The Profitability of Horizontal Mergers: A General Model

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Abstract

The profitability of a horizontal merger between symmetric firms is examined using a general oligopoly model that nests Cournot and Bertrand competition. General sufficient conditions for a merger to be profitable or unprofitable are derived. Mergers under Cournot and strategic substitutes are shown to be profitable if demand is convex enough or if commodities are differentiated enough. Mergers under Bertrand with strategic complements are always profitable. The model is applied in a strategic trade setting. It is shown that conditions for a profitable merger are less stringent when the governments do not commit to trade policies before firms merge.

1. Introduction

It is well known that mergers almost always raise industry profits but that merger participants typically do not capture all of the ensuing gains. If as a result of a merger the prices charged by the participants rise, rivals can expand their market share and thus capture some rents. Because of this externality a merger that raises industry profit may not benefit and may even harm the participants.

The key early paper on the profitability of mergers is Salant, Switzer and Reynolds (1983). They employed a symmetric homogeneous good Cournot model with linear demand, identical constant marginal costs for each of n firms. Even though industry profits are higher with fewer firms, in the absence of fixed cost saving a two-firm merger is never profitable in their model. The exception to this is when duopolists merge into monopoly. Instead of focusing on two-firm mergers Salant *et al* look for the threshold percentage of the industry that must participate in a merger to make it profitable. They find that the breakeven value never falls below 80%. Fauli-Oller (1997) reexamined the profitability of mergers in a homogenous product Cournot industry. He considered a wider class of demand functions with constant curvature and found that an increase in convexity of demand reduces the threshold market share of participants required for a merger to be profitable.

Both of these papers concentrated on mergers of more than one firm at the same time. However, it is reasonable to question the plausibility of simultaneous mergers involving many firms. Negotiations involving many players are likely to be very difficult to carry out in practice. It seems much more sensible to think of mergers taking place sequentially. This suggests that two-firm mergers, which we focus on here, are more relevant from a real-world perspective.

Perry and Porter (1985) and others have shown that two-firm mergers are profitable under Cournot with linear demands and homogenous products provided there is sufficient cost savings. Huck and Konrad (2001) using a Cournot model with linear demands showed that a merger is more profitable when governments are choosing strategic export subsidies. Deneckere and Davidson (1985) consider Bertrand competition with differentiated products and linear demand. They find that a merger is profitable even in the absence of cost savings.

Although there is now a large theoretical literature on the profitability of horizontal mergers in an oligopoly setting. It is widely assumed that little can be said about the issue unless specific functional forms are used. In fact in most of the earlier work on horizontal mergers linear demands has been used.

The purpose of this paper is to examine the incentives for bilateral horizontal mergers in a general setting that allows for general demands, quantity or price competition and differentiated goods. The focus here is on mergers to increase market power rather than to achieve synergies[1]. General sufficient conditions for a merger to be profitable or unprofitable are derived. Mergers under Cournot and strategic substitutes are shown to be profitable if demand is convex enough or if commodities are differentiated enough. Mergers under Bertrand with strategic complements are always profitable. Having examined the incentive to merge in the absence of government policy the model is then applied to a strategic trade policy setup[2]. Using a general oligopoly model I show that surprisingly general results can be obtained in such a strategic export subsidisation setting.

The structure of the rest of the paper is as follows. In Section 2 the general model is set up and general sufficient conditions for a merger to be profitable or unprofitable are derived and discussed. Section 3 applies the model in two strategic trade policy settings. Section 4 is a short conclusion.

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[1]Such synergies naturally increase the profitability of merger. In addition to Perry and Porter (1985) see Van Long and Vousden (1995) who also consider cross-border mergers that save on transport costs.

[2] Collie (1997) considered a Strategic trade model with Cournot competition, homogenous products and general demands. He considered the effect on optimal policy of exogenous changes in the number of firms. He did not examine the *profitability* of

2. The Model

There are *n+2* firms producing commodities that are perfect or imperfect substitutes for each other. Two of the firms are considering a merger. I will refer to these firms as the insiders and call them the *A* firm and the *B* firm. The remaining *n* firms are the outsiders. All the firms simultaneously choose their market actions, which could be quantity or price. The two insiders choose actions x_i (i=*A,B)* and the outsiders choose actions y_j *(j=1...n).* Marginal production costs are non-decreasing in own output. All firms, insiders and outsiders face symmetric demand and identical cost functions. In the absence of a merger each firm produces one variety of the good and we assume that the costs of introducing as new variety is prohibitive. This implies that in the absence of a merger all firms will produce the same quantities and charge the same prices at a non-cooperative equilibrium. (Note that this does not imply that they produce identical goods). The profits of a typical insider are:

$$\boldsymbol{p}^{i} = \boldsymbol{p}^{i}(\boldsymbol{x},\boldsymbol{y}) \qquad i = A,B \qquad (1)$$

where $\boldsymbol{x}$ is the vector of actions of the insiders and $\boldsymbol{y}$ is the vector of actions of the outsiders.The profits of a typical outsider are similarly written as:

$$\boldsymbol{p}^{j} = \boldsymbol{p}^{j}(\boldsymbol{x},\boldsymbol{y}) \qquad j = 1,...,n \qquad (2)$$

