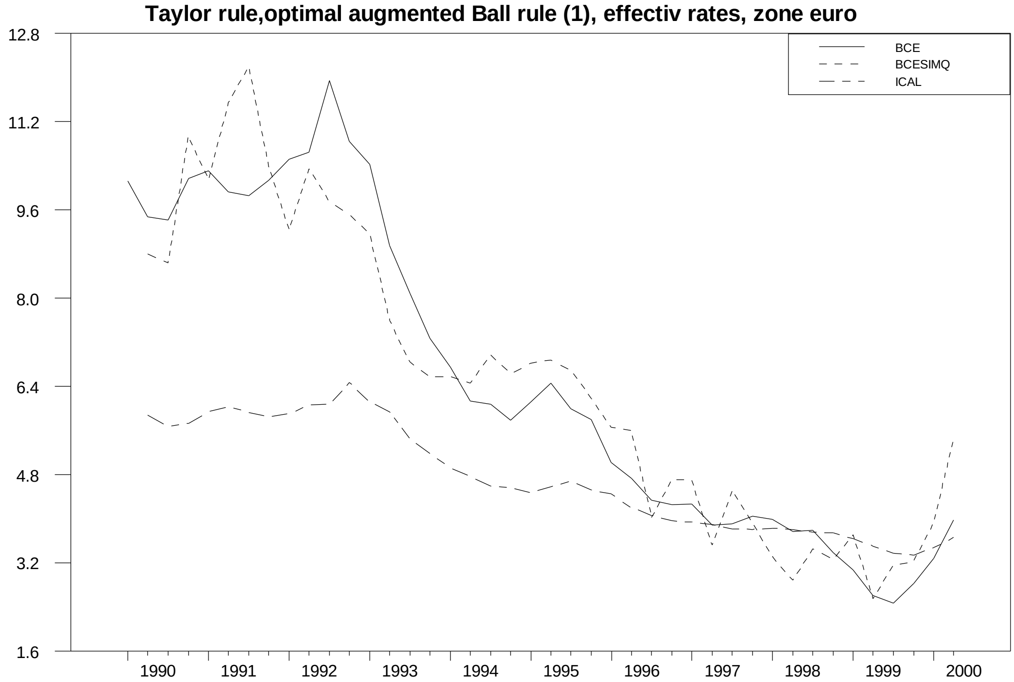
The rule of optimal monetary policy would be then :

$$i_t = 0{,}05\pi_{t-1} + 0{,}32y_{t-1} + 0{,}13i_{t-1}$$

with Lmini=0,24741.

The following graph will illustrate this situation.

[8] The difference between the results obtain in sections II and III comes probably from the model specifications. The most reliable results must be the results of section II because the model has been validated by the estimations. In contrast, model in section III can't have been estimated and we sould have calibrated it.



with :
BCE= effective rates in the euro zone.
BCESIMQ=Taylor's rates calculated with a quadratic output-gap.
ICAL1=optimal rates obtained with an augmented Ball's rule with γ=0.5 et ν=0,25.

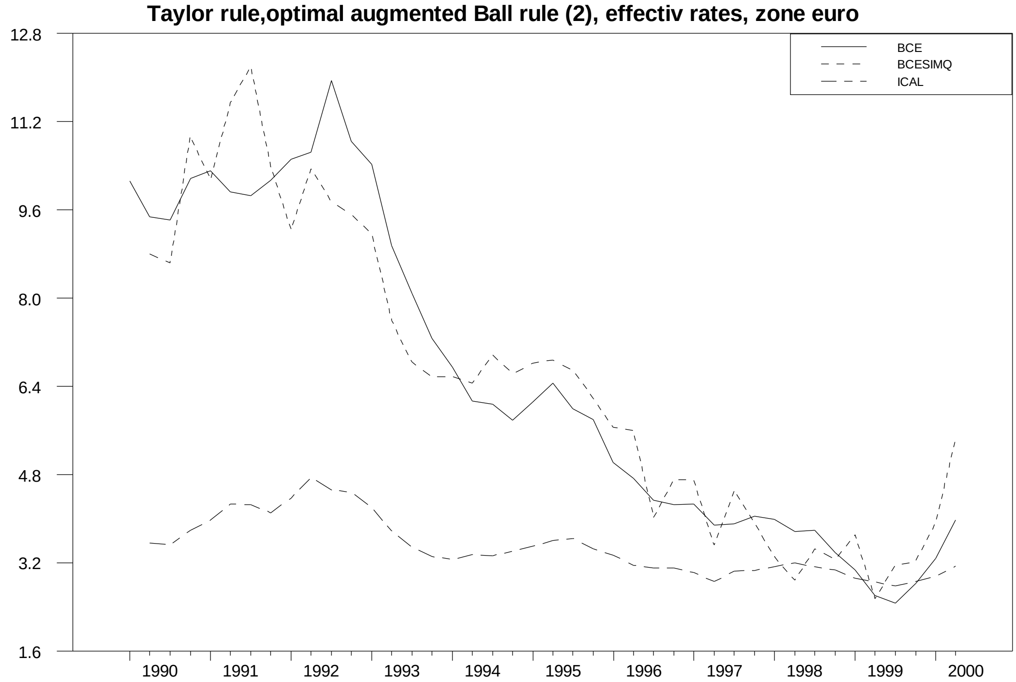
b - Case 2 : γ=1 et ν=0,25

The rule of optimal monetary policy would be then:

$$i_t = 0{,}07\pi_{t-1} + 0{,}02 y_{t-1} + 0{,}31 i_{t-1}$$

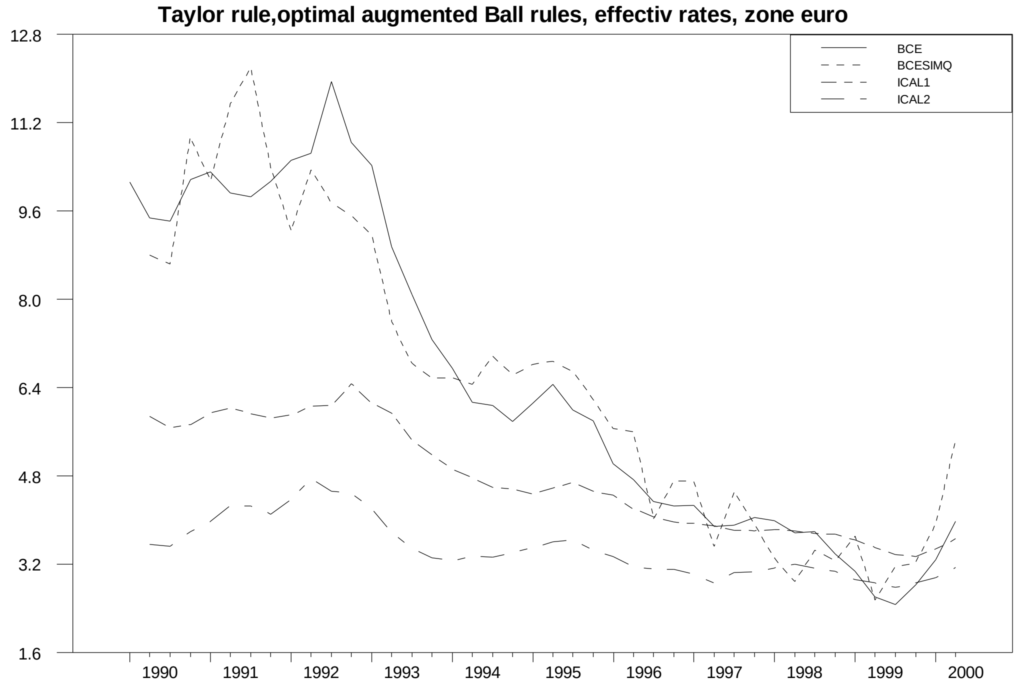
with Lmini=0,29535.

The following graph presents the behavior of this rule



with :
BCE=effective rates in the euro zone.
BCESIMQ=Taylor's rates calculated with a quadratic output-gap.
ICAL2=optimal rates obtained with an augmented Ball's rule with γ=1 et ν=0,25.

The following graph represents the confrontation of the two obtained optimal rules and compares them to the effective interest rate and to the Taylor's rule in the area of the Euro.



with :
BCE=effective rates in the euro zone.
BCESIMQ=Taylor's rates calculated with a quadratic output-gap.
ICAL1=optimal rates obtained with an augmented Ball's rule with γ=0.5 et ν=0,25.

ICAL2=optimal rates obtained with an augmented Ball's rule with γ=1 et ν=0,25.

In the two cases addressed here, it is remarkable that the variability of interest rates, taken into account in the loss function, naming the consideration of interest rates as well asthe argument in the rule itself, brings the interest rates to much weaker levels and are thus much more realistic.