In the absence of a merger each firm chooses its action independently. The first-order condition for a typical insider is:

$$\boldsymbol{p}^{i}_{x_i}(\boldsymbol{x},\boldsymbol{y}) = 0 \qquad (3)$$

After a merger has taken place and provided that the good is differentiated, the merged entity produces two varieties of the good. I assume the costs of introducing a new variety are prohibitive. The insiders profits are the sum of the profits from producing variety *A* and variety *B*: $\boldsymbol{p}^{A}(\boldsymbol{x},\boldsymbol{y})+\boldsymbol{p}^{B}(\boldsymbol{x},\boldsymbol{y})$. The action x_i (i=*A,B*) is chosen so as to maximise the total profits of the merged entity:

$$\boldsymbol{p}^{i}_{x_i}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{p}^{k}_{x_i}(\boldsymbol{x},\boldsymbol{y}) = 0 \qquad k = A,B \ ; \ k \neq i \qquad (4)$$

Each of the *n* outsiders has a first-order condition: $\boldsymbol{p}^{j}{}_{y_j}(\boldsymbol{x},\boldsymbol{y}) = 0$ that can be written as a reaction function $y_j = y^{j}(\boldsymbol{x},\boldsymbol{y}_{-j})$. The optimal action chosen by outsider *j* depends on the actions of the insiders and the actions of all the other outsiders, $\boldsymbol{y}_{-j}$. There are clearly *n* such reaction functions. Assume that there is a unique solution to this system

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mergers. Horn and Levinsohn (2001) examined merger policy under trade liberalisation in a strategic trade setting with quantity competition and linear demands.

of n equations. It is then possible to write the vector of Nash equilibrium values of the actions of the outsiders as a function of the actions of the insiders only. I will write this Outsiders Reaction Function ORF as:

$$\boldsymbol{y} = \Psi(\boldsymbol{x}) \tag{5}$$

I assume throughout the paper that all Nash equilibria are unique. Let $\{\boldsymbol{x}^N,\boldsymbol{y}^N\}$be the Nash equilibrium when the insiders do not merge. Then, using (5) this can be rewritten as: $\{\boldsymbol{x}^N, \Psi(\boldsymbol{x}^N)\}$. Similarly let $\{\boldsymbol{x}^M, \boldsymbol{y}^M\} = \{\boldsymbol{x}^M, \Psi(\boldsymbol{x}^M)\}$ be the equilibrium when the insiders merge.

To assess the profitability of a merger we need to be able to compare the insider profits at the two equilibria. To do this it is useful to find the $\boldsymbol{x}$ that maximises insiders profits subject to the reaction of outsiders given in (5). Consider the following maximisation problem:

$$\underset{\boldsymbol{x}}{Max}\, H(\boldsymbol{x}) \equiv \boldsymbol{p}^A(\boldsymbol{x}, \Psi(\boldsymbol{x})) + \boldsymbol{p}^B(\boldsymbol{x}, \Psi(\boldsymbol{x})) \tag{6}$$

I will assume $H(\boldsymbol{x})$ is concave with first-order condition:

$$H_{x_i}(\boldsymbol{x}) \equiv \boldsymbol{p}^i_{x_i}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{p}^k_{x_i}(\boldsymbol{x},\boldsymbol{y}) + 2\sum_{\mathrm{j}}^{\mathrm{n}} \boldsymbol{p}^i_{y_j}(\boldsymbol{x},\boldsymbol{y})\Psi^j_{x_i}(\boldsymbol{x}) = 0 \qquad \textit{for} \quad i = A,B \;;\; k = A,B \;;\; i \neq k \;;\; j = 1,\ldots,n \tag{7}$$

where $\boldsymbol{p}^i_{yj} = \boldsymbol{p}^k_{yj}$ I will write the solution to this maximisation problem as $\boldsymbol{x}^S$. The outcome $\{\boldsymbol{x}^S, \Psi(\boldsymbol{x}^S)\}$ can, of course be interpreted as a Stackelberg equilibrium in which the merged firm chooses $\boldsymbol{x}$ before the outsiders move. However, instead I will continue to assume that all firms move simultaneously and use $\boldsymbol{x}^S$ as a reference point with which to compare $\boldsymbol{x}^N$ and $\boldsymbol{x}^M$.

First consider "normal" Bertrand competition. I will define this as Bertrand competition with differentiated products and strategic complements. (This includes the differentiated linear demand case.) In that case $\boldsymbol{p}^k_{xi}>0$, $\boldsymbol{p}^i_{yj}>0$ and $\Psi^j_{xi}>0$ then $\boldsymbol{x}^S > \boldsymbol{x}^M > \boldsymbol{x}^N$ which from the concavity of $H(\boldsymbol{x})$ implies $H(\boldsymbol{x}^M)>H(\boldsymbol{x}^N)$ that is a merger is profitable. This generalises Deneckere and Davidson (1985) who demonstrated that a merger was profitable under Bertrand with linear demands.

Next consider "normal" Cournot competition that is Cournot with strategic substitutes. The following is a sufficient condition for a merger to be unprofitable under Cournot:

$$p^{k}_{x_i}(\boldsymbol{x},\boldsymbol{y}) + 2\sum_{j}^{n} p^{i}_{y_j}(\boldsymbol{x},\boldsymbol{y})\Psi^{j}_{x_i}(\boldsymbol{x}) \geq 0 \qquad (8)$$

when evaluated at: $\{\boldsymbol{x}^N, \Psi(\boldsymbol{x}^N)\}$. Under Cournot competition the first term on the left hand of (8) is negative while the second term is positive.

Case 1: the sufficient condition in (8) is met. This is illustrated in figure 1a. In this case $\boldsymbol{x}^S \geq \boldsymbol{x}^N > \boldsymbol{x}^M$ which from the concavity of $H(\boldsymbol{x})$ implies $H(\boldsymbol{x}^N) > H(\boldsymbol{x}^M)$ so that a merger is unprofitable. This is more likely to hold the *more outsider output rises when output of the insiders fall due to the merger*. This is larger the more outside firms there are and the more negative is the slope of the reaction functions.

Case 2: the sufficient condition in (8) is not met (but quantitites remain strategic substitutes) as illustrated in figure 1b. Then we get $\boldsymbol{x}^N > \boldsymbol{x}^S > \boldsymbol{x}^M$ and we cannot unambiguously rank $\boldsymbol{x}^N$ and $\boldsymbol{x}^M$ as they are on opposite sides of the optimum. Clearly if $\boldsymbol{x}^N$ is close enough to $\boldsymbol{x}^S$ then $H(\boldsymbol{x}^N)$ is still larger than $H(\boldsymbol{x}^M)$ and a merger is still unprofitable. A merger is certainly profitable $(H(\boldsymbol{x}^M) > H(\boldsymbol{x}^N))$ if $\boldsymbol{x}^N > \boldsymbol{x}^M \geq \boldsymbol{x}^S$. But this occurs if the outputs are strategic complements when $\boldsymbol{x}^M \geq \boldsymbol{x}^S$ or the Outsiders Reaction Function is locally flat when $\boldsymbol{x}^M = \boldsymbol{x}^S$. Clearly by continuity there must be cases in which $\boldsymbol{x}^S - \boldsymbol{x}^M > 0$ is sufficiently small that a merger is profitable. This suggests that a merger will be profitable under Cournot with strategic substitutes provided that the outsiders reaction function is sufficiently flat locally. Thus the key to finding a profitable two-firm merger under "normal Cournot" is to look for situations in which the Outsider Reaction Function is sufficiently flat. Convexity of demand and product differentiation work towards this. We will examine each in turn.

2.1 Merger profitability and the curvature of demand

To focus on the effect of the curvature of demand on merger profitability it proves easiest to abstract from product differentiation. In the case of homogenous product Cournot (8) simplifies to:

$$-q_i p'(Q)\left(2\frac{n+\alpha_o r}{n+1+\alpha_o r} - 1\right) \geq 0 \qquad (9)$$

where q_i $(i=A,B)$ is the output of an insider, (thus total insider output is $2q_i$), Q is total industry output, α_o is the total market share of outsiders and $r=Qp''/p'$ is the elasticity of the slope of demand and a measure of the concavity of the demand function. The sign of (9) is determined by the term in brackets. This is always positive for *r³0*. Thus with homogenous products weakly concave demand is sufficient to ensure that a bilateral merger is unprofitable. In the linear case $r=0$ and two-firm mergers are unprofitable unless they lead to monopoly (see Salant et al (1983)). This means that if we were to find circumstances in which a bilateral merger *is* profitable we must look at cases in which demand is convex ($r<0$). However, if demand is sufficiently convex then quantities are strategic complements and, as we have seen, mergers are always profitable. The interesting case then is profitable mergers under Cournot with convex demand but with strategic substitutes. I will now give an illustrative example of this.