Gap between the effective rates and the rates obtained with the optimal rules (type Ball augmented rule) in the euro zone

Gap between effective and optimal rates		Average gap	Standart-error	Minimum	Maximum
Rule 1	1990:1-2000:4	1.7346	1.9242	-0.9113	5.8649
γ=0,5	1996:2-2000:4	-0.0711	0.4349	-0.9113	0.5275
ν=0,25	1999:1-2000:4	-0.4590	0.4637	-0.9113	0.3131
Rule 2	1990:1-2000:4	2.9758	2.4061	-0.3103	7.4172
γ=1	1996:2-2000:4	0.6582	0.5491	-0.3103	1.5687
ν=0,25	1999:1-2000:4	0.1194	0.4233	-0.3103	0.8345

Gap between the Taylor's rates and the rates obtained with the optimals rules (type Ball augmented rule) in the euro zone

Gap between Taylor's rates and optimal rates		Average gap	Standart-error	Minimum	Maximum
Rule 1	1990:1-2000:4	1.7776	1.8386	-0.9554	6.2631
γ=0,5	1996:2-2000:4	0.1249	0.7634	-0.9554	1.7945
ν=0,25	1999:1-2000:4	0.1721	0.9191	-0.9554	1.7945
Rule 2	1990:1-2000:4	3.0188	2.2611	-0.3137	7.9404
γ=1	1996:2-2000:4	0.8541	0.8179	-0.3137	2.4435
ν=0,25	1999:1-2000:4	0.7506	0.8844	-0.3137	2.3159

As previously, the European monetary policy seems to be relatively close, since 1996:2, to the two rules of optimal monetary policy.

After this date, elsewhere, as we have mentioned earlier, optimal rule n°1, which corresponds to a strategy where the central bank grants value to the output-gap stabilisation and the evolution of interest rates, seems therefore to best describe the monetary policy in the area of the Euro.

Conclusion

The objective of our work has been here to calculate an optimal reaction function for the European Central Bank and this from the inflation and the output-gap formation model in the euro area.

In the first part, we present the model inspired by Peersman and Smets (1998) of prices and of output-gap formation in the euro area during the period 1990:1-2000:4. Thus, the output-gap is produced by an IS curve and depends on the past interest rate, the past output-gap and a demand shock. Inflation is formed according to a Phillip's curve. It is a function of past inflation, of past output-gap and a supply shock. So, as Ball (1997) specifies, all movement of interest rate by the central bank will have an effect on the economic variables of inflation and production, which in their turn will engender a reaction by the central bank. In the specific case of the European Central Bank, such as we are studying here, a movement of interest rate will first modify the evolution of the output-gap which depends on it directly, and modify only after the inflation, that is itself, a function of the output-gap and not directly of interest rates. In order to define its monetary policy, the European Central Bank must then take account of the before mentioned's channels of transmission. By defining a loss function for the European Central Bank, we define then what will have to be its optimal monetary policy taking into consideration the preceding interactions. Generally, the loss function of central banks is a function of three known arguments, the difference of inflation regarding its target, the output-gap and the interest rate difference regarding its level during the preceding period.

The second part of our work has first necessitate an estimation of the model previously defined. The obtained results correspond to the results generally presented in the economic literature, even if surprisingly, the IS curve estimated for the euro area seems closer than those estimated by other authors for the United States than those estimated for Germany. One also remarks that inflation, in the euro area, is particularly sensitive to the level of inflation attained in the trimester preceding the current period. By thus using an estimation method of constrained minimization, we are able, thanks to this model and the definition of the loss function associated with the monetary policy led by the European Central Bank, to define the rule of optimal monetary policy. We have calculated two rules of optimal monetary policy for the European Central Bank. These two rules correspond to two canonical cases. The first refers to the works of Rudebusch and Svensson (1998) and consider that the European Central Bank grants equal value in its strategy of stabilization of inflation and to that of output-gap. The second, on the contrary, supposes that the European Central Bank is not concern with inflation stabilization and ignores the evolution of output-gap. In any case, these two rules imply that the European Central Bank considers the effects of its policy on the evolution of interest rates. The optimal evolution of obtained rates through these two types of rules, during the 1990:1-2000:4, have thus been compared with the evolution of effective rates and with those of rates obtained throught the rule of Taylor, estimated for the same period in the euro area. From this confrontation, it thus appears that monetary policy in the euro area reveals itself to be close to these two types of rules of optimal monetary policy after 1996:2. After this date, the rule of optimal monetary policy taking into consideration output-gap seems to better describe the monetary policy directed in the euro area that the one we had qualified as a pure inflation target. It is moreover the same if one studies the links existing between the optimal rules and the rule of Taylor that describes the behavior of effective rates in the euro area for the same period. One also notices that even the rule of optimal monetary policy of target inflation takes into account the output-gap as argument. In fact, this result emerges because of the mechanisms of transmission of the monetary policy previously described and the nature of the advanced indicator of inflation potentially attributed to the output-gap. Moreover, it is noteworthy that the two rules of optimal monetary policy incorporate past interest rates.

Finally, the third part of this work consists of calculating all over again what would be the rules of optimal monetary policy, for the euro are, by calibrating the Ball (1997) model, for the same period. This alternative scenario thus permits giving priority to the role of interest rate adjustment in monetary policy that is not able to be summarized, due to the volatility that would result on the rates, from a simple arbitrage between inflation variability and the output-gap volatility.

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