An example: Cournot with constant elasticity of demand

Consider the following constant elasticity inverse demand:

$$p=\left(\frac{1}{Q}\right)^{\frac{1}{h}} \tag{10}$$

where $h>1$ is the constant elasticity of demand. The elasticity of the slope of demand is then $r=-(1+h)/h<0$, a negative constant. Assume, for simplicity, that all firms face the same constant marginal cost, c. The first-order condition for one of the $n+2$ firms in the absence of a merger is:

$$p\left(\frac{(n+2)h-1}{(n+2)h}\right)=c \tag{11}$$

From (10) and (11) it is straightforward to solve for industry output before the merger takes place:

$$Q^N=\left(c\frac{(n+2)h}{(n+2)h-1}\right)^{-h} \tag{12}$$

The profit for an individual firm at the no-merger equilibrium is: $p^N=-p'Q^2/(n+2)^2$.
Use of (12) and the derivative of (10) in this gives:

$$p^N=\frac{1}{h(n+2)^2}\left(c\frac{(n+2)h}{(n+2)h-1}\right)^{1-h} \tag{13}$$

Profits of a typical firm after the merger is then:

$$p^M = \frac{1}{h(n+1)^2}\left(c\frac{(n+1)h}{(n+1)h-1}\right)^{1-h} \qquad (14)$$

The condition for a profitable merger is:

$$G = \frac{c^{1-h}}{h}\left[\frac{1}{(n+1)^2}\left(\frac{(n+1)h}{(n+1)h-1}\right)^{1-h} - \frac{2}{(n+2)^2}\left(\frac{(n+2)h}{(n+2)h-1}\right)^{1-h}\right] > 0 \qquad (15)$$

The sign of this depends on that of the term in square brackets. It can be shown that a merger is profitable for *n=1* that is for two firms in a three firm- industry provided that the h is low enough. (With this demand function it is never profitable if *n>1*.) For *n=1* there is a range of h at which outputs are strategic substitutes for both firms at the no-merger and merger equilibrium and yet a merger is profitable.

2.2 Merger profitability and product differentiation

Like demand convexity, product differentiation also tends to reduce the slope of the outsiders reaction function and so would appear to work towards a profitable merger. However *all* the components of (8) tend to fall as goods become more different. In the limit when goods are totally different the left-hand side is zero and $H(\mathbf{x}^M)=H(\mathbf{x}^N)$. I will now examine the effect of product differentiation on the incentive to merge more formally. Let all firms produce a different variety and let the inverse demand faced be: $p^j(q_j, \mathbf{q}_{-j})$ where p^j and q_j are the price and outputs of *j=(A,B,1....n)* and $\mathbf{q}_{-j}$ is the vector of outputs of the other firms. The first derivatives $\partial p^j/\partial q_j$ and $\partial p^j/\partial q_k$ where $k \neq j$ are both negative. Let $e = (\partial p^j/\partial q_k)/(\partial p^j/\partial q_j)$ be an inverse measure of the degree of product differentiation with $0 \leq e \leq 1$. I will restrict attention to demand systems in which the measure of differentiation can be written as $e = ef(\mathbf{q})$ where $\mathbf{q}$ is the vector of outputs and *e* is a constant and equal for all firms. This includes the special linear case[3].

Assume for simplicity that the outsiders are symmetric. With a demand function of this form there will always be a level e>0 small enough that a merger is profitable. The left-hand side of (8) can be rewritten as:

[3] It also includes inverse demand functions of the form $p^j = p^j(q_j + e\Sigma q_{-j})$ and $p^j = z(q_j) + ei(\mathbf{q}_{-j})$ $j=A,B,1,...,n$.

$$p^i_{q_k}\left(1+2n\frac{p^i_{q_j}}{p^i_{q_k}}\Psi^j_{q_i}\right)=-e\boldsymbol{q}(1-e\boldsymbol{l}) \tag{16}$$
$$\text{where}\quad \boldsymbol{q}=\boldsymbol{q}(\boldsymbol{q},n,e)>0 \quad\text{and}\quad \boldsymbol{l}=\boldsymbol{l}(\boldsymbol{q},n,e)>0$$

in the differentiated Cournot case. When $e=0$ the term on the right of (16) is zero but when e>0 falls and approaches zero then if $e\boldsymbol{l}$ falls and approaches zero the term in brackets approaches unity. What is happening here is that at low enough e we have $\boldsymbol{x}^N$ $>\boldsymbol{x}^S>\boldsymbol{x}^M$ and as e falls further $\boldsymbol{x}^N$ and $\boldsymbol{x}^M$ both converge towards $\boldsymbol{x}^S$ but $\boldsymbol{x}^M$ approaches the optimum faster than $\boldsymbol{x}^N$ does. So when $e>0$ is sufficiently small a merger is profitable. To illustrate this I will use the following simple example:

An example: differentiated Cournot competition with linear demand

Consider the following inverse demand system:

$$p^i=1-q_i-e\sum_{j\neq i}q_j \tag{17}$$
$$\text{where}\quad i,j=A,B,1,\dots,n$$

Assume all firms, both insiders and outsiders face identical constant marginal cost. We can normalise the marginal cost at zero. In the absence of a merger each firm's first-order condition can be written as $p^i=q_i$. The Cournot equilibrium output is:

$$q_i^N=\frac{1}{2+e(n+1)}\quad\text{where}\quad i=A,B,1,\dots,n \tag{18}$$

The superscript N refers to the no-merger equilibrium. When firm A and B merge the first-order condition for the merged firm implies: $p^A=q_A+eq_B$ and $p^B=q_B+eq_A$. (The first-order condition for an outsider is unchanged.) The merged firm produces the following outputs of the two varieties:

$$q_A^M=q_B^M=\frac{2-e}{2(1+e)[2+e(n-1)]-2ne^2} \tag{19}$$

From (18) and (19) and the firms' first-order conditions it is easy to derive the gains from merger:

$$G=2\left(\frac{2-e}{2(1+e)[2+e(n-1)]-2ne^2}\right)^2(1+e)-2\left(\frac{1}{2+e(n+1)}\right)^2 \tag{20}$$

The negative relationship between e and n at $G(e,n)=0$ is illustrated in figure 2 . Note that for any n there is an $e>0$ at which a merger is profitable.

3. An Application: Strategic trade policy

Mergers in a strategic trade policy setting is an interesting application of the model. In such a situation much can be said about the incentive to merge while making few assumptions. Consider a setup in which the two insiders are located in a country that is using a strategic export subsidy. The outsiders are located in other countries. As is standard in strategic export policy models the government is concerned with domestic firms' profits and government revenue[4]. Before a merger takes place the profits of a typical insider (home firm) are:

$$\Pi^i(\boldsymbol{x},\boldsymbol{y},s) = \boldsymbol{p}^i(\boldsymbol{x},\boldsymbol{y}) + S^i(\boldsymbol{x},\boldsymbol{y},s) \qquad i = A, B \tag{21}$$

where $S = sq$ is the subsidy payments with s the per-unit subsidy given. Profits net of subsidy payments are represented by $\boldsymbol{p}$. Profits of outsider (foreign) firms which do not receive a subsidy or pay a tax are given in (2). The government chooses the trade policy to maximise domestic welfare before firms choose their actions. Welfare is simply profits net of subsidies:

$$W(\boldsymbol{x},\boldsymbol{y}) = \sum_i \left(\Pi^i(\boldsymbol{x},\boldsymbol{y},s) - S^i(\boldsymbol{x},\boldsymbol{y},s)\right) = \sum_{\mathrm{i}} \boldsymbol{p}^i(\boldsymbol{x},\boldsymbol{y}) \qquad i = A,B \tag{22}$$

In the absence of a merger the first-order condition for a typical insider is:

$$\Pi^i_{x_i}(\boldsymbol{x},\boldsymbol{y},s) = \boldsymbol{p}^i_{x_i}(\boldsymbol{x},\boldsymbol{y}) + S^i_{x_i}(\boldsymbol{x},\boldsymbol{y},s) = 0 \tag{23}$$

If a merger has taken place then the merged firm chooses actions (quantity or price) for each variety it produces to maximise its total profits. The first-order condition implies:

$$\Pi^i_{x_i}(\boldsymbol{x},\boldsymbol{y},s) + \Pi^j_{x_i}(\boldsymbol{x},\boldsymbol{y},s) = 0 \quad \text{for} \quad i,j = A,B \ \text{with} \ \ i \neq j \tag{24}$$

3.1 The government cannot commit before the merger decision

Suppose the government cannot commit to its trade policy before the firms decide whether to merge or not. The sequence of the game is now as follows: In stage 1 the insiders decide whether to merge or not in stage 2 the government chooses it optimal trade policy and in stage 3 all the firms simultaneously choose their market actions. The government will set the same subsidy/tax for each home firm. As the two firms are identical it does not matter whether or not the government can *ex ante* discriminate between them. The government can effectively control $\boldsymbol{x}$, the (symmetric) market actions of the home insiders. It cannot control $\boldsymbol{y}$, the actions of the foreign (outsider) firms. However, it knows that their actions will change as $\boldsymbol{x}$ changes according to the outsider reaction function represented in (5). The government effectively chooses $\boldsymbol{x}$ to maximise the welfare function in (22). This optimisation problem is the same as maximising $H(\boldsymbol{x})$ and the first-order condition is given in (7) above. The optimal subsidy in the absence of a merger is then obtained by combining (7) and (23) to get:

$$S^N_{x_i} = p^k_{x_i} + 2\sum_j^n p^i_{y_j}\Psi^j_{x_i} \qquad (25)$$
$$i = A,B;\ k = A,B\ ;\ \ i \neq k\ \ ;\ \ j = 1,...,n$$

To interpret (25) first note that S_{xi} is equal to s in the Cournot case and $s\partial q_i/\partial p^i$ where $\partial q_i/\partial p^i<0$ in the Bertrand case. The optimal per unit subsidy under Cournot is thus positively related to the right-hand side of (25) but is negatively related to the right-hand side under Bertrand competition. All the terms on the right-hand side are positive under Bertrand and so the optimal policy is always a tax. Under Cournot the right-hand side of (25) has the same form as the left-hand side of (8). Thus it is more likely to be positive the more concave is demand, the closer the degree of substitutability between goods and the more foreign firms there are in the market. Use of (24) in (7) gives the optimal policy if the home firms merge:

$$S^M_{x_i} = p^k_{x_i} - \Pi^k_{x_i} + 2\sum_j^n p^i_{y_j}\Psi^j_{x_i} \qquad (26)$$
$$i = A,B;\ k = A,B\ ;\ \ i \neq k\ \ ;\ \ j = 1,...,n$$

The optimal subsidy if a merger takes place is always larger than the corresponding subsidy in the absence of a merger. To see this first note that (25) and (26) are both evaluated at the $\boldsymbol{x}$ that maximises $H(\boldsymbol{x})$. Since they are both evaluated at the same point they can be directly compared. The difference between the optimal per-unit export subsidies is:

$$s^M - s^N = -\frac{\Pi^k_{x_i}}{\partial q_i / \partial x_i} > 0 \qquad (27)$$

To see this under Cournot competition note that: $\partial q_i/\partial x_i=1$ and $\Pi^k_{xi}=p^k_{xi}=q_k(\P p^k/\P q_i)<0$ while under Bertrand competition $\partial q_i/\P x_i=\P q_i/\P p^i$ and $\Pi^k_{xi}=[p^k-C^{k}{'}(q_k)+s]\P q_k/\P p^i>0$ (where is $C^{k}{'}(q_k)$ marginal production costs). The real equilibrium at the policy optimum is the same with and without a merger. All the prices and quantities are the same. The profits net of subsidies are thus the same and hence welfare is independent

[4]See Brander (1995) for an excellent survey the strategic trade literature.

of whether firms merge or not. It is immediately clear from this and (27) that the insiders always gain from a merger if they take the decision before the government chooses it trade policy.

3.2 The government commits before the merger decision

Suppose the government commits to its trade policy before the firms decide whether to merge or not. The sequence of the game is now as follows: In stage 1 the government chooses it optimal trade policy, in stage 2 the two home firms decide whether to merge or not and in stage 3 all the firms simultaneously choose their market actions. When deciding to merge the firms face given export subsidies. This means that they must make the same calculations as was described in section 2 above. The firms will always find it profitable to merge when firms compete in prices but will only find it optimal to merge under Cournot if output the response of outsiders is sufficiently weak. In stage 1 the government chooses the subsidy taking account of the effect of this on the decision to merge. The government is actually indifferent as to whether the firms merge or not provided it can reach the desired real equilibrium: $\{\boldsymbol{x}^s, \Psi(\boldsymbol{x}^s)\}$. This may not be possible when the government commits to its subsidy and cannot alter it after the merger decision is made. If $G(s^N)>0$ while $G(s^M)<0$ then the government cannot reach the first best without another instrument such as anti-trust policy to directly effect the merger decision. In the absence of another such policy the government is better off if it does not commit to its trade policy before the firms choose whether to merge or not.

3.3 More than two home firms

Returning now to the setup in which the merger decision is made before trade policy is selected. The introduction of more home firms does not alter the main results provided that the goods are differentiated and the government is able to optimally discriminate between the firms. The firms that merge reduce output under Cournot and increase price under Bertrand and the government corrects for this with a higher subsidy. Again the merging firms always gain and the government is indifferent about the merger. If the goods are not differentiated and the firms play Cournot then discrimination between the firms does not yield any welfare benefits. Suppose then that the government sets a uniform subsidy. This will be chosen so as to set total home output at the optimal level given the response of the foreign firms. Total output, the price and welfare will be the same at the optimum regardless of whether the firms merge or not. The subsidy will be larger if the two insiders merge but the total output of the outsiders will be larger. Let the number of home firms be n and the number of foreign firms be n^* with $n+n^*=n+2$. Let the total output of the home firms at the optimum be written as q^s and the corresponding total output of the foreign firms is q^{*s} the market output is then $Q^s = q^s + q^{*s}$. The profits of a typical home insider if it does not merge is: $-p'(Q^s)(q^s/n)^2$ and the profits of home merged firm is: $-p'(Q^s)[q^s/(n-1)]^2$. Hence the gain from a merger can be written as:

$$G = -p'(Q^s)(q^s)^2\left(\frac{1}{(v-1)^2} - \frac{2}{v^2}\right) \qquad (28)$$

This is positive for $n=2$ and $n=3$ but negative for more home firms. Note that this holds independently of the form of the demand function.

4. Concluding remarks

I have examined the profitability of horizontal mergers in an oligopoly setting using general demands and have shown that more can be said about this problem than was previously thought. A general sufficient condition for mergers to be unprofitable was derived. It was shown that mergers are profitable under Cournot competition with strategic substitutes provided that product differentiation is high enough and/or demand convex enough. The model was then applied in a strategic trade setting and it was shown that conditions for a profitable merger are less stringent when the governments do not commit to trade policies before firms merge.

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Figure 1a: Case 1

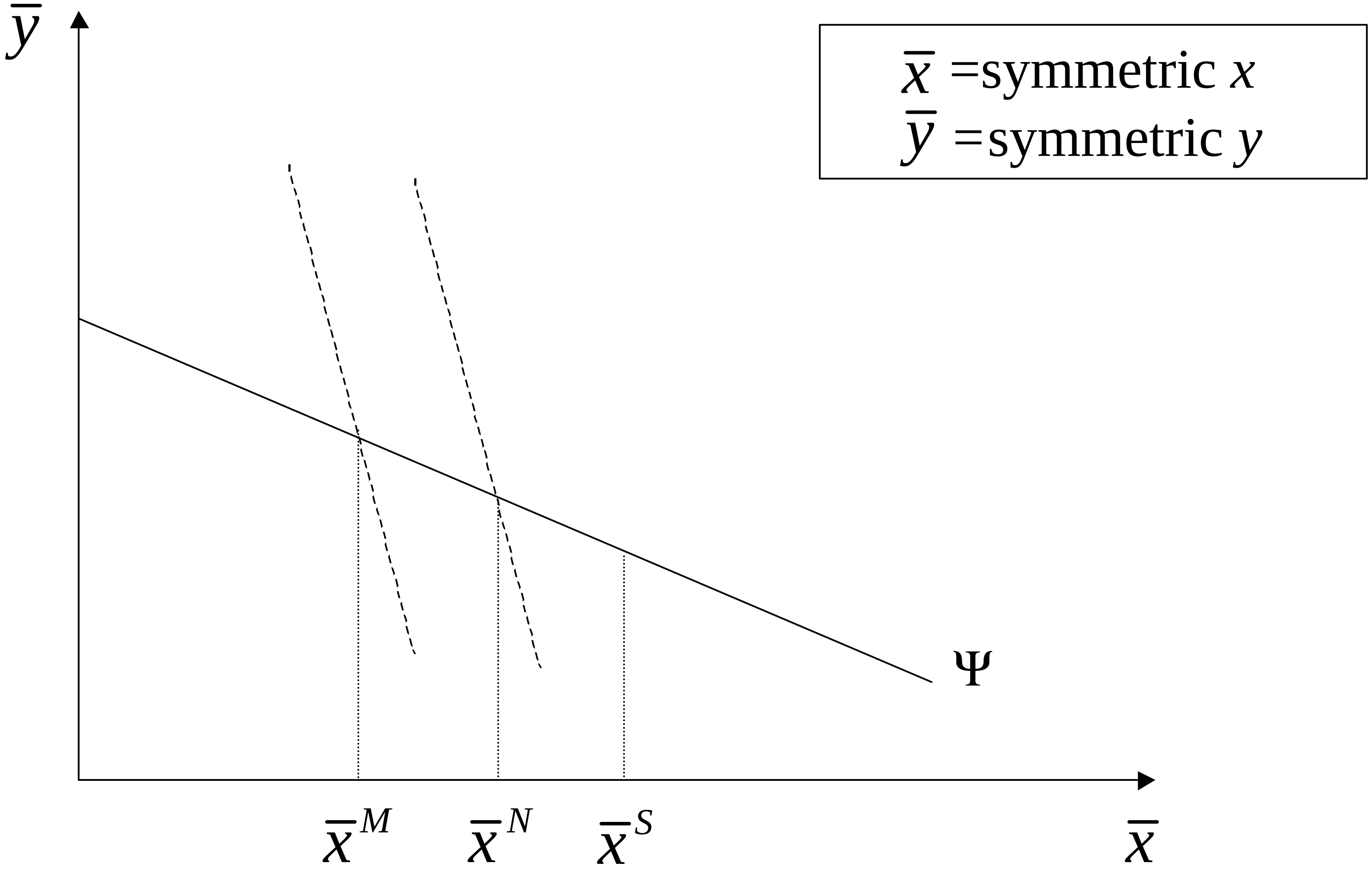

Figure 1b: Case 2

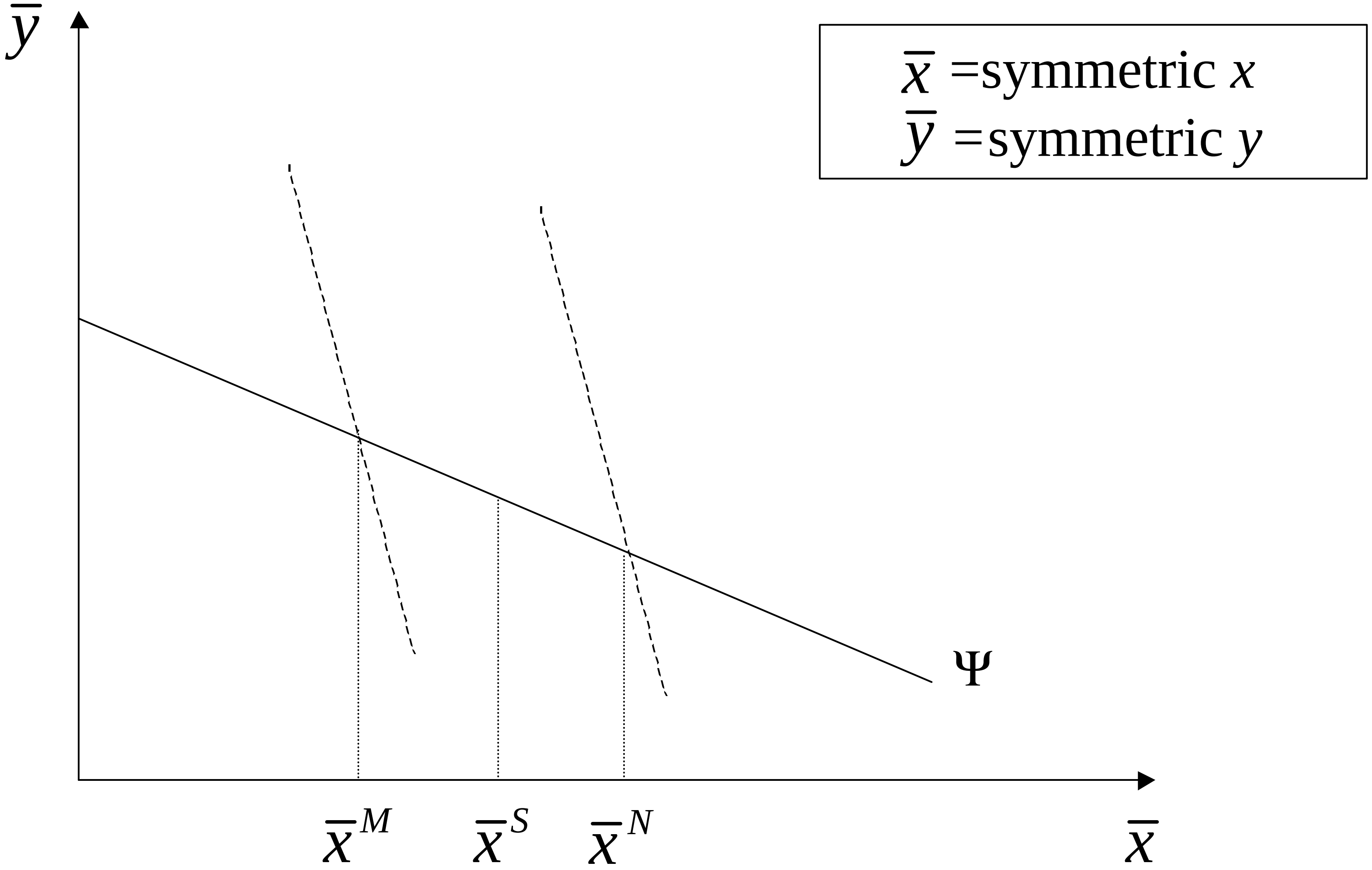

Figure 2: G(e,n)=